

CALCULATION OF REFERENCE FEATURES IN SPHERICITY MEASUREMENTS BY THE RADIAL METHOD WITH RESPECT TO THE DENSITY OF SAMPLING POINTS

S. Adamczak, D. Janecki, K. Stepień

Kielce University of Technology, Poland, kstepien@tu.kielce.pl

Abstract: At the Kielce University of Technology a novel approach to measurements of sphericity has been developed. The approach involves application of traditional radial measuring instrument equipped with a special-purpose fixing unit allowing accurate positioning of measured ball. The new concept required developing of the set of mathematical equations allowing, for example, the calculation of the reference sphere parameters. The paper assumptions of the new concept and the mathematical model of calculation of the reference feature by the radial method with respect to sampling points density. The theoretical analysis is supported by the practical example.

Keywords: sphericity, measurement, radial method, reference feature.

1. INTRODUCTION

According to the existing industrial standards, the strategy for sphericity assessment consists in measuring roundness profiles in two or three mutually perpendicular planes. The results obtained for the individual cross-sections are represented graphically in polar coordinates, which makes it possible to determine sphericity deviations. This approach to sphericity measurement is very simplified because a significant part of the spherical surface is not measured [1].

Most industrial standards state that the best reference in sphericity evaluation is the minimum circle circumscribed on a measured profile. Sphericity measurements are generally conducted using radial devices, i.e. ones determining changes in the radius. In radial measurement, the workpiece is placed on a table, which can be rotary or non-rotary, according to the design of the device. If the measuring table is non-rotary, the measuring sensor rotates [2].

Recent advances in metrology have caused that researchers are working on the development of other methods for sphericity measurement. A large number of institutions are investigating the use of coordinate measurement systems to assess sphericity deviations. As the accuracy of coordinate measuring machines is increasing, this technique may soon be applied not only to assess sphericity but also other form deviations [3]. Nowadays,

however, it is still the radial devices that provide the highest measurement accuracy. One of the most original methods proposed recently for measuring sphericity was that developed by Gleason and Schwenke [4]. This measurement strategy employs the three-point method. An advantage of this concept is that the measuring signal is not affected by the spindle errors. The most important drawback is that some harmonic components of the profile cannot be detected by the measuring system. Another interesting concept of measurement of spherical parts is the use of optical systems [5]. However, measurement accuracy of optical systems is still relatively low, when form deviations are measured. This suggests that optical methods are not suitable for accurate sphericity measurements.

A large number of academic and industrial research centres are engaged in developing and improving methods for the measurement of spherical surfaces as well as methods for the evaluation of sphericity deviations. Traditionally, the methods used for assessing deviations from an ideal sphere are similar to those used for analyzing out-of-roundness.

2. THE CONCEPT OF SPHERICITY MEASUREMENT BY THE RADIAL METHOD

The fundamental of the concept proposed by authors is applying a typical radial measuring instrument equipped with a special-purpose unit for the accurate positioning the workpiece. Such system allows measurements of deviations of roundness profiles in precisely defined cross-sections of measured balls. The concept is based on various solutions presented in the literature as well as the authors' experience concerning accurate measurement of roundness and cylindricity.

The research work involving the development of the method was divided into two parts: theoretical (including computer simulations) and experimental. The theoretical investigations included: defining the spherical surface, selecting a relevant measurement strategy, generating and superimposing profiles, filtering the profiles, calculating the reference sphere and the sphericity parameters.

The authors assumed the following strategy of measurement. First, a selected sphere is measured on a

measuring table to determine its roundness profiles in several equally-spaced cross-sections. Next, the object is rotated at a right angle about the vertical axis so that more cross-sections can be measured (see Fig. 1). The limitation of the approach proposed by authors is that the fixing unit may not rotate the workpiece accurately, which can influence measurement data. However, authors assumed that in the preliminary stage of the research the influence of the errors of the positioning unit may be neglected.

