

## UNEQUALIZED CURRENTS IN TWO TERMINAL-PAIR COAXIAL CAPACITANCE BRIDGES

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**Abstract:** It is necessary to maintain conditions of zero net currents in the cables of ac coaxial bridges. Current equalizers are used for dealing with this problem. It is shown here how they were applied in the construction of a two terminal-pair capacitance bridge with an uncertainty of about one part in  $10^8$ .

**Keywords:** current equalizers, unequalized currents, coaxial bridges.

### 1. INTRODUCTION

The two terminal-pair coaxial ratio bridge built recently at Inmetro operates mainly at 1 kHz and 1.592 kHz and compares decadic capacitors in the range from 10 pF to 1 nF at the ratios 1:-1, 10:-1, and -1:10, with an uncertainty of about one part in  $10^8$ . The bridge operating principle is discussed in [1].

A simplified scheme of a two terminal-pair coaxial bridge for comparing two standard capacitors of the same nominal value ( $C_N$  and  $C_X$ ) is shown in Fig. 1.  $C_X$  is the capacitor under calibration and  $C_N$  is the reference standard.

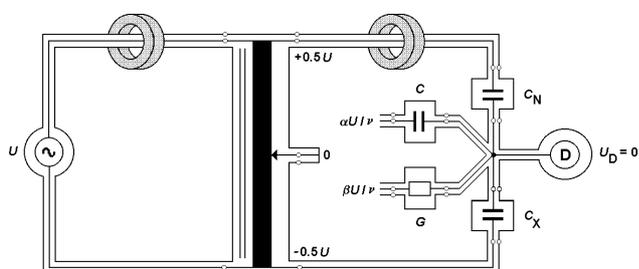


Fig. 1. Simplified scheme of the capacitance bridge.

The main inductive voltage divider (IVD) operates with a voltage  $U$  across the taps. Its actual ratio deviates from the nominal 1:-1 ratio by a small complex amount. Further the

two capacitances are not ideal and do not have precisely the same value. The complex voltage  $U_D$  detected at the node between the two capacitances is therefore not null. The real part of this voltage is then compensated by injecting an adjustable in-phase current. This is done by applying an adjustable voltage  $\alpha U/v$  to capacitor  $C$ . Likewise, the imaginary part is compensated by injecting an adjustable quadrature current. This is done by applying an adjustable voltage  $\beta U/v$  to conductance  $G$ . When the detector voltage is nullified in this way, we obtain the in-phase balance equation

$$C_X = C_N + (\alpha_1 - \alpha_2) \frac{C}{v} + (\beta_1 - \beta_2) \frac{C'}{v} \quad (1)$$

where  $C'$  is the parasitic capacitance (not shown in the figure) that shunts the conductance  $G$ . The dividing factors  $\alpha_1$  and  $\beta_1$  are obtained by balancing the bridge with the capacitors  $C_N$  and  $C_X$  positioned as shown in Fig. 1, and  $\alpha_2$  and  $\beta_2$  are obtained by rearranging the cables so that  $C_N$  and  $C_X$  are interchanged. The complex ratio error of the main IVD is cancelled by this technique.

The constructive details of the bridge built recently at Inmetro are discussed in the next section. The principles for finding the proper arrangement of current equalizers in coaxial bridges to achieve equal return currents in the whole bridge network are described in section 3. The conclusions are drawn in section 4.

### 2. CONSTRUCTION DETAILS

A more detailed scheme of the coaxial ac bridge is shown in Fig. 2 (for clarity, we preferred not to draw the coaxial cable shields). The cable arrangement for the 1:-1 ratio bridge is shown. A 1:-1 ratio bridge is easily changed to a 10:-1 ratio bridge by a cable rearrangement. A photo of the coaxial capacitance bridge is shown in Fig. 3. The bridge comprises an isolation transformer, a two-stage main IVD, a voltage predivider, a T-network box, a battery-operated switch box,

a short-circuit box, and a Wagner current transformer. The following commercial equipment complement the bridge: an ultra-pure sinusoidal oscillator, a wideband power amplifier,

a low-noise preamplifier, a lock-in amplifier, a 1-pF fused-silica standard capacitor and two double, six-decade IVDs.

