XXI IMEKO World Congress “Measurement in Research and Industry”
August 30 – September 4, 2015, Prague, Czech Republic

MEASURING FREQUENCY OF NOISY SIGNALS THROUGH A STOCHASTIC RESONANCE APPROACH

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Abstract − The paper deals with the problem of measuring the frequency of sinusoidal signals hardly buried in noise. Stemming from their past experience on the topic, the authors suggest a new method based on the stochastic resonance capable of outperforming their previous proposal when acquired signals characterized by very low signal-to-noise-ratio (even lower than zero). In particular, the method exploits the peculiar resonance condition that double threshold system exhibits in the presence of an optimal noise level superimposed to the useful signal.

Preliminary tests carried out on numerical signals confirm the promising performance of the method and its possible implementation on cost-effective platform, thanks to its reduced computational burden.

Keywords: frequency measurements, noisy signals, negative SNR, stochastic resonance.

1. INTRODUCTION

Measuring the frequency of signals turns out to be the key operation in several application field [1-3]; however, the presence of superimposed noise results in a harmful degradation of the signal, thus making the considered measurement unreliable or even impossible to be carried out. This is particularly true when the noise sources cannot be made negligible by means of traditional filtering approaches.

The authors recently presented a suitable solution to the considered measurement problem in [4], where a revised version of the traditional histogram technique was exploited to execute reliable frequency measurements also in the presence of SNR as low as 5 dB. A deep performance analysis highlighted that the method was capable of estimating the frequency as good as some of the best time [5]-[10] or frequency [11]-[20] domain methods. Authors’ proposal specialty was reduced computational burden, thanks to the agile algorithm involved. Unfortunately, the performance of the method rapidly degraded when the signal-to-noise-ratio (SNR) approached zero or negative values.

To overcome the considered problem and extend the measurement range in terms of SNR of the input signals, the authors propose hereinafter a new method, based on the stochastic resonance, for the frequency measurement of sinusoidal signal hardly buried in noise.

The key idea underlying the stochastic resonance is the possibility of exploiting noise, generally considered a harmful nuisance, to improve the performance of large classes of both natural and artificially designed systems. In particular, stochastic resonance defines a favourable condition of double threshold systems embedded in which the presence of an optimal amount of noise should provide a sensitivity enhancement to low-amplitude external time-dependent forcing signals.

In a similar way, the proposed method aims at determining the optimal value of a double threshold that, applied to the input noisy signal, allows a reliable measurement of its frequency by means of the abovementioned-revised histogram.

2. THEORETICAL BACKGROUND

Some fundamental remarks about the basic idea underlying the stochastic resonance along with some examples of its exploitation in measurement applications are given in the following.

Stochastic resonance was born from the consideration that *"Certain complex systems can generate phenomena that classical theory cannot explain"* [21]-[24]. In particular, several studies focused their attention on the attractive behavior that bistable systems, characterized by double threshold structure, exhibits when their forcing signal is summed with optimal amplitude noise. More specifically, let us consider a periodic forcing signal whose amplitude is not sufficient to overcome the two thresholds of the system of interest (Fig.1.a); if a suitable noise is added to the input signals, the output of the bistable system is generated (Fig.1.b). When stochastic resonance occurs, the noise amplitude determines an advantageous condition in which the output frequency of the system turns out to be very close (ideally equal) to that of the low-amplitude input signal.

The very first application of stochastic resonance was related to study about climatology to explain the origin of dramatic climate changes [28]. In particular, it has been demonstrated that the combination of (*i*) weak gravitational interaction (i.e. the low-amplitude input signal) with the sun and solar systems planets and (*ii*) random contributions due to the Earth local weather conditions (i.e. the additive noise) gave rise to cyclic climatic changes as the glaciations.

