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# UNCERTAINTY IN MEASUREMENT: A KEY CRITERION FOR DEFINING THE COVERAGE FACTOR ASSOCIATED WITH THE MANUFACTURING PROCESS OF WOVEN FABRICS

[1]*Cristian S. Castillo*; [1]*Maurício N. Frota,* [2]*Ignácio Lira*

[1]Catholic University, Rio de Janeiro, Postgraduate Metrology Programme, Brazil, cristiancastillosantos@gmail.com; mfrota@puc-rio.br
[2]Catholic University of Chile, Mechanical Engineering Department, Chile, ilira@ing.puc.cl

**Abstract:** This study evaluates the *fractional coverage factor* ($Cf_T$) of a woven fabric before and after the manufacturing process is completed. The uncertainty associated with $Cf_T$ was evaluated for specific conditions of mechanical strain of the thread yarns during the weaving process. The economic impact from undesirable geometric deviations of $Cf_T$ is performed by making use of the *Taguchi's Loss Function.* The proposed method proved to be effective to correlate the limits of acceptability of geometrical deviations with the *coverage factor* for a given tolerance of the manufacturing process, thereby reducing the degree of waste.

**Key words**: Fractional coverage factor; fabric manufacturing; uncertainty in measurement; economic impact; Taguchi's Loss Function.

## 1. INTRODUCTION

This work deals with the deformation of a woven fabric resulting from the manufacturing process. It investigates whether or not the Taguchi's Loss Function [1] is capable of estimating the economic impact resulting from undesirable geometrical deviations of yarns during the weaving manufacturing process.

The yarns of a woven fabric are usually arranged in the longitudinal and transverse directions. The warp yarns (aligned in the longitudinal direction) of the fabric interlaced with filling yarns (transverse direction) define the structure of the woven fabric (depicted in Figure 1) while the spaces between them, the c*overage factor* (also known as *fractional coverage factor*, $Cf_T$). This dimensionless factor is defined by the fraction of free space in a defined sample of the woven fabric. Deficiencies in the fabric manufacturing process result in undesirable geometric deviations in the fabric structure. These deviations are critical to define the degree of acceptability of a batch of manufactured fabric. The coverage factor indicates the extent to which an area of a fabric is covered by one set of yarns.

The concept of *coverage factor*, still in use, was conceived in the first half of the last century through the pioneering work of Pierce [2], Kemp [3], Galcerán [4] followed by Hamilton [5]. As documented in [6], Grosberg, in 1969, proposed an improved model to measure the *fractional coverage factor* $Cf_T$. Two decades later Jinlian & Newton [7] introduced image analysis techniques to measure deformation and fabric structure characteristics. De Castellar [8] finally applied the image technique to obtain $Cf_T$. More recently, Bertrán et al. [9], in their work on the effect of the coverage factor in mechanical properties of woven fabrics, correlated the geometric and optical coverage factors proposed by Pierce, Galcerán and Grosberg.

## 2. FRACTIONAL COVERAGE MODEL

Table 1, extracted from [9], resumes the correlation of the geometric and optical coverage factors proposed in the literature.

Table 1. Correlation among geometric and optical coverages

| | Optical | Geometrical factors | | |
|---|---|---|---|---|
| | factor | Galcerán | Pierce | Grosberg |
| **Optical factor** | 1.000 | 0.3258 | 0.0739 | **0.8133** |
| Geometrical factor (**Galcerán**) | | 1.000 | 0.5706 | 0.6296 |
| Geometric factor (**Pierce**) | | | 1.000 | 0.0961 |
| Geometric factor (**Grosberg**) | | | | 1.000 |

The data shown in Table 1 validate the adoption of Grosberg's model [6]. It is the geometric model that exhibits the highest correlation (0.8133) of the geometric factor with the optical factor by means of the image analysis method, known to produce the most reliable prediction of the coverage factor. The woven fabric tested was manufactured using cotton yarns (*ring spun*), interlaced in a plain weave fashion.