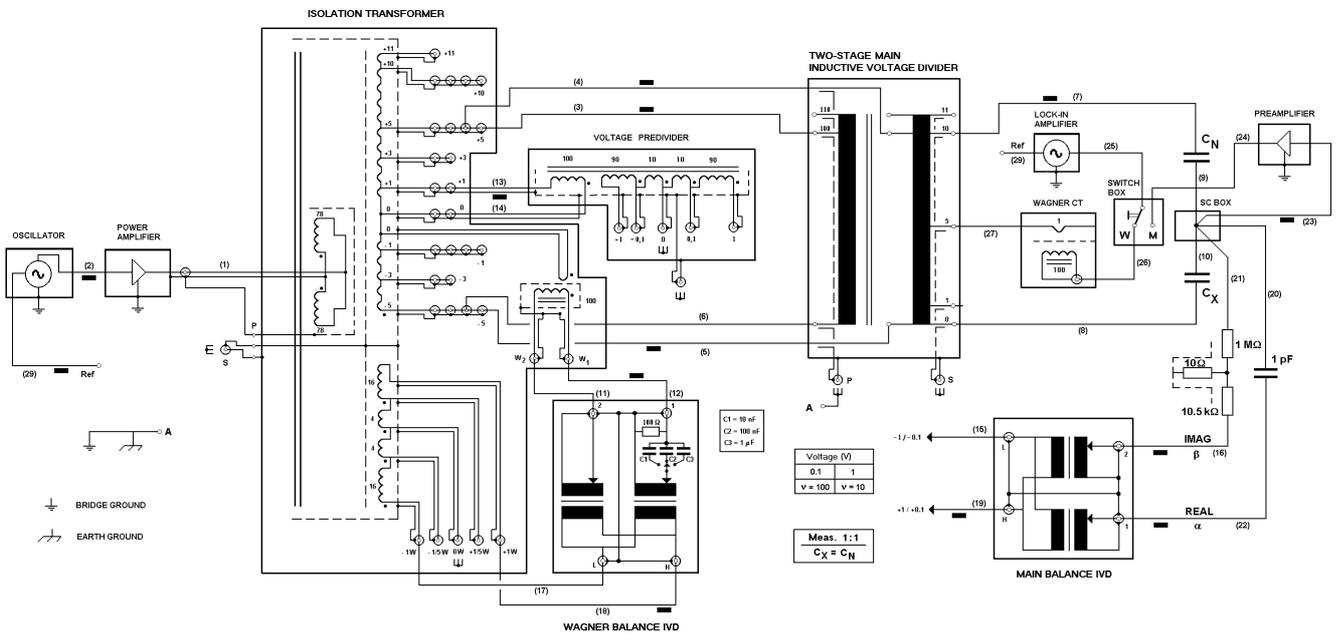


Fig. 2. Detailed scheme of the coaxial capacitance bridge (equalizers shown as black rectangles).