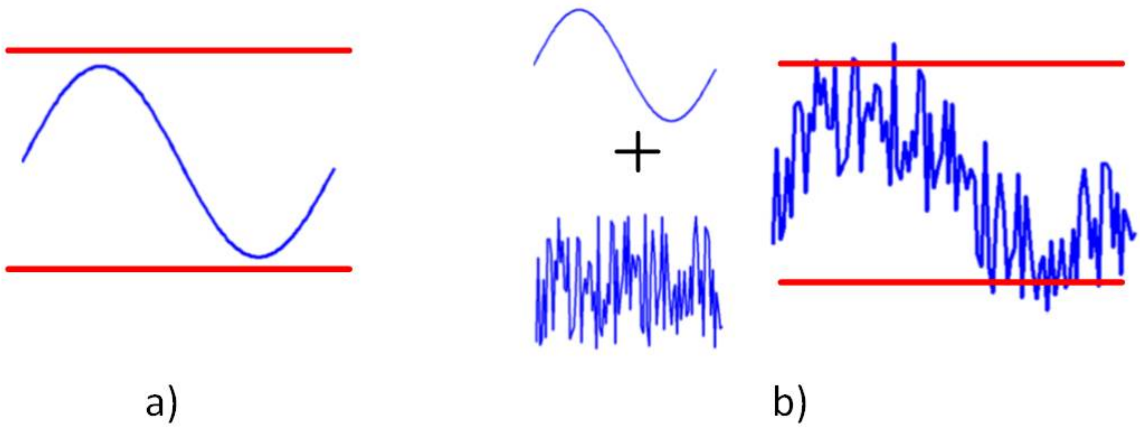

Fig. 1. Low-amplitude forcing signals (a) become capable to overcome defined threshold levels if a suitable amount of noise is added (b)..

3. PROPOSED METHOD

As stated above, in typical applications of stochastic resonance a suitable noise is injected in a bi-stable system to make it switch between two different states, separated by a defined threshold level, also in the presence of weak periodic forcing [25]-[27]. It is worth remember that, in resonance condition, the frequency of the output signal corresponds to that of the forcing, according to (1).

The key idea underlying the proposed method is a whole roles inversion between noise level and threshold with respect to the traditional approach. In particular, in actual measurement conditions the noise content of the signal cannot be selected. The stochastic resonance has to be reached by optimally selecting the values of the double thresholds. For the sake of the clarity, the operating steps of the method (Fig.2) are given in the following with reference to an application example.

The first step is the acquisition of the signal whose frequency has to be measured. As it can be expected, sample rate f_s and number of acquired samples N are the most influencing parameters; the sample rate directly affects the measurement resolution, while the higher the value of N the more accurate is the successive stage of frequency estimation through the histogram. As for the application example, it refers to a numeric sinusoidal signal, $x[n]$ whose amplitude and frequency are set equal respectively to 1 V and 50 Hz, and digitized at a sample rate of 10 kS/s with 5 kSamples. A white uniform sequence with maximum amplitude equal to 1 V has then been added to the signal of interest (Fig.3).

A double threshold (+*TH* and –*TH*, with *TH* varying in the interval from zero up to the noisy signal peak) non-linear system is applied to the acquired signal in order to gain the output signal $x_{TH}[n]$. In particular, when the signal first overcomes the positive threshold +*TH*, the output value is set to the high level and so remains until the signal undergoes the negative threshold (-*TH)*.

The successive step of the operating procedure accounts for the preliminary frequency $f[k]$ estimation of the output signal x_{TH} through the revised version of the histogram presented in [4]. Once all the threshold values have been

