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A SIMPLE METHOD OF 2D AND 3D PROFILE FILTERING USING B-SPLINE APPROXIMATION

D. Janecki

Kielce University of Technology, Faculty Mechatronics and Machinery Design,
al. Tysiąclecia PP 3, 25-314 Kielce, Poland, djanecki@tu.kielce.pl

Abstract: The aim of the analysis was to develop an algorithm for one- and two-dimensional filtering of profiles using approximation by means of B-splines. The theory of B-spline functions introduced by Schoenberg and extended by Unser et al. was used. Unlike the spline filter proposed by Krystek, which is described in the ISO standards, the algorithm does not take account of the bending energy of the filtered profile in the functional whose minimization is the principle of the filter. Appropriate smoothness of a filtered profile is achieved by selecting an appropriate distance between the nodes of a spline function. The paper includes examples of separation of 2D and 3D roughness.

Keywords: one and two-dimensional filtering, B-spline functions, 2D and 3D roughness

1. INTRODUCTION

The smoothing and low-pass filtration of measurement data are performed in many areas of science and technology. In surface metrology, filtration is used to separate form, waviness and roughness components. Today, the most common are digital filters because of the application of computer-based measurement methods. The problem of filtration can be easily solved in the analysis of periodic profiles registered at a constant sampling interval. In this case, the most convenient is first to determine a discrete Fourier transform of a profile, and then filter it in the frequency domain according to the required filter frequency characteristic. Filtering non-periodic profiles is definitely more difficult. The major problem is to determine the filtered profile at the ends of the profile. This is due to the fact that the value of the filtered profile at a given point is obtained through averaging of the measured profile in a certain neighborhood of this point. However, at the ends of the profile, the values of the profile lying outside the area measured are not known. In the literature, this problem is called the end problem.

The simplest method to solve the problem is to register a sufficiently long fragment of a profile and use additional end fragments for determining the filtered profile in its central part. Many methods have been developed to determine a filtered profile, also at the profile ends [1]-[4], [9]. Many of them have good theoretical background. In Refs. [1], [2], a variational approach was used. It assumes that the filtered profile minimizes a certain functional made up of two parts. One part is responsible for the approximation of the primary profile by the filtered profile, whereas the other, called bending energy, is to ensure appropriate smoothness of the filtered profile. In Ref. [3], [4], a filtered profile is determined by means of local approximation of the primary profile with a polynomial of degree 1 or higher. In the literature, such filters are called spline and regression filters, respectively, and they are described in ISO standardization documents [5], [6].

Data smoothing can also be achieved by approximating the profile by a linear combination of certain basis functions. In this paper, we assumed that the basis functions were appropriately shifted B-spline functions (B-splines). We used the theory of B-spline functions developed by Schoenberg [7] and extended by Unser et al. [8]. We included examples of the application of the filter to separate 2D and 3D surface roughness.

It should be emphasized that, unlike spline filters described in the standard, the algorithm does not take account of the bending energy of the filtered profile in the functional whose minimization is the principle of the filter. The approximating function is smoothed by selecting an appropriate distance between the nodes of the spline function.

2. PROFILE APPROXIMATION BY MEANS OF B-SPLINE FUNCTIONS

The problem of approximation of the set of points (t_m, x_m) by means of a spline function can be formulated as follows: determine the spline function $s(t)$ that minimizes the functional

$$J = \sum_{m=1}^{M} (x_m - s(t_m))^2 \ . \tag{1}$$

A very convenient and elegant method for constructing spline functions is to apply B-spline functions (B-splines, for short) [7], [8]. B-splines can be defined in a few ways. The definition provided below is a recursive definition using the concept of convolution. Let $\hat{\beta}_0(t)$ be a characteristic function of the interval $[-1/2, 1/2]$, that is:

$$\hat{\beta}_0(t) = \begin{cases} 1 & \text{for } |t| \le 1/2, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

and assume that

$$\hat{\beta}_n(t) = \hat{\beta}_{n-1}(t) * \hat{\beta}_0(t) \text{ for } n = 1, 2, \ldots \tag{3}$$

For example, a cubic B-spline function, which is commonly applied, is described with the following relationship

$$\hat{\beta}_3(t) = \frac{1}{6}\begin{cases} t^3 + 6t^2 + 12t + 8 & \text{for } -2 \le t \le -1, \\ -3t^3 - 6t^2 + 4 & \text{for } -1 \le t \le 0, \\ 3t^3 - 6t^2 + 4 & \text{for } 0 \le t \le 1, \\ -t^3 + 6t^2 - 12t + 8 & \text{for } 1 \le t \le 2, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Fig. 1 shows a plot of the B-spline functions for several initial values of n.