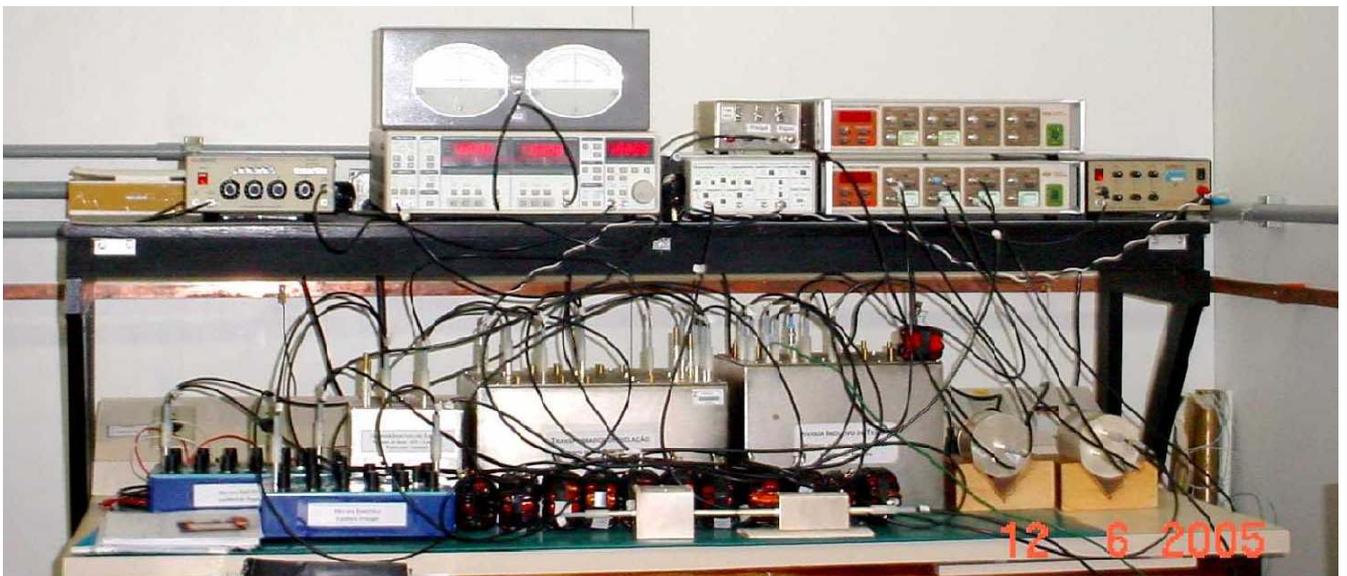


Fig. 3. Two terminal-pair coaxial capacitance bridge (a group of current equalizers is shown at bottom center).

The isolation transformer is used to isolate the bridge from the mains. The two-stage main IVD was constructed to provide an uncertainty in the 1:–1 and 10:–1 ratios of a few parts in  $10^8$ . The specific design and construction details of these devices were already published [2][3].

The sinusoidal oscillator is connected to the power amplifier to apply the required voltage to the isolation transformer primary taps, so that up to 200 V is available across the +5

and –5 secondary taps (when two 10 pF capacitors are compared).

The main balance voltage is derived from the +1 and 0 taps of the isolation transformer secondary winding and reduced by the 10:1 voltage predivider. The voltages at the  $\pm 0.1$  outputs of the voltage predivider, namely  $\pm U/v$  where  $v = 100$ , are applied to double, six-decade IVDs. The adjustable voltages at the outputs of these IVDs, namely  $\alpha U/v$  and  $\beta U/v$  (see also Fig. 1), are then applied to a 1 pF fused silica

capacitor and a 1 nS T-network box (conductance box), thus generating the in-phase and quadrature main balance currents, respectively. The T-network box is built with metal film resistors.

The short-circuit box is a multiport connector in which the inners are brought together at a point, as are the outers. A suitable construction is illustrated in [4] where the inners are soldered symmetrically to a disc. The short-circuit box receives the currents flowing through the standards being compared and the in-phase and quadrature main balance currents. It has also an output port for null voltage detection.

The main balance is adjusted by monitoring the voltage at the short-circuit box while changing the settings of the double, six-decade IVDs. The low-noise preamplifier is cascaded to the lock-in amplifier to monitor the balance voltage. With the preamplifier gain set to  $10^3$ , it has been possible to balance the bridge so that the lock-in amplifier reading is within  $\pm 20 \mu\text{V}$  (which means  $\pm 20 \text{ nV}$  at the short-circuit box).

We have verified that the Wagner balance is crucial to the accuracy of the main balance. The isolation transformer has a separate secondary winding for the Wagner balance [2]. The voltage between the taps of this separate winding is applied to double, six-decade IVDs with selectable fixed impedances and the generated current is injected into the isolation transformer zero tap connection to ground using an injection transformer with a 1:100 ratio (installed inside the isolation transformer frame). This changes the potential between the zero tap and ground. The current in the shorted tap of the main IVD is changed accordingly. A required defining condition for the main IVD is that there should be no current flowing through the shorted tap. The Wagner balance is adjusted by monitoring the current in the shorted tap of the main IVD with the 1:100 Wagner current transformer (see Fig. 2) while changing the taps of the Wagner balance IVDs. It has been possible to balance the Wagner network so that the lock-in amplifier reading is within  $\pm 5 \mu\text{V}$ . The main and Wagner balances are done alternately until the voltage monitored at the short-circuit box and the current monitored at the shorted tap of the main IVD are both nullified (the battery-operated switch box is used to select the balances to be monitored).

