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METALLIC 2-WIRE PARAMETRIC LINE MODELS - A SURVEY

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Abstract: A survey of frequency domain models to describe cables of the access network of operators is presented. The cable characteristics of some commercial cables are measured and the parameters of the used models identified. From this, the properties and application areas of physical models as well as rational functions are discussed. It will be demonstrated and explained why some frequently used cable models show an unrealistic impulse behavior.

Keywords: Transmission lines, cable modelling, access network, identification

1 INTRODUCTION

The access network of an operator contains a variety of cable types, which differ in wire diameter (gauge), twist rate and insulator material such as polyethylene, paper and polyvinylchloride. Most of them are twisted pairs but sometimes parallel wires are used as well, e.g. drop wires. A wire pair can be screened or be located in a cable bundle where it is surrounded by neighboring metallic conductors that act as a screen. Originally, the access network has been installed for telephony services, yet it is being used by broadband modems. Modem manufacturers still expend a lot of effort to increase the data rate and the quality of service. Hereto, cable models are frequently used in order to simulate the throughput of a system. Recently, operators demand single ended line testing (SELT) tools to pre-qualify a subscriber line or use it for plain fault location. Such tools depend on good line models [1].

There exist three groups of cable models. Firstly, one can distinct analytical cable models which are based on the physical geometry and material properties. They facilitate investigation of the electrical behavior when a design parameter, e.g. the wire diameter, is modified. Secondly, a number of models use a mix of physical behavior and some heuristics. These models are described in the standards [2, 3, 4] and in contrast with their unsound mathematical construction, frequently promoted. These semi-empirical models exhibit some inconsistencies which appear as pulse-like fore-runners in the impulse response. A third group are the rational functions, which are very flexible and accurate but not well suited if inherent cable behavior is required such as for SELT-applications [1].

It can be emphasized that the exact twisted pair model has not yet been obtained. However, using a few assumptions, a novel cable model will be introduced for an unshielded and shielded twisted pair. This model can be classified in the first group. It contains a minimal number of parameters and approximates the true frequency response very close.

The characteristics of different cables have been measured, i.e. Belgacom, France Telecom and British Telecom cables, using a network analyzer. The parameters of all models will be identified, the accuracy and approximation behavior discussed and their application domain indicated.

2 PRELIMINARY

Throughout this text $s = j\omega$ and $\omega = 2\pi f$. Also, cable functions such a the series-impedance Z_s and the parallel admittance Y_p (see fig. 1) are considered as per-unit-length quantities throughout this text, unless otherwise stated. The transfer function of a line with length l is given by $H = e^{-\gamma l}$ with $\gamma = \sqrt{Z_s Y_p}$. The characteristic impedance can be obtained with $Z_c = \sqrt{Z_s/Y_p}$. Some elementary physical constants that are used in the subsequent sections are $\mu_0 = 4\pi 10^{-7}$ H/m, $\epsilon_0 = (36\pi)^{-1}10^{-9}$ F/m.

3 PHYSICAL MODELS

3.1 *Kelvin model*

The well-known model for the series impedance $Z_s(\omega)$ of a 2 wire line with cylindrical conductors takes into account the skin effect as investigated by Thomson (Lord Kelvin) around 1887. The problem formulation and solution of it, can be found in any good text book about electromagnetism. This basic model will be named the Kelvin model. Sometimes, a high frequency approximation for the skin-effect is used. It provides simple formulas but can not be

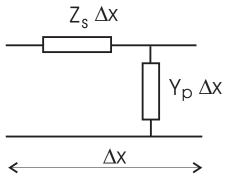

Figure 1: The infinitesimal line section

used to correctly derive impulse responses.

4 HEURISTIC MODELS

4.1 *British Telecom*

The most popular model is the British Telecom model BT0. It is used and described in all xDSL standards. Two BT models were introduced by Cook [4]. By the knowledge of the authors, no prove for these models exist, therefore, they should be considered as empiric. The BT1 model will not be considered because it is over-parameterized when used for plain access network copper cable.

It was found that the series impedance of the BT0 model is non-causal. This is because the resistivity (real part of Z_s) and the reactance (imaginary part of Z_s) were modelled separately without respecting the Cauchy-Rieman conditions for an analytical function. The parallel admittance Y_p of the popular BT0 model is also non-causal except when some parameters of the model are omitted.