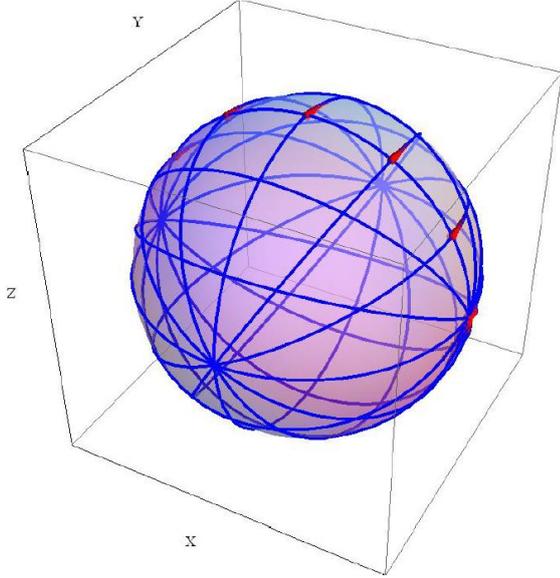


Fig. 1. Strategy for sphericity measurement proposed by the authors – “combined” strategy

3. THE PROBLEM OF SAMPLING POINTS DENSITY

In order to determine form deviations of the element correctly it is necessary to establish parameters of the reference feature, first. The parameters are calculated from measurement data with the use of assumed method. It is usually the least squares or minimum zone method. In the research work described here the method used for the calculation of the reference sphere parameters was the least squares one. Taking into account measurement strategy it is clear that in areas of the surface where roundness profiles cross there are more sampling points than in other areas. Therefore, if parameters of the reference sphere are to be calculated accurately, it was necessary to develop the equation allowing calculation of the coefficient of the sampling points density.

In order to illustrate this question, the problem of calculation of the value of the mean square deviation of the sphericity profile Sq was considered. Let $\Delta R_{sphere}(\theta, \varphi)$ be the deviation from the ideal sphere. Mean square deviation of the profile Sq is equal to

$$Sq = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi (\Delta R_{sphere}(\theta, \varphi))^2 \sin \theta d\theta d\varphi \quad (1)$$

where R, θ, φ are spherical coordinates.

The coefficient 4π is the area of the sphere. If $\Delta r_{sphere}(\varphi') = \Delta R_{sphere}(\theta_i(\varphi'), \varphi_i(\varphi'))$, $i = 1, \dots, M$ are roundness profiles in cross-sections of the sphere, then the parameter Sq should be calculated from the equation

$$Sq = \frac{1}{2\pi M} \sum_{i=1}^M \int_0^{2\pi} (\Delta r_{sphere}(\varphi'))^2 \eta_i(\varphi') d\varphi' \quad (2)$$

where $\eta_i(\varphi')$ is the inverted sampling points density coefficient. Thus, it is necessary to define relevant density function $\zeta_i(\varphi')$ and its inversion $\eta_i(\varphi')$.

Let $(\theta_i(\varphi'), \varphi_i(\varphi'))$, $i = 1, \dots, M$ be parametric equations of the scanning trajectory, $v_i(\varphi')$ be the unit vector with coordinates equal to $(\theta_i(\varphi'), \varphi_i(\varphi'))$ defined in the coordinate system of the sphere XYZ , thus

$$v_i(\varphi') = A^T(\alpha_i, \beta_i, \gamma_i) \begin{pmatrix} \cos \varphi' \\ \sin \varphi' \\ 0 \end{pmatrix} \quad (3)$$

Two unit vectors v_1, v_2 indicate neighboring points on the sphere if the scalar product (v_1, v_2) is approximately equal to 1. It was assumed that the point v_1 lies near to the point v_2 , if $(v_1, v_2) \geq \kappa$, for suitably chosen constant value κ . This constant should depend on an applied measuring strategy, for example, in the “bird-cage” strategy, it depends on the value δ , which is the angle between subsequent profiles or on the coefficient of the conditioning of the strategy. The constant κ can be also chosen experimentally. Assuming that the inverted density function $\eta_i(\varphi')$ depends on the angle δ between vectors following equations may be written:

$$\delta_{i,j}(\varphi', \varphi'') = \arcsin(v_i(\varphi')^T v_j(\varphi'')) \quad (4)$$

and

$$\eta_i(\varphi') = \sum_{i=1}^M \int_0^{2\pi} e^{-\ln 2(\delta_{i,j}(\varphi', \varphi'')/\kappa)^2} d\varphi'' \quad (5)$$

It is obvious that the equations given above apply Gaussian function to the calculation of the inverted density coefficients.