## 3. THEORETICAL FOUNDATIONS FOR $Cf_T$

Grosberg [6] describes the *fractional coverage factor* of a woven fabric by the schematic representation shown in Figure 1. It illustrates the position of yarns placed in the loom in the manufacturing position. For the purpose of this work, yarns are modeled as filaments exhibiting an elastic modulus and resistance to tension and bending. As can be seen, $d_w$ refers to the diameter of the longitudinal *warp yarns*; $d_f$ is the diameter of the transverse *filling yarns*; $g_w$ the lateral free space (gap) between warp yarns and $g_f$, the free space between filling yarns.

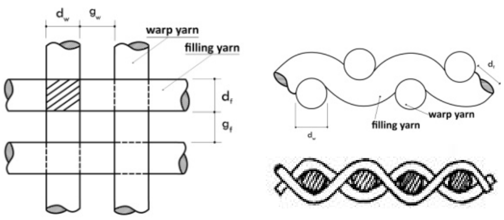


Fig. 1. Longitudinal and transverse yarns of the fabric [6]

The dimensionless *fractional coverage factor* ($Cf_T$) can be calculated by Grosberg's equation (1), as given in reference [6]:

$$Cf_T = Cf_w + Cf_f - (Cf_w\, Cf_f) \qquad (1)$$

Determination of $Cf_T$ requires knowledge of the following physical quantities:

$\mathcal{N}$ (a discrete number): amount of yarns per unit length. ($\mathcal{N}_w$: warp yarns and $\mathcal{N}_f$: filling yarns).

$\mathcal{L}$: length of the fabric; $\mathcal{L}$ becomes $\mathcal{L}_w$ (along the warp yarn) or $\mathcal{L}_f$: (along the filling yarn).

**Tex**: linear density of the yarn, given by the mass ($\mathcal{M}$) of 1,000 meters of textile yarn per unit length ($\ell$). $\ell_w$ refers to the length measured in the longitudinal warp direction and $\ell_f$ in the transverse filling direction. Similarly, $\mathcal{M}_w$ and $\mathcal{M}_f$ denote the mass of a warp and filling yarn of lengths $\ell_w$ and $\ell_f$, respectively.

$\mathbf{P_y}$: porosity of the yarn, a dimensionless parameter given by the free space between fibers within the structure of the yarn. The value assumed here ($\mathbf{P_y}$ = 0.60) was taken from [6].

$\mathbf{D_f}$: density of the fiber used to manufacture the yarn; (mass per volume of fiber). The value adopted in this work ($\mathbf{D_f}$=1.52 g/cm$^3$) was found in [6].

**d**: diameter of the yarn (becomes $d_w$ or $d_f$ depending on the direction considered). The yarn diameter ($d_w$ or $d_f$) can be calculated by equation (2), where the λ's factors account for specific conversion of units.

$$d = \lambda_1 \left(\frac{Tex}{P_y\, D_f}\right)^{\frac{1}{2}} \qquad (2)$$

In Equation (1), $\mathbf{Cf_w}$ is the fractional coverage of the warp yarn defined by the product of two parameters: the diameter of the warp yarns and their amount per unit of length, positioned along the width of the fabric. Similarly, $\mathbf{Cf_f}$ refers to the fractional coverage of the filling yarn, i.e.:

$$Cf_w = d_w\, \mathcal{N}_w; \quad Cf_f = d_f\, \mathcal{N}_f \qquad (3\text{-}4)$$

## 4. EQUATION FOR CALCULATION OF $\mathbf{Cf_T}$

The calculation of the fractional coverage factor is based on results of measurements of the physical quantities described above.

Replacing equations (2), (3), (4) in equation (1), one obtains a practical equation for the calculation of the coverage factor.

$$Cf_T = \left[\lambda_2 \left(\frac{1}{P_y\, D_f}\right)^{\frac{1}{2}} \left(\frac{\mathcal{M}_w}{\ell_w}\right)^{\frac{1}{2}} \mathcal{N}_w\, (\mathcal{L}_f)^{-1}\right] + \left[\lambda_2 \left(\frac{1}{P_y\, D_f}\right)^{\frac{1}{2}} \left(\frac{\mathcal{M}_f}{\ell_f}\right)^{\frac{1}{2}} \mathcal{N}_f\, (\mathcal{L}_w)^{-1}\right] -$$

$$- \left[\lambda_3 \left(\frac{1}{P_y\, D_f}\right)^{\frac{1}{2}} \left(\frac{\mathcal{M}_w}{\ell_w}\right)^{\frac{1}{2}} \mathcal{N}_w\, (\mathcal{L}_f)^{-1} \left(\frac{1}{P_y\, D_f}\right)^{\frac{1}{2}} \left(\frac{\mathcal{M}_f}{\ell_f}\right)^{\frac{1}{2}} \mathcal{N}_f\, (\mathcal{L}_w)^{-1}\right] \qquad (5)$$