### 3. CURRENT EQUALIZATION

The capacitance bridge uses a two terminal-pair coaxial design (see Fig. 1). One way of looking at a coaxial bridge is to see it as two superposed networks. The first of these consists of straightforward meshes of components and the interconnecting wires between them. The second network comprises the shields of the components and the outer, coaxial shield of the connecting cables. The configurations of the two networks are identical and by providing every independent mesh with an equalizing device, the current in the outer shield is constrained to be equal in magnitude and shifted  $180^\circ$  to the current in the components and central conductors. The current in any cable as a whole is zero and no external magnetic field is created. The second network of

shields and cable outer conductors has a low impedance, and it is all at nearly the same potential, so that there is no significant external electric field. This construction has the further advantage that such networks do not respond to fields from external sources. Otherwise, the bridge balance conditions could be affected.

Passive current equalization is achieved by threading a coaxial cable through a high permeability (typically Supermalloy) toroidal core so that core and cable act as a 1:-1 transformer [4]-[7] (Fig. 4).



Fig. 4. Passive current equalizer (a few turns of copper wire were also wound for assessing the equalizer effectiveness – see [4]).

A coaxial circuit with a current equalizer and its equivalent circuit are shown in Fig. 5. The impedance  $z$  of the outer conductor is mainly the copper resistance of the outer conductor which is several orders of magnitude smaller than the impedance  $Z$  of the inner conductor (which includes impedance standards, transformers, etc.). A current equalizer has therefore no direct effect on the current flowing along the inner conductor; it just gives rise to an equal current flowing along the outer conductor which is supplied from the same source as the current in the inner conductor.

Though only one turn is depicted in Fig. 5, the effectiveness of the device can be enhanced by winding the cable through the core  $N$  times. A rule of thumb is to select the number of turns and core dimensions to obtain an inductance in the range 25 mH – 50 mH at 1 kHz. The self-inductance of either conductor of the coaxial cable is given approximately by  $L = \mu_0 \mu N^2 A / l_m$ , where  $\mu_0$  is the vacuum permeability,  $\mu$  is the relative permeability,  $A$  is the cross-sectional area and  $l_m$  is the effective magnetic length of the core.

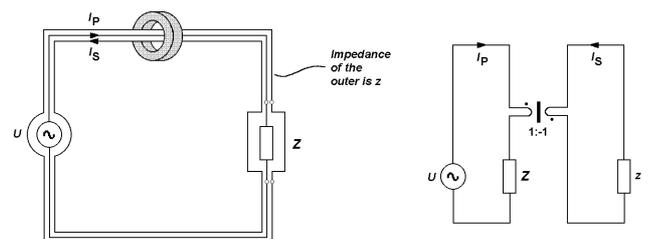


Fig. 5. One mesh of a coaxial network threaded through a core and its equivalent circuit.

The emf induced across the secondary winding of the equalizer is  $(R_L + j\omega L)(I_p - I_s)$ , where  $R_L$  is the resistance related to the core loss,  $\omega$  is the angular frequency and  $I_p$  and  $I_s$  are the primary and secondary currents. It is possible to neglect  $R_L$  since equalizers are often built with 0.0254 mm thick Supermalloy cores. This emf is equal to  $zI_s$ . Therefore,

$$I_p = I_s + zI_s / j\omega L \quad (2)$$

and since  $j\omega L \gg z$  we have  $I_p \cong I_s$  as desired. Incidentally, if the secondary impedance is reflected to the primary, we have  $U \cong I_p(Z + z)$ . In general, in equalized bridge networks, the measured impedance of a two terminal-pair standard is that of the impedance in the circuit of the inner conductor increased by the corresponding small impedance of the outer conductor [4]. Fortunately, this is not a concern for capacitance bridges as the reflected impedance is mainly resistive and one is mainly interested in the in-phase balance (see Eq. (1)).