4. REFERENCE SPHERE PARAMETERS

The reference sphere can be calculated very easily for the whole surface of the element. The surface of the ideal sphere with the radius r_0 and at the centre given by e_x, e_y, e_z is described by the equation (6)

$$R_{ref}(\theta, \varphi) = r_0 + e_x \cos \varphi \sin \theta + e_y \sin \varphi \sin \theta + e_z \cos \theta = \Psi(\theta, \varphi)^T p_{ref}$$

$$\Psi(\theta, \varphi) = [1 \quad \cos \varphi \sin \theta \quad \sin \varphi \sin \theta \quad \cos \theta]^T \quad (6)$$

$$p_{ref} = [r_0 \quad e_x \quad e_y \quad e_z]^T$$

(assuming that all vectors are column ones).

It is obvious that it is not the equation of the ideal geometrical sphere but only the approximation if the radius of the sphere is much larger than the coordinates of its centre. If the whole surface of the sphere $R_{sphere}(\theta, \varphi)$ is given, then the parameters of the reference sphere can be calculated with use of the least square method by minimization of the sum of squares of distances of points of the real sphere from the reference, which is described by the equation

$$J(r_0, e_x, e_y, e_z) = \int_0^\pi \int_0^{2\pi} (R_{sphere}(\theta, \varphi) - R_{ref}(\theta, \varphi))^2 \sin \theta d\theta d\varphi \quad (7)$$

Functions $1, \cos\theta, \cos\varphi\sin\theta, \sin\varphi\sin\theta$ are orthogonal on the sphere S^2 , thus values of parameters r_0, e_x, e_y and e_z can be calculated from the following equation

$$p_{ref,k} = \frac{\int_0^\pi \int_0^{2\pi} R_{sphere}(\theta, \varphi) \Psi_k(\theta, \varphi) \sin \theta d\theta d\varphi}{\int_0^\pi \int_0^{2\pi} (\Psi_k(\theta, \varphi))^2 \sin \theta d\theta d\varphi}, \quad k = 1, 2, 3, 4 \quad (8)$$

It should be taken into account that the value of the expression $\int_0^\pi \int_0^{2\pi} (\Psi_k(\theta, \varphi))^2 \sin \theta d\theta d\varphi$ for $k = 1$ equals 4π , and for $k = 2, 3, 4$ it equals $4\pi/3$.

If number of sampling points or the number of roundness profiles is finite, then the equation (8) should be modified. In order to make equations simpler it was assumed that measured profiles are continuous. In further considerations following indicator was used:

$$J(r_0, e_x, e_y, e_z) = \sum_{i=1}^M \int_0^{2\pi} (r_{sphere_i}(\varphi') - R_{ref}(\theta_i(\varphi'), \varphi_i(\varphi'))^2 \eta_i(\varphi') d\varphi' \quad (9)$$

The equation (9) can be also written in the following form:

$$J(r_0, e_x, e_y, e_z) = \sum_{i=1}^M \int_0^{2\pi} (r_{sphere_i}(\varphi') - \Psi(\theta_i(\varphi'), \varphi_i(\varphi'))^T p_{ref})^2 \eta_i(\varphi') d\varphi' \quad (10)$$

where $(\theta_i(\varphi'), \varphi_i(\varphi')), \varphi' \in (0, 2\pi)$ is the scanning trajectory depending on ball orientations $(\alpha_i, \beta_i, \gamma_i)$, $i = 1, \dots, M$.

Accordingly, $r_{sphere_i}(\varphi') = R_{sphere}(\theta_i(\varphi'), \varphi_i(\varphi'))$ is the profile of the cross-section of the ball.