## 5. MEASUREMENT UNCERTAINTY

According to ISO GUM [10], the *final standardized uncertainty $u_f$ associated with* $Cf_T$ is given by equation (6). Component type A of the uncertainty ($u_{random}$) accounts for the random nature of the measurand while component type B accounts for the uncertainty associated with factors not directly associated with measurement of the desired physical quantities. In this study, type B uncertainty —taken as a rectangular probability distribution— accounts for the uncertainty associated with the resolution of the measurement equipment used ($u_{resolution}$) and that stated in the calibration certificate ($u_{certified}$). Equation 6 can then be re-written (equation 7):

$$u_f = \sqrt{[(u_{random})^2]_{TypeA} + [(u_{resolution})^2 + (u_{certified})^2]_{TypeB}} \qquad (6)$$

$$u_f = \sqrt{[(s)^2]_{TypeA} + \left[\left(\frac{resolution}{\sqrt{12}}\right)^2 + \left(\pm\frac{U_c}{k_c}\right)^2\right]_{TypeB}} \qquad (7)$$

Given the nature of the physical quantity considered, type A or type B components of the uncertainty may or may not be meaningful. In this case, type A component is not associated with $\mathcal{N}_w$ and $\mathcal{N}_f$ while type B uncertainty is not present in $\ell_w$ and $\ell_f$ values. Both components are relevant in the calculation of $\mathcal{M}_w$ and $\mathcal{M}_f$.

For the above described (eight) physical quantities one can obtain the sensitivity coefficients required to propagate the uncertainties associated with each quantity. Once the eight sensitivity coefficients are obtained, one can calculate the combined standardized uncertainty associated with $Cf_T$, given by equation (12). For the sake of simplicity, the eight sensitivity coefficients can be re-written in four equations if the correspondent subscript indexes associated with the physical quantities are suppresses. These equations become:

$$\frac{\partial Cf_T}{\partial \mathcal{M}} = [\lambda_4(\mathcal{M})^{-\frac{1}{2}} (\ell)^{-\frac{1}{2}} \mathcal{N}(\mathcal{L})^{-1}] - [\lambda_5(\mathcal{M})^{-\frac{1}{2}} (\ell)^{-\frac{1}{2}} \mathcal{N} (\mathcal{L})^{-1}(\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{1}{2}} \mathcal{N}(\mathcal{L})^{-1}] \tag{8}$$

$$\frac{\partial Cf_T}{\partial \ell} = -[\lambda_4 (\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{3}{2}} \mathcal{N} (\mathcal{L})^{-1}] + [\lambda_5(\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{3}{2}} \mathcal{N}(\mathcal{L})^{-1} (\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{1}{2}}\mathcal{N} (\mathcal{L})^{-1}] \tag{9}$$

$$\frac{\partial Cf_T}{\partial N} = [\lambda_6 (\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{1}{2}}(\mathcal{L})^{-1}] - [\lambda_7 (\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{1}{2}} (\mathcal{L})^{-1} (\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{1}{2}} \mathcal{N} (\mathcal{L})^{-1}] \tag{10}$$

$$\frac{\partial Cf_T}{\partial \mathcal{L}} = -[\lambda_6 (\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{1}{2}} \mathcal{N}(\mathcal{L})^{-2}] + [\lambda_7(\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{1}{2}}\mathcal{N}(\mathcal{L})^{-2}(\mathcal{M})^{\frac{1}{2}} (\ell)^{-\frac{1}{2}} \mathcal{N} (\mathcal{L})^{-1}] \tag{11}$$