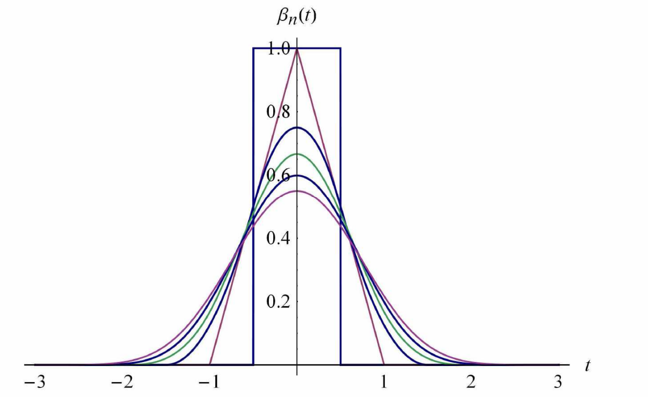

Fig. 1. B-spline functions for $n = 0,1,2,3,4,5$.

It is easy to check that the B-spline function $\hat{\beta}_n(t)$ is a spline function of degree n, with the node-to-node distance $T = 1$. The nodes coordinates equal $\ldots,-2,-1,0,1,2,\ldots$ for odd n, and $\ldots,-3/2,-1/2,1/2,3/2,\ldots$ for even n. Therefore, the function $\hat{\beta}_n(t)$ is also called a normalized B-spline function. The B-spline function with an arbitrary node-to-node distance T is a function in the form

$$\beta_{n,T}(t) = \frac{1}{T}\hat{\beta}_n(t/T) . \quad (5)$$

Each n-th degree spline function determined over the interval $\mathcal{P} = [0, T_f]$ with nodes at the points kT, $k = 1,2,\ldots,K-1$, where $T_f = TK$, can be written as

$$s(t) = \sum_{k=-(n-1)/2}^{K+(n-1)/2} a_k \beta_{n,T}(t - kT) \text{ for odd } n \quad (6)$$

and

$$s(t) = \sum_{k=-n/2}^{K+n/2-1} a_k \beta_{n,T}(t - kT - 1/2) \text{ for even } n . \quad (7)$$

The summation range in Eqs. (6) and (7) was selected in such a way that the shifted basis functions in Eqs. (6) and (7) had non-zero values in the interval $\mathcal{P}$. Thus, for instance, for $n = 3$, the functional (1) can be written as

$$J = \mathbf{a}^T\mathbf{B}\mathbf{a} - 2\mathbf{c}^T\mathbf{a} + d , \quad (8)$$

where $\mathbf{B} \in \mathbb{R}^{(K+3)\times(K+3)}, \mathbf{a},\mathbf{c} \in \mathbb{R}^{K+3}$, d is a scalar, independent of the coefficients a_k, and

$$\mathbf{a} = [a_{-1}\ a_0\ \ldots a_{K+1}]^T , \quad (9)$$

$$\boldsymbol{\beta}(t) = [\beta_{3,T}(t+T)\ \beta_{3,T}(t)\ \ldots \beta_{3,T}(t-(K+1)T)]^T , \quad (10)$$

$$\mathbf{B} = \sum_{m=1}^{M} \boldsymbol{\beta}(t_m)\boldsymbol{\beta}(t_m)^T , \quad (11)$$

$$\mathbf{c} = \sum_{m=1}^{M} \boldsymbol{\beta}(t_m)x(t_m) . \quad (12)$$

Finally, the parameters of the spline function which ensures the best approximation of the measuring points (t_m, x_m) in the sense of a minimum of (1) are as follows:

$$\mathbf{a} = \mathbf{B}^{-1}\mathbf{c} . \quad (13)$$

3. THE PROPERTIES OF THE B-SPLINE APPROXIMATION FILTER

Now, it is essential to determine how to select the distance between the nodes of the spline function, T. It is understandable that the greater the distance between the nodes, the smoother the spline curve. Probably, the smoothness of the curve is also dependent on the degree n. The representation $(t_m, x_m) \to s(t)$ can be treated as a low-pass filter. Another important objective is to answer the question what the cutoff frequency of the defined filter is.