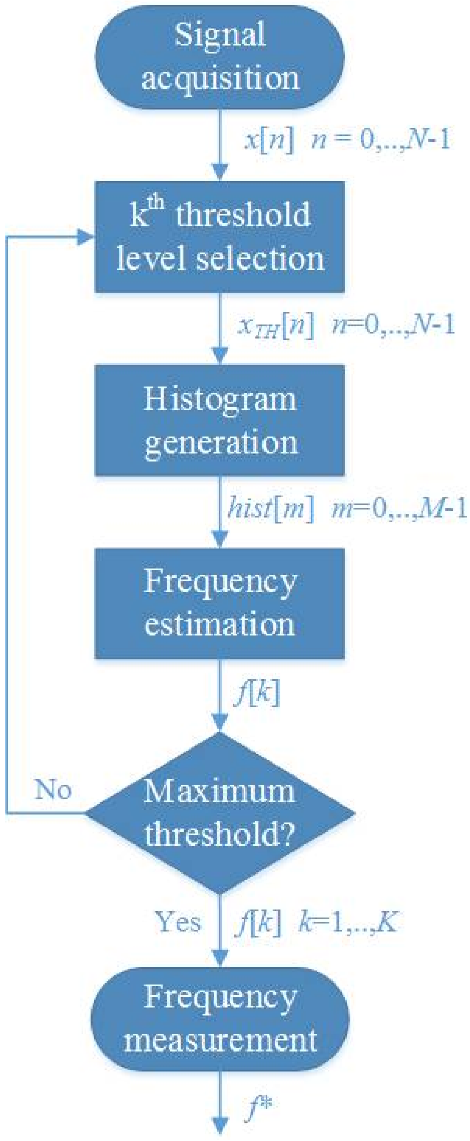

Fig. 2. Block diagram of the proposed method.

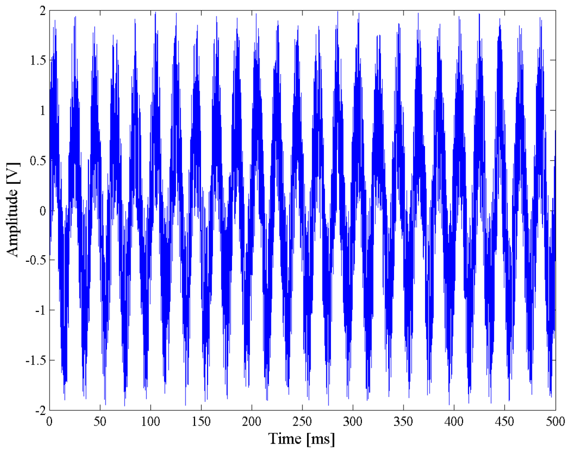

Fig. 3. Noisy signal adopted for the application example.

taken into account, the sequence of estimated frequency can be attained by means of the occurrence histogram approach. Moreover, the evolution versus threshold level of both estimated frequency and SNR is given in Fig.4. The frequency measurements is gained within the large portion of curve showing an almost constant value. The SNR has been defined as

$$SNR(f_0) = \frac{S_p(f_0)}{S_n(f_0)} \tag{1}$$

i.e. the ratio between the amplitude $S_p(f_0)$ of the output at the frequency f_0 of interest and the amplitude $S_n(f_0)$ of the noise at the same frequency. As an example, the threshold value associated with the maximum of the SNR curve provide the frequency measure of 49.9 Hz.

PRELIMINARY EXPERIMENTAL RESULTS

The performance of the proposed methods has preliminary been assessed through a number of tests conducted in actual experimental tests. At this aim, a suitable measurement station has been realized (Fig.5); it enlisted a digital oscilloscope, an arbitrary waveform generator and a personal PC.

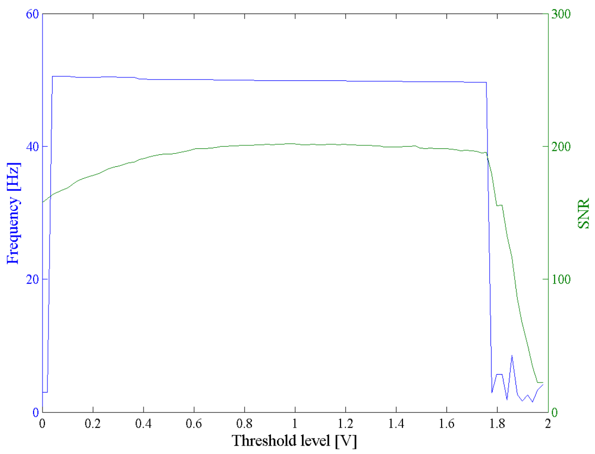

Fig. 4. Evolution both of the estimated frequency and SNR versus the threshold value.