$$u_c\{Cf_T\} = \sqrt{\begin{aligned}&\left(\frac{\partial Cf_T}{\partial \ell_w} u_f\{\ell_w\}\right)^2 + \left(\frac{\partial Cf_T}{\partial \ell_f} u_f\{\ell_f\}\right)^2 + \left(\frac{\partial Cf_T}{\partial \mathcal{M}_w} u_f\{\mathcal{M}_w\}\right)^2 + \left(\frac{\partial Cf_T}{\partial \mathcal{M}_f} u_f\{\mathcal{M}_f\}\right)^2 + \\ &+ \left(\frac{\partial Cf_T}{\partial \mathcal{N}_w} u_f\{\mathcal{N}_w\}\right)^2 + \left(\frac{\partial Cf_T}{\partial \mathcal{N}_f} u_f\{\mathcal{N}_f\}\right)^2 + \left(\frac{\partial Cf_T}{\partial \mathcal{L}_w} u_f\{\mathcal{L}_w\}\right)^2 + \left(\frac{\partial Cf_T}{\partial \mathcal{L}_f} u_f\{\mathcal{L}_f\}\right)^2\end{aligned}} \tag{12}$$

The effective degree of freedom ($\nu_{eff}$) associated with the fractional coverage factor $Cf_T$ is given by the equation of Welch-Satterwaite (ISO GUM [10]). Applying this equation to each physical quantity ($\ell$, $\mathcal{L}$, $\mathcal{N}$ and $\mathcal{M}$), and taking into account the applicable components of type A and type B uncertainties, one obtains the following four equations related to the effective degree of freedom ($\nu_{eff}$) associated with $Cf_T$. After involving algebraic manipulations, the required cumbersome calculations yielded:

$$\frac{u_f^4\{\ell\}}{\nu_{eff}\{\ell\}} = \frac{u^4\{\ell_{resolution}\}}{\nu_{resolution}} + \frac{u^4\{\ell_{certified}\}}{\nu_{certified}} = \frac{\left(\frac{resolution}{\sqrt{12}}\right)^4}{\nu_{resolution}} + \frac{\left(\pm\frac{U_c}{k_c}\right)^4}{n-1} \tag{13}$$

$$\frac{u_f^4\{\mathcal{M}\}}{\nu_{eff}\{\mathcal{M}\}} = \frac{u^4\{\mathcal{M}_{random}\}}{\nu_{random}} + \frac{u^4\{\mathcal{M}_{resolution}\}}{\nu_{resolution}} + \frac{u^4\{\mathcal{M}_{certified}\}}{\nu_{certified}} = \frac{(s)^4}{n-1} + \frac{\left(\frac{resolution}{\sqrt{12}}\right)^4}{\nu_{resolution}} + \frac{\left(\pm\frac{U_c}{k_c}\right)^4}{n-1} \tag{14}$$

$$\frac{u_f^4\{\mathcal{N}\}}{\nu_{eff}\{\mathcal{N}\}} = \frac{u^4\{\mathcal{N}_{random}\}}{\nu_{random}} = \frac{(s)^4}{n-1} \tag{15}$$

$$\frac{u_f^4\{\mathcal{L}\}}{\nu_{eff}\{\mathcal{L}\}} = \frac{u^4\{\mathcal{L}_{random}\}}{\nu_{resolution}} + \frac{u^4\{\mathcal{L}_{certified}\}}{\nu_{certified}} = \frac{\left(\frac{resolution}{\sqrt{12}}\right)^4}{\nu_{resolution}} + \frac{\left(\pm\frac{U_c}{k_c}\right)^4}{n-1} \tag{16}$$

In these equations, $U_c$ and $k_c$, refer, respectively, to the expanded uncertainty and to the coverage factor associated with the calibration certificate of the equipment used to perform the required measurements. Whenever an uncertainty of type B arises from a known probability distribution function (e.g.: a rectangular distribution): $\nu_{eff} \to \infty$ (uncertainty free). These four equations, in fact, represent eight equations as each physical quantity should be considered in both fabric yarn directions (warp and filling). Thus, the combined effect yields the overall effective degree of freedom ($\nu_{eff}$) associated with the fractional coverage factor $Cf_T$:

$$\frac{{u_c}^4\{Cf_T\}}{\nu_{eff}\{Cf_T\}} = \frac{{u_f}^4\{\ell_w\}}{\nu_{eff}\{\ell_w\}} + \frac{{u_f}^4\{\ell_f\}}{\nu_{eff}\{\ell_f\}} + \frac{{u_f}^4\{\mathcal{M}_w\}}{\nu_{eff}\{\mathcal{M}_w\}} + \frac{{u_f}^4\{\mathcal{M}_f\}}{\nu_{eff}\{\mathcal{M}_f\}} + \frac{{u_f}^4\{\mathcal{N}_w\}}{\nu_{eff}\{\mathcal{N}_w\}} + \frac{{u_f}^4\{\mathcal{N}_f\}}{\nu_{eff}\{\mathcal{N}_f\}} + \frac{{u_f}^4\{\mathcal{L}_w\}}{\nu_{eff}\{\mathcal{L}_w\}} + \frac{{u_f}^4\{\mathcal{L}_f\}}{\nu_{eff}\{\mathcal{L}_f\}} \tag{17}$$

Once $\nu_{eff}$ is calculated, the coverage factor k (not to be confused with the fractional coverage factor $Cf_T$) can be determined through the t-distribution table, for a given coverage probability (here taken as p = 95.45%) [10].

Finally, the expanded uncertainty ($U$) associated with the fractional coverage factor $Cf_T$ is given by:

$$\boldsymbol{U} = \pm(\boldsymbol{k} * \boldsymbol{u_c}) \tag{18}$$

$$\boldsymbol{MR}\ (Measurement\ Result) = (\boldsymbol{Cf_T} \pm \boldsymbol{U}) \tag{19}$$

## 6. ECONOMIC IMPACT RESULTED FROM UNDESIRABLE DEVIATIONS OF $Cf_T$

*Taguchi's Loss Function* is a statistical method conceived to improve the quality of manufactured goods [1]. Recently, *Taguchi's* idea was rediscovered and applied to engineering problems. Equation (20) is its mathematical representation:

$$L(x)\left[\frac{US\$}{m^2}\right] = \frac{A\left[\frac{US\$}{m^2}\right]}{\Delta^2}(x-m)^2 \qquad (20)$$

Figure 2 (a contribution of this study) depicts an expansion of Taguchi's loss function to incorporate the uncertainty associated with the specification limits (tolerances) of the manufacturing process. The key parameters to be considered in this expanded Taguchi analysis are:

**A**: The total manufacturing cost (or maximum allowable) of the produced product per unit area of fabric, designed before the weaving process takes place. It refers to the minimum acceptable financial gain capable to provide the economic sustainability of the investment.

$\Delta$(non-dimensional): tolerance with respect to the applicable nominal value of $Cf_T$, planned before the weaving process takes place.

**x** (non-dimensional): deviation with respect to nominal value of $Cf_T$, after the weaving process.

**m** (non-dimensional): an assigned value to quantify quality of the weaving process.

**L(x)**: financial losses estimated after the weaving process takes place.

**LIE** and **LSE** (non-dimensional): lower and upper bound limits of compliance (tolerance) that were considered before the weaving process takes place.

**LIC** and **LSC** (non-dimensional): lower and upper bound limits of compliance considered before the weaving process takes place [11]. If the measured value of $Cf_T$ exceeds these limits, it will fall within the uncertainty interval.

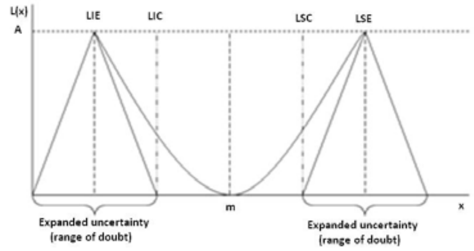


Fig. 2. Representation of the proposed expanded analysis based on *Taguchi's Loss Function*

## 7. RESULTS

The above derived equations establishes the required framework to calculate the economic impact that results from undesirable deviations of the fractional coverage factor $Cf_T$. The unit conversion factors $\lambda$ ($\lambda_1 \ldots \lambda_7$) were introduced to simplify the above governing equations, decoded in Table 2.