The change of the angle φ' will result in even displacement along the trajectory, therefore it is not necessary to apply the coefficient $\sin\theta$, which indicates the area of the element $d\theta d\varphi$. Instead of this coefficient a certain function $\eta_i(\varphi')$ is introduced that describes local density of the surface

scanning. The value of this function should be in inverse proportion to local density of sampling points.

Finally, the following equation can be obtained from the condition of minimization of the indicator J

$$\left(\sum_{i=1}^M \int_0^{2\pi} \Psi(\theta_i, \varphi_i) \Psi^T(\theta_i, \varphi_i) \eta_i(\varphi') d\varphi' \right) p_{ref} = \sum_{i=1}^M \int_0^{2\pi} \Psi(\theta_i, \varphi_i) r_{sphere_i}(\varphi') \eta_i(\varphi') d\varphi' \quad (11)$$

which is equivalent to

$$G p_{ref} = g \quad (12)$$

5. THE EXPERIMENT

The concept of the calculation of reference sphere parameters with respect to the sampling points density was tested through computer simulations. The research was divided into four main stages: generating the spherical surface with some imperfections, simulation of the roundness profiles obtained by the radial method, calculation of the reference sphere parameters assuming that sampling points are distributed uniformly and calculation of the reference sphere parameters with use of the density coefficient.

Figure 2 presents the spherical surface to be investigated. The surface was generated with use of spherical harmonics and associated Legendre functions and polynomials [6].

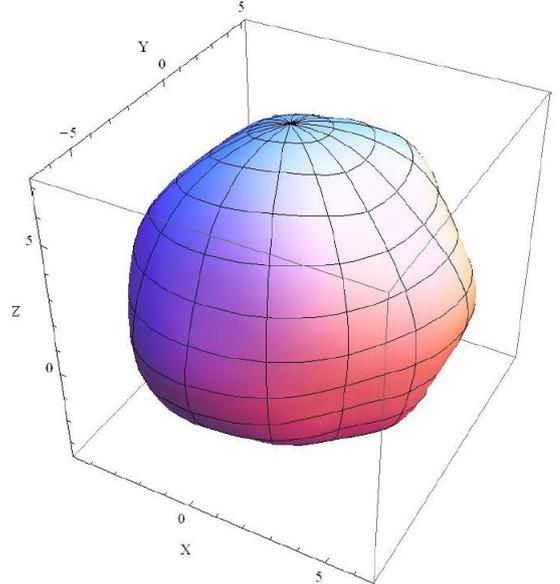


Fig. 2. The spherical surface being the subject of the research

After the generation of the surface the choice of the relevant measuring strategy was made. Authors decided that the bird-cage strategy will be used in this case.

In order to evaluate the correctness of equations presented in the previous chapter the errors of determination of the sphericity deviation were calculated. Diagram of the difference between the deviation obtained from the whole

surface and from the roundness profiles without respecting the sampling points density is shown in Figure 3.

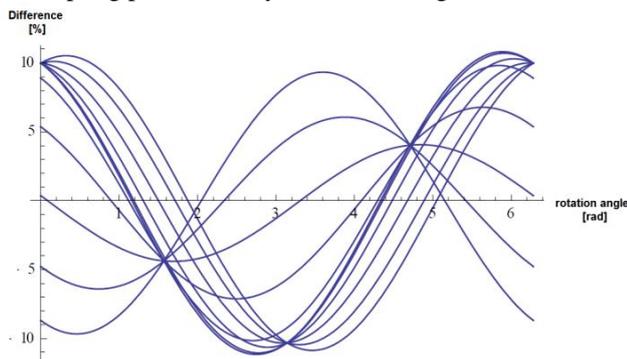


Fig. 3. Difference between deviation calculated for the whole surface and for the roundness profiles (uniform distribution of sampling points assumed)

The diagram shown in Fig. 3 indicates that the maximum difference between values of sphericity deviation is about 10% if uniform distribution of the sampling points is assumed. Taking to account the strategy presented in Fig. 1 it is obvious that the sampling points are not distributed randomly on the whole surface of the sphere. In points where roundness profiles cross each other the density of the sampling points is higher.

In order to decrease the difference between the deviation it is necessary to apply the coefficient of the sampling points density (see equation (12)-(13)).