The properties of the filter were analyzed using a Fourier transform. For this purpose, let us consider the interval over which the spline function is defined. We assume that the approximated function is a whole set of real numbers, i.e. $\mathcal{P} = \mathbb{R}$. Moreover, the sampling period is constant and much smaller than the distance between the nodes of the spline function, T. Now we can assume that the approximated function $x(t)$ is continuous. Accordingly, the approximation can be performed by first finding a minimum of the functional:

$$J = \int_{-\infty}^{\infty} (x(t) - s(t))^2 dt \quad (14)$$

with respect to the parameters of the spline function

$$s(t) = \sum_{k=-\infty}^{\infty} a_k \beta_{n,T}(t - kT) .$$

The mapping $x(t) \to s(t)$ defined by a minimum of the functional (14) is a spatially-varying linear system. It can be described with an integral equation

$$s(t) = \int_{-\infty}^{\infty} x(\tau) h_\tau(t - \tau) d\tau , \quad (16)$$

where $h_\Delta(t - \Delta)$ is the system response to a shifted Dirac impulse function $\delta(t - \Delta)$. (In other words, the function $h_\Delta(t)$ is a response of a system in which the nodes of the spline function are shifted by $-\Delta$).

Statement: *The function $h_\Delta(t)$ satisfies the following properties:*

1) the functions $h_\Delta(t)$ and $h_{T/2}(t)$ are even functions,

2) the function $h_\Delta(t)$ is a periodic function with respect to the variable Δ with a period T,

3) $$H_0(\omega) \le |H_\Delta(\omega)| \le |H_{T/2}(\omega)| , \quad (17)$$

where $H_\Delta(\omega)$ is the Fourier transform of $h_\Delta(t)$ as a function of variable t,

4) $$\lim_{n\to\infty} H_\Delta(\omega) = \begin{cases} 1 \text{ for } \omega < \pi/T, \\ 0 \text{ for } \omega > \pi/T, \end{cases} \quad (18)$$

5) $$\lim_{n\to\infty} H_0(\pi/T) = 1 , H_{T/2}(\pi/T) = 0 . \quad (19)$$

Fig. 2 shows plots of families of amplitude characteristics $\left|H_\Delta(\omega)\right|$ for different values of the degree n of the spline curve. We can see that the characteristics resemble a low-pass filter characteristic with the cutoff frequency $\omega_c = \pi / T$, and thus with the length $\lambda_c = 2T$. Therefore, for the predetermined cutoff length λ_c, the distance between the nodes of the spline function should be $\lambda_c / 2$. Note that the higher the degree n, the better the approximation of the ideal filter response. Obviously, one should realize that the higher the degree of the spline functions, the more numerically complex the filter becomes.