Table 2. Unit conversion factors $\lambda$

| $\lambda$ Factor | unit | Magnitude | $\lambda$ Factor | unit | Magnitude |
|---|---|---|---|---|---|
| $\lambda_1$ | $cm^{-1/2}$ | 0.0036 | $\lambda_5$ | $cm^3g^{-1}$ | 0.6990 |
| $\lambda_2$ | nd | 1.1290 | $\lambda_6$ | $cm^{3/2}g^{-1/2}$ | 1.1820 |
| $\lambda_3$ | nd | 1.2750 | $\lambda_7$ | $cm^3g^{-1}$ | 1.3970 |
| $\lambda_4$ | $cm^{3/2}g^{-1/2}$ | 0.5910 | | | |

nd = non-dimensional

The physical quantities expressed in the governing equations were either estimated or measured. Table 3 summarizes the results obtained. Before the manufacturing process, they were either estimated or measured. After, they were measured by calibrated equipment based on the applicable standard [12, 13].

The final standardized uncertainty ($u_f$) — calculated based on $u_{random}$, $u_{resolution}$ and $u_{certified}$, given in Equation (7)— associated with the results shown in Table 3 is summarized in Table 4.

Table 3. Estimated and measured entry values (before and after the manufacturing process takes place)

| Physical quantities required for the calculation of $Cf_T$ | Before the manufacturing process | | After the manufacturing process | Reference Standard | Calibrated Equipment Used |
|---|---|---|---|---|---|
| | Estimated Value | Measured Value | Measured Value | | |
| $N_w$ [afy/cm] | 20 | — | 22 | ISO 7211-2 | Counting glass |
| $N_f$ [afy/cm] | 18 | — | 21 | ISO 7211-2 | Counting glass |
| $L_f$ [cm] | 1.0 | — | 1.0 | ISO 7211-2 | Counting glass |
| $L_w$ [cm] | 1.0 | — | 1.0 | ISO 7211-2 | Counting glass |
| $\ell_w$ [cm] | — | 25.0 | 25.0 | ASTM D1059 | Twistmeter |
| $\ell_f$ [cm] | — | 25.0 | 25.0 | ASTM D1059 | Twistmeter |
| $M_w$ [g] | — | 0.0098 | 0.0094 | ASTM D1059 | Analytic scale (0.1 mg) |
| $M_f$ [g] | — | 0.0070 | 0.0071 | ASTM D1059 | Analytic scale (0.1 mg) |

awy/cm: amount of warp yarns/cm     afy/cm: amount of filling yarns/cm

Based on the estimated and measured results given in Table 3, one can finally calculate (through equations 8, 9, 10 and 11) the sensitivity coefficients of $Cf_T$ with respect to the critical quantities analyzed. The final results are given in Table 5. Those are the sensitivity coefficients needed to calculate the combined standardized uncertainty ($u_c$) associated with the $Cf_T$ (shown at the bottom of Table 5).

Table 4. Associated uncertainties and final standardized uncertainty

| Physical quantities required for the calculation of $Cf_T$ | Uncertainties associated with physical quantities *before* manufacturing process | | | | Uncertainties associated with physical quantities *after* manufacturing process | | | |
|---|---|---|---|---|---|---|---|---|
| | $u_{random}$ | $u_{resolution}$ | $u_{certified}$ | $u_f$ | $u_{random}$ | $u_{resolution}$ | $u_{certified}$ | $u_f$ |
| $N_w$ [awy/cm] | — | — | — | 0 [awy/cm] | 0.837 [awy/cm] | — | — | 0.837 [awy/cm] |
| $N_f$ [afy/cm] | — | — | — | 0 [afy/cm] | 0.548 [afy/cm] | — | — | 0.548 [afy/cm] |
| $L_f$ [cm] | — | — | — | 0 [cm] | — | $0.01/12^{1/2}$ [cm] | $0.08/2.01^{1/2}$ [cm] | 0.0399 [cm] |
| $L_w$ [cm] | — | — | — | 0 [cm] | — | $0.01/12^{1/2}$ [cm] | $0.08/2.01^{1/2}$ [cm] | 0.0399 [cm] |
| $\ell_w$ [cm] | — | $0.01/12^{1/2}$ [cm] | $0.08/2.01^{1/2}$ [cm] | 0.0399 [cm] | — | $0.01/12^{1/2}$ [cm] | $0.08/2.01^{1/2}$ [cm] | 0.0399 [cm] |
| $\ell_f$ [cm] | — | $0.01/12^{1/2}$ [cm] | $0.08/2.01^{1/2}$ [cm] | 0.0399 [cm] | — | $0.01/12^{1/2}$ [cm] | $0.08/2.01^{1/2}$ [cm] | 0.0399 [cm] |
| $M_w$ [g] | 0.0010 [g] | $0.0001/12^{1/2}$ [g] | $0.0002/2.00^{1/2}$ [g] | 0.0010 [g] | 0.0015 [g] | $0.0001/12^{1/2}$ [g] | $0.0002/2.00^{1/2}$ [g] | 0.0015 [g] |
| $M_f$ [g] | 0.0005 [g] | $0.0001/12^{1/2}$ [g] | $0.0002/2.00^{1/2}$ [g] | 0.0005 [g] | 0.0003 [g] | $0.0001/12^{1/2}$ [g] | $0.0002/2.00^{1/2}$ [g] | 0.0003 [g] |