In order to maximize the effectiveness of the current equalizers, one should analyze the ground network of the bridge to define the correct number of them and their best location. The following steps should be followed: (a) evaluate the number of nodes  $n$  and branches  $b$  of the graph representing the bridge, (b) ensure that the network contains all nodes, that is, the number of components  $c = 1$ , (c) evaluate the number of independent meshes  $m = b - n + c$ . This is the number of current equalizers required. Every independent mesh of the bridge network must be provided with one current equalizer.

A condition needs to be fulfilled: every node should be connected to the central ground point with just one path without a current equalizer. This central point is connected to earth ground. This condition suggests a procedure to fix the arrangement of the equalizers: start with those branches where the presence of an equalizer is to be preferably avoided, that is, the branches which are directly attached to  $C_X$  and  $C_N$  (one equalizer will be required at the loop around the capacitors) and the branches attached to the main detector node. Everything connected here will decrease the effective input impedance of the detector (preamplifier) and therefore affect the signal-to-noise ratio. This is particularly important for the measurement of large impedances. After having chosen those branches without equalizers – they are not allowed to form meshes! – branches will have to be added to the nodes not yet connected, again without forming meshes. In this way one arrives at a complete tree linking all nodes without meshes. This tree contains  $n-1$  branches. The remaining  $m$  branches then must be equipped with current equalizers.

The ground network of the bridge constructed at Inmetro is shown in Fig. 6. The reader may compare it with the scheme in Fig. 2. There are 16 nodes and 28 branches. The network contains all nodes, i.e.,  $c = 1$ . Therefore, a total of 13 current equalizers are needed. It is seen that every node is connected to the central ground point (node 16) with just one path without a current equalizer. The ground tree linking all nodes without meshes is depicted in Fig. 7 for easy reference. The tree contains 15 branches as expected.