Figure 4 presents the difference between values of the deviation if local density of sampling points is taken to account.

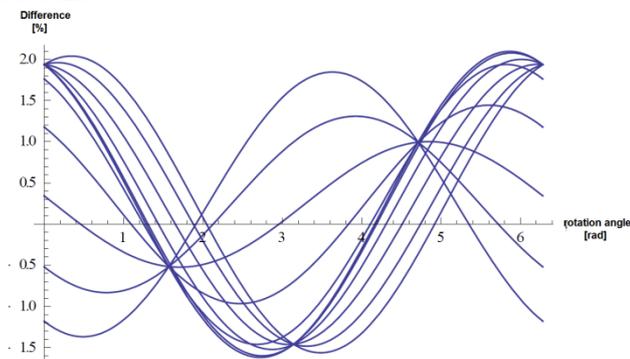


Fig. 4. Difference between deviation calculated for the whole surface and for the roundness profiles (the coefficient of the sampling points density applied)

The diagram presented in Fig. 4 shows that if one applies the coefficient $\eta_s(\varphi)$, then the difference between deviations is about five times smaller than in the previous case. It proves that the concept presented in Chapter 4 is correct. It may be assumed that the difference would be even smaller if more roundness profiles had been measured.

6. SUMMARY

Since the existing measurement strategies applied to assess sphericity deviations are not sufficiently accurate, the

authors developed a combined method, which involves measuring the changes in radius with radial devices. The measuring instrument is a computer-aided radial device for measuring radius changes in roundness profiles. It is equipped with a special-purpose fixing unit for accurate positioning of the measured element. The research involved solving the problem of calculation of the reference sphere. In authors opinion this problem should be considered with respect to nonuniform local density of sampling points. Thus, authors developed mathematical equations allowing calculation of the reference sphere parameters where the coefficient of sampling points density is applied. The mathematical model was verified through computer simulations with the use of Mathematica. Results of the simulations show that the application of the coefficient of the sampling points density results in significant increasing of measurement accuracy. If one does not respect the sampling points density, then the maximum difference between the sphericity deviation calculated for the whole surface and for roundness profiles is about 10 %. By applying the coefficient describing the density of sampling points, this difference decreases to about 2 %.

Thus, it can be assumed that developed equations are correct and can be used in the experimental part of the research work.

The next stage of the research will be the experimental verification of the concept. It will be carried out at the Kielce University of Technology at the Laboratory of Computer-Aided Measurements of Geometrical Quantities. The measuring device will be a computer-aided instrument Talyrond 73 by Taylor Hobson equipped with a unit allowing positioning of spherical parts. The spherical parts will be measured by the designed system and by the conventional one and obtained results will be compared then. The number of parts to be measured will be equal to 50 in order to evaluate the accuracy of proposed method with the use of statistical parameters.

5. REFERENCES

- [1] Adamczak S., Janecki D., Stępień K.: "Recent trends in measurements of sphericity", *Annals of DAAAM for 2011*, pp. 1569 – 1570, 2011.
- [2] Kanada T.: "Estimation of sphericity by means of statistical processing for roundness of spherical parts", *Precision Engineering*, vol. 20, no. 2, pp. 117-122, 1997.
- [3] Poniatowska M.: "Parameters for CMM contact measurements of free-form surfaces". *Metrology and Measurement Systems*, vol. 18, no. 2, pp. 199-208, 2011.
- [4] Gleason E., Schwenke H.: "A spindless instrument for the roundness measurement of precision spheres", *Precision Engineering*, vol. 22, no. 1, pp. 36-42, 1998.
- [5] Halkaci H. S., Mavi Ö., Yigit O.: "Evaluation of form error at semi-spherical tools by use of image processing", *Measurement*, vol. 40, no. 9-10, pp. 860-867, 2007.
- [6] Janecki D., Stępień K., Adamczak S.: "Investigating methods of mathematical modelling of measurement and analysis of spherical surfaces", *Proceedings of the X International Symposium on Measurement and Quality Control – ISMQC 2010, Osaka, Japan, 2010*.