awy/cm=amount of warp yarns/cm; afy/cm=amount of filling yarns/cm

Table 5. Calculated values for the sensitivity coefficients and combined standardized uncertainty associated with $Cf_T$

| Sensitivity coefficients and combined standardized uncertainty associated with $Cf_T$ | Magnitudes | |
|---|---|---|
| | Before manufacturing | After manufacturing |
| $\partial Cf_T/\partial N_w$ [awy/cm]$^{-1}$ | — | 0.013 |
| $\partial Cf_T/\partial N_f$ [afy/cm]$^{-1}$ | — | 0.010 |
| $\partial Cf_T/\partial L_w$ [cm]$^{-1}$ | — | (-) 0.208 |
| $\partial Cf_T/\partial L_f$ [cm]$^{-1}$ | — | (-) 0.293 |
| $\partial Cf_T/\partial \ell_w$ [cm]$^{-1}$ | (-) 0.006 | (-) 0.006 |
| $\partial Cf_T/\partial \ell_f$ [cm]$^{-1}$ | (-) 0.004 | (-) 0.004 |
| $\partial Cf_T/\partial M_w$ [g]$^{-1}$ | 15.356 | 15.587 |
| $\partial Cf_T/\partial M_f$ [g]$^{-1}$ | 13.510 | 14.585 |
| $u_c\{Cf_T\}$ [nd] | 0.017 | 0.030 |

awy/cm = amount of warp yarn/cm; afy/cm = amount of filling yarn/cm; nd = non-dimensional

Once the value of the combined uncertainty ($u_c$) is known, equation (18) can be used to calculate the expanded uncertainty (U) associated with $Cf_T$. Before that, the coverage factor k needs to be determined from the t-distribution table [10], using the effective degree of freedom value as an entry data for a given probability coverage (taken as 95.45%). Table 6 summarizes the calculations developed to obtain the k coverage factor required in equation 18.

Table 6. Calculation of the Effective Degree of Freedom

| Physical quantities required for the calculation of $Cf_T$ | **Degree of freedom** (before the manufacturing process) | | | | **Degree of freedom** (after the manufacturing process) | | | |
|---|---|---|---|---|---|---|---|---|
| | $v_{random}$ [nd] | $v_{resolution}$ [nd] | $v_{certified}$ [nd] | $v_{eff}$ [nd] | $v_{random}$ [nd] | $v_{resolution}$ [nd] | $v_{certified}$ [nd] | $v_{eff}$ [nd] |
| $N_w$ [awy/cm] | — | — | — | — | 4 | — | — | 4 |
| $N_f$ [afy/cm] | — | — | — | — | 4 | — | — | 4 |
| $L_f$ [cm] | — | — | — | — | — | ∞ | 50 | 50.4994 |
| $L_w$ [cm] | — | — | — | — | — | ∞ | 50 | 50.4994 |
| $\ell_w$ [cm] | — | ∞ | 50 | 50.4994 | — | ∞ | 50 | 50.4994 |
| $\ell_f$ [cm] | — | ∞ | 50 | 50.4994 | — | ∞ | 50 | 50.4994 |
| $M_w$ [g] | 9 | ∞ | ∞ | 9.1866 | 9 | ∞ | ∞ | 9.0907 |
| $M_f$ [g] | 9 | ∞ | ∞ | 9.8068 | 9 | ∞ | ∞ | 10.8444 |