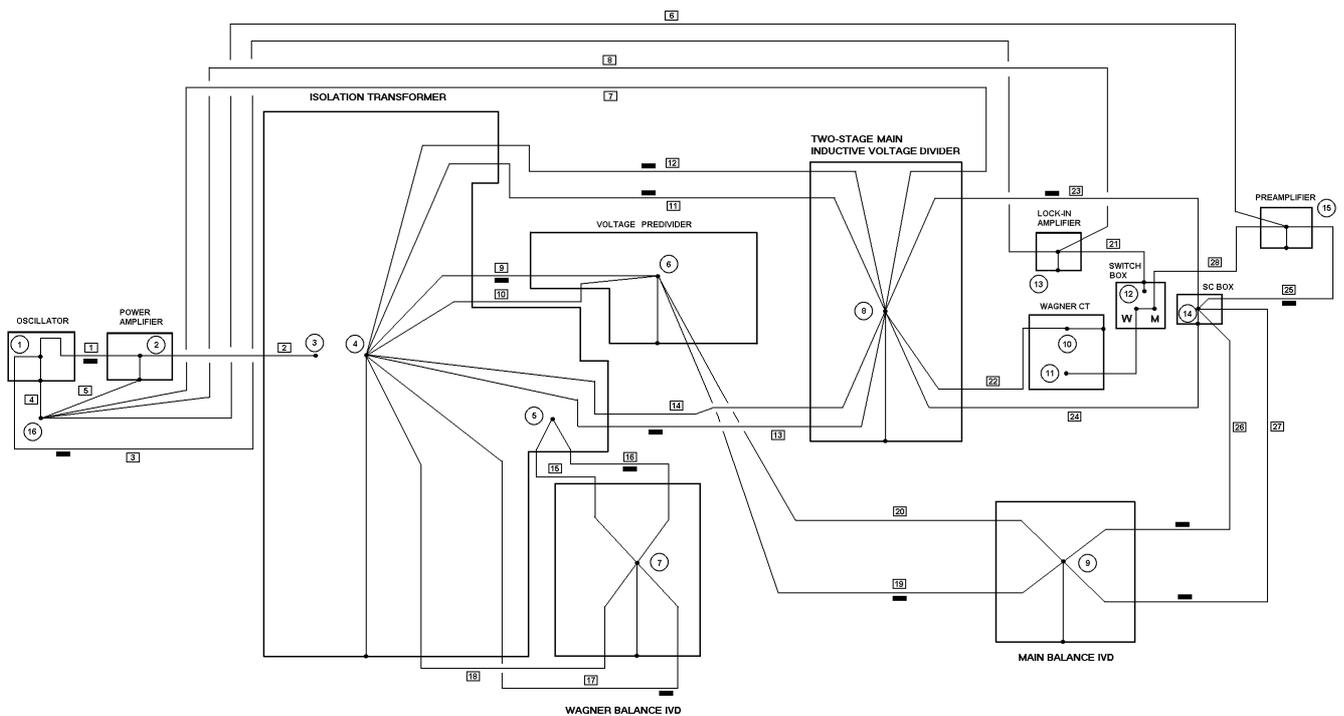


Fig. 6. Ground scheme of the capacitance bridge (equalizers shown as black rectangles).

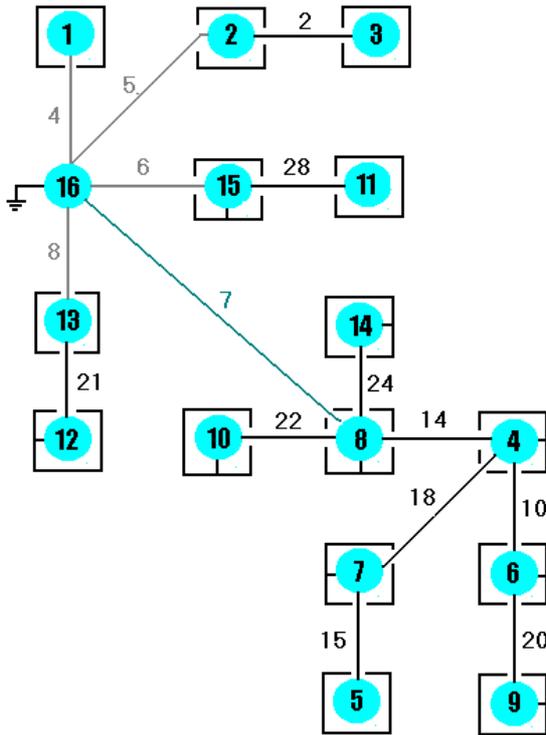


Fig. 7. Ground tree linking all nodes without meshes. Some nodes are connected to the metal boxes. The grounding conductors of ac power cords are represented by gray lines.

An attempt was made to suppress the current equalizer in branch 25 which is attached to the main detector node (node 14). Thus,  $m = 12$  and the  $b$  needs to be reduced to 27. An arrangement that does not change significantly the ground network is to use a battery-operated preamplifier for suppressing branch 6. The arrangement is illustrated in the scheme of Fig. 8. It was verified that both arrangements yield the same results for 10 pF to 1000 pF standards.

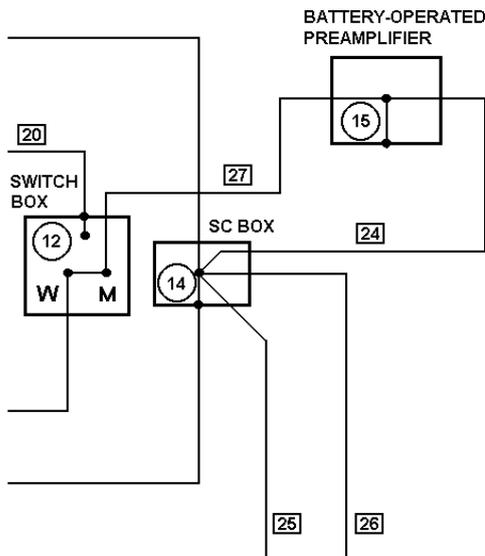


Fig. 8. Partial scheme of the bridge illustrating the arrangement.

#### 4. CONCLUSIONS

It is shown here how passive current equalizers were applied in the construction of a two terminal-pair capacitance bridge with an uncertainty of about one part in  $10^8$ . The construction technique of coaxial bridges was briefly reviewed. The principles for finding the proper arrangement of current equalizers in coaxial bridges to achieve equal return currents in the whole bridge network were described.

#### 5. ACKNOWLEDGMENTS

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