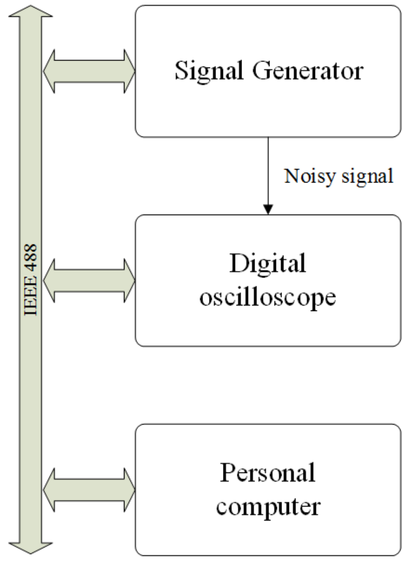

Fig. 5. Block diagram of the measurement station for performance assessment of the proposed method

Preliminary results involving sinusoidal signals characterized by SNR values varying in the interval from 3 down to -11 dB are shown and discussed in following. In particular, they refer to a sinusoidal signal whose nominal frequency and amplitude were equal respectively to 2 kHz and 0.5 V, and digitized with a sample rate of 1 GHz; as an example, a noisy signal whose SNR was equal to 3 dB is shown in Fig.6.

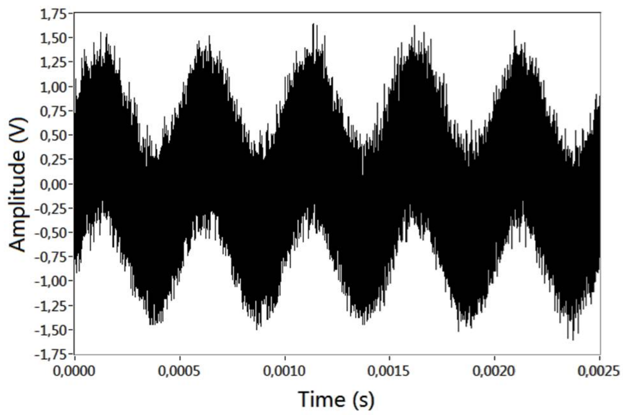

Fig. 6. Example of acquired noisy signal; in particular, SNR equal to 5 dB

Obtained results through the proposed method are given in Fig.7; in particular, the evolution of the difference, expressed in relative percentage terms, between nominal and measured frequency versus the considered values of SNR is plotted. Values never greater than 1% have been epxerinced in the whole investigated range of SNR, thus confirming the promising performance of the proposed method in the presence of highly noisy signals.

CONCLUSIONS

The paper presents the proof-of-concept of a new method for the measurement of frequency of sinusoidal signals in severe noise conditions. The method has taken advantages from some peculiar features of stochastic resonance to make it possible to overcome limitations experienced in authors' previous solution. Preliminary results obtained in tests conducted on actual noisy signals highlighted the promising performance of the proposed method; differences, expressed in relative percentage terms, between nominal and measured frequency never greater than 1% has, in particular, been experienced for SNR values down to -11 dB.

On-going activities are mainly addressed to (i) extend the method also to periodic signals, (ii) carry out an extensive performance assessment of the method in both numerical and actual experiments, and (iii) define and implement an analytical expression capable of automatically determine the value of the optimal threshold value.

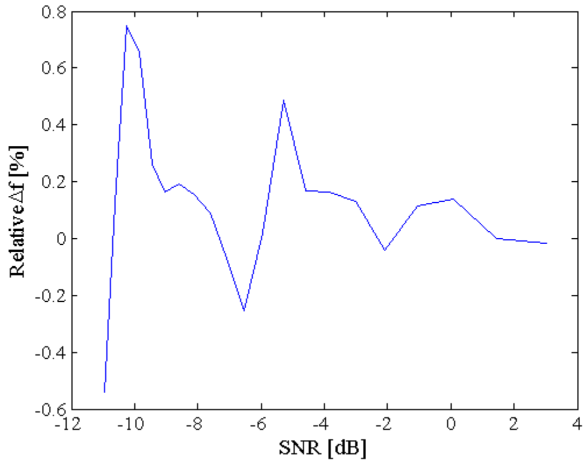

Fig. 7. Differences between nominal and measured frequency for SNR values from -11 up to 3 dB.

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