From t-distribution: p= 95.45%; k=13.97

awy/cm = amount of warp yarns/cm; afy/cm = amount of filling yarns/cm; nd = [non-dimensional]

Considering a coverage probability of 95.45% and taking into account that all calculated values (related to the effective degree of freedom) are less than 1, the only acceptable value for the coverage factor in the t-distribution table is k = 13.97 (the lowest tabulated value). Finally, multiplying k by $u_c$ (given in table 5) one obtains the desired expanded uncertainty associated with $Cf_T$, before and after the manufacturing process. These are given in Table 7.

Table 7. Impact induced by the weaving process on the calculated uncertainty associated with $Cf_T$

| | Before Manufacturing | After Manufacturing |
|---|---|---|
| **Calculated $Cf_T$ [nd]** | 0.66 | 0.71 |
| **Expanded Uncertainty, U [nd]** | 0.24 | 0.42 |

[nd] = non-dimensional

Knowing the calculated value of the coverage factor $Cf_T$ and its associated expanded uncertainties after the manufacturing process is completed one can make use of Taguchi's Function to estimate the economic impact resulting from undesirable deviations of it. Table 8 summarizes actual parameters (obtained in the market) needed to estimate the desired economic impact.

Table 8. Market parameters used in Taguchi's function

| Market parameters [unit] and quoted prices in US$ | Magnitude |
|---|---|
| Client tolerance wrt nominal value of $Cf_T$ [nd] | (0.60 ± 0.25) |
| Overall manufacturing cost/m$^2$ [US$/m$^2$] | 1.70 |
| Compliance range related to nominal value of $Cf_T$ [nd] | 0.50 to 0.70 |
| Expanded uncertainty associated to both, LIE and LSE [nd] | ± 0.15 |
| Deviation of $Cf_T$ outside compliance range [nd]* | (+) 0.85 |
| Coverage probability [%] | 95.45 |
| Coverage factor [nd] | 13.97 |
| Degree of freedom [nd] | 1 |
| Width of the manufactured fabric [m] | 2 |
| Monthly fabric manufacturing production [m] | 4,000,000 |
| Relevant deviation for $Cf_T$ (monthly production of fabric, %) | 2 |

nd = non-dimensional; *not exceeding bound limits

Limits of compliance were defined based on technical specifications of the fabric and on the calculated expanded uncertainty. In this calculation it is assumed that: (i) the manufactured fabric area = 8,000,000 m$^2$ (4,000,000 m long and 2 m wide); (ii) only 2% of its length (80,000 m) displayed a significant deviation of $Cf_T$; i.e.: 160,000 m$^2$ (80,000 m long by 2 m wide) and (iii) the deviation of the nominal value of $Cf_T$ remained outside the range of compliance but never exceeded the upper limit of LSE. Values exceeding these limit would not be financially viable.

Based on the above assumptions and making use of the Taguchi loss function [1], it became possible to estimate that the above described control of deviation of the nominal value of $Cf_T$ resulted in an economy of US$ 228,800.00. Calculations have shown that the financial loss due to deviation of the geometric structure of the woven fabric reached US$ 0.27 (for the upper compliance limit) and US$ 1.70 (for the upper limit of specification). From the difference of these values (US$ 1.70 - US$ 0.27 = US$ 1.43), it follows that US$ 1.43 per square meter of fabric applied to the 160,000 square meters of fabric manufactured per month would result in a maximum financial gain of US$ 228,800.00.

## 8. CONCLUSION

The *fractional coverage factor* ($Cf_T$) is a key controlling parameter in the manufacturing of woven fabrics. Little is known of the uncertainty in measurement associated with it. A proper evaluation of the associated expanded uncertainties proved to be a valuable tool to control undesirable deviations of the coverage factor, understood to be an adequate descriptor of the effectiveness of the manufacturing process of woven fabrics.

The application of Taguchi's concept, originally conceived to improve the quality of manufactured goods, has shown to be a useful tool to assess the economic impact of undesirable geometric deviations of threads in the textile industry.

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