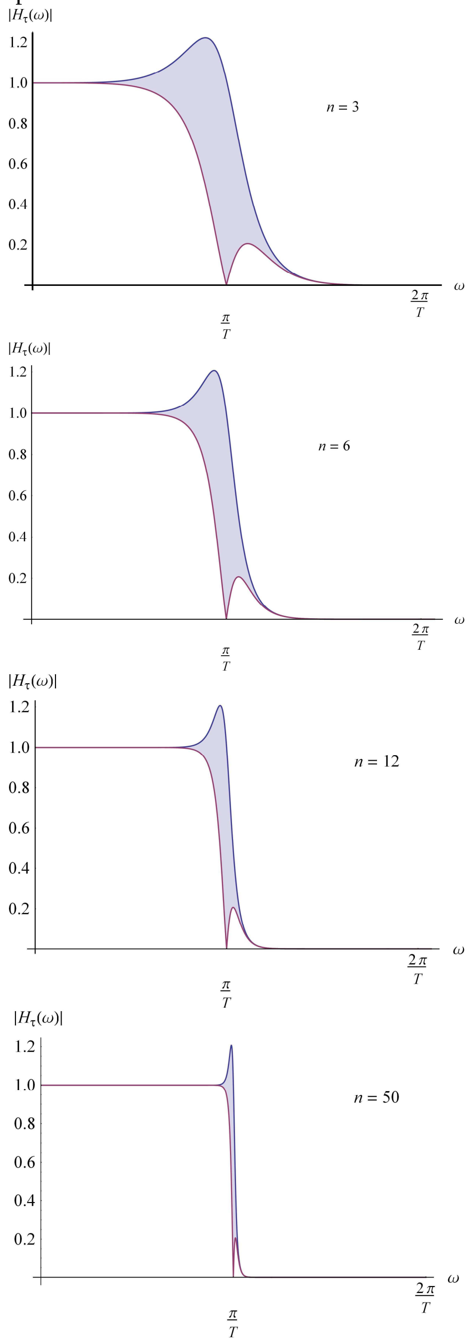

Fig. 2. Families of the filter amplitude characteristics for different filter orders.

5. AN EXAMPLE OF APPROXIMATION OF A 2D AND 3D PROFILE

To illustrate the performance of the approximation algorithm, let us determine, for example, a 2D and 3D roughness profile of the inner ring race of the 608 series ball bearing. Nominally, the race cross-section has the shape of a circle with the radius $r = 2.05$. Here, the spatial variable is denoted by x and the values of the profile by y. In Fig. 3, we can see the measured detail. The surface topography was measured using a contact profilometer with an adjustable table, which enables measurement of 2D and 3D profiles.

Fig. 4 shows a measured profile of a cross-section of the bearing race. The profile was measured using the sampling interval $\Delta x = 1\ \mu\text{m}$. Our objective was to determine the roughness of the central part of the profile, with the filter cutoff length being $\lambda_c = 0.25$ mm, and the profile length $8\lambda_c$. The profile was approximated using cubic spline functions with the node-to-node distance $T = \lambda_c / 2$. Since the profile was sampled at a constant sampling interval, it was assumed that $\eta_m = 0$. Fig. 5 presents the roughness profile obtained by subtracting the measured profile from the approximated profile representing the mean line. We can see that the roughness profile was determined correctly; there was no end effect, despite the fact that the amplitude of the form component of the measured profile was a thousand times higher than the amplitude of the roughness component.

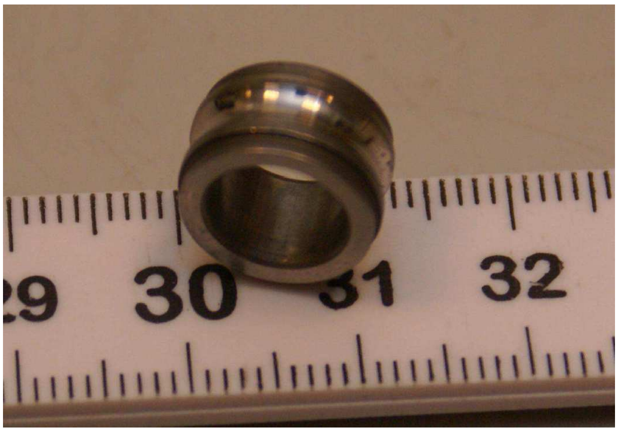

Fig. 3. Measured detail.

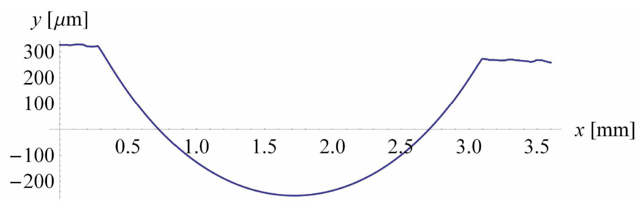

Fig. 4. Cross-section of the bearing race.

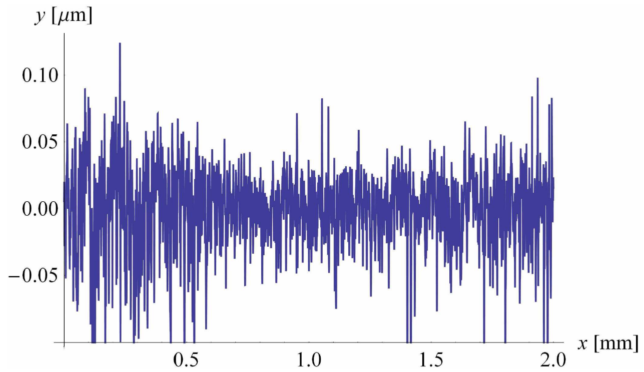

Fig. 5. 2D roughness profile.

Let us, for example, analyze surface roughness of the inner ring race in a ball bearing. The race surface has the shape of a torus with the major and minor radii being $R = 7.36$ mm and $r = 2.05$ mm, respectively. The profile was measured using a contact profilometer with an adjustable table. It was assumed that the sensor needle moves along the X axis, whereas the table enables the object shift along the Y axis. Uniform sampling was performed along the axes X and Y. The sampling intervals along the two directions were $\Delta x = \Delta y = 1\,\mu$m. The length and width of the registered profile fragment were 1.0 mm each. Fig. 6 shows a graph of the registered profile.

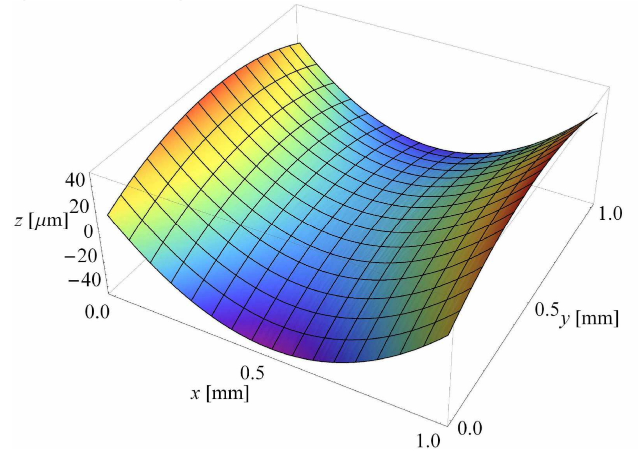

Fig. 6. Registered profile of the inner ring race of a ball bearing.

A three-dimensional third-degree spline surface was used to approximate the profile. It was assumed that $\lambda_{cx} = \lambda_{cy} = 0.25$ mm, thus the distances between the nodes were equal: $X = Y = 0.125$ mm. Note that there are 11 B-spline functions per each of the variables x and y, which gives a total of 121 two-dimensional B-splines and the same number of coefficients of approximation. Fig. 7 shows the corner fragment of the roughness profile $r_m = z_m - s(x_m, y_m)$ with dimensions of one measuring segment. Note that the height of roughness is about 200 times smaller than the height of the measured profile. In spite of that, no end effect is observed. This confirms very good approximation of the form components by means of spline functions.

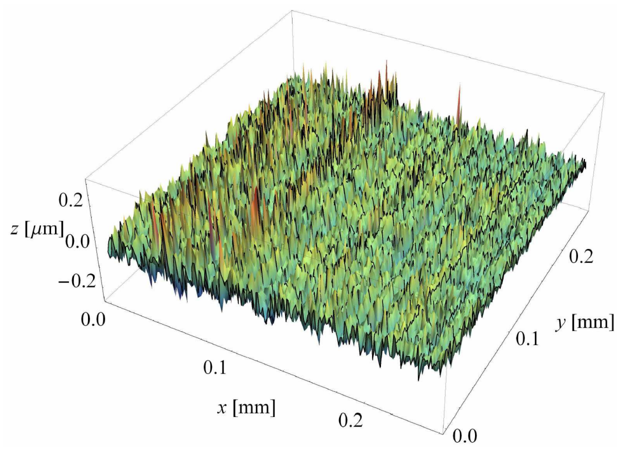

Fig. 7. Roughness profile.

6. CONCLUSION

The study aimed at developing a method for the approximation of measurement data for 2D and 3D profiles using B-spline functions. The smoothness of the lines and surfaces was achieved by selecting an appropriate distance between the nodes of a spline function. It was shown that the node-to-node distance of the spline should be equal to half of the desired cutoff length of the filter. Moreover, it is possible to obtain an arbitrarily large inclination of the frequency characteristic in the transition band if a sufficiently high degree of the spline function is selected.

The paper includes examples of separation of 2D and 3D roughness by means of the developed approximation method using B-spline functions. The experiments show that a roughness profile can be determined correctly, without any end effect, even if the amplitude of the form component of the measured profile is very much higher than the amplitude of the roughness component.

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