The non-causal behavior of the BT0 model was clearly demonstrated in [6]. Physically impossible impulse responses are generated by this BT0 model. Therefore, the authors propose to remove this model from the standards and replace it with a causal cable model.

4.2 *KPN*

The Koninklijke Posterijen Nederland (KPN) models were described by Van Den Brink [4]. No prove was given that those models are derived from the electromagnetic wave theory. Hence, they must be considered as empiric. Two models exists namely KPN0 and KPN1.

Just as the BT0 model, this KPN0 model is non-causal. This is due to the non-causal construction of $Y_p(\omega)$, although $Z_s(\omega)$ is causal. The non-causal behavior was demonstrated in [6].

A more advanced model using the KPN0 model as a basis model resulted into the KPN1 model. Eleven parameters were introduced but after applying KPN1 to measurements, it was concluded that at least 3 parameters should be fixed. The KPN1 model is non-causal. This was also demonstrated in [6].

4.3 *Marconi*

The Marconi MAR1 and MAR2 models were introduced by Musson [4]. By the knowledge of the authors, no in-depth argumentation was provided for these models, therefore, they should be considered heuristic. A model reduction of the MAR1 model was carried out. This reduced model was called MAR2 and contains only 5 free parameters. MAR2 is a causal model. It was numerically verified in [6]. The MAR2 model was successfully applied in Austria to characterize a 50 pair copper cable [9].

5 RATIONAL FUNCTION MODELS

Rational functions are an alternative to model the electrical behavior of transmission lines [5]. The transfer function $e^{-\gamma l}$ and the characteristic impedance $Z_c(\omega)$ can be approximated very accurately with a rational function. The rational function with delay has the following form:

$$H(\Omega) = \frac{\sum\limits_{n=0}^{N} b_n \Omega^n}{\sum\limits_{n=0}^{M} a_n \Omega^n} D(\Omega) \tag{1}$$

If the line transfer function has to be identified, one has to take the delay function $D(\Omega)$ into account. If the characteristic impedance has to be identified, one must set $D(\Omega) = 1$. Three domains were investigated in the past and each domain has its own application area. They are the following:

S-Domain: If $\Omega = s$, then the rational function is operative in the Laplace domain. It uses the Laplace variable s where $s = j\omega$. The delay function is given by $D(\Omega) = e^{-s\tau}$ and $\tau > 0$. If the rational function must be stable, its poles must lie in the left halve plane.

Z-Domain: If $\Omega = z$, then the rational function is operative in the discrete-time domain. It uses the variable z where $z = e^{j\omega T_s}$. The delay function is given by $D(\Omega) = z^{-\tau}$, the delay τ is relative with respect to the sampling period T_s and $\tau > 0$. If the rational function must be stable, its poles must lie within the unit-circle.

Warburg-Domain: If $\Omega = w$, then the rational function is operative in the Warburg domain. It uses the Warburg variable w where $w = \sqrt{s}$. The delay function becomes $D(\Omega) = e^{-w^2\tau}$ and $\tau > 0$. If the rational function must be stable, the absolute values of the angles of the poles must be bigger than 45 degrees.

6 A NOVEL CABLE MODEL (VUB)

A new analytical physical line model 'VUB' will be derived. Firstly, an analysis of the series-impedance Z_s will be made and at the end of this section the parallel admittance Y_p will be treated.

The series-impedance is obtained by truncating the infinite series solution given by Belevitch [7] for the uniform parallel wire system with cylindrical conductors. In fact, the uniformity of a twisted pair transmission line is violated. However, if the twist-period of the pair is small compared with the cross-section dimensions the uniformity is quasi fulfilled. The physical geometry of the 2-wire line

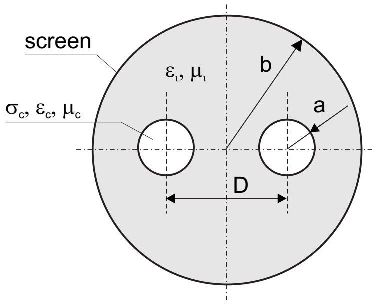

Figure 2: A cross section of a 2-wire line and the corresponding dimensions.

Table 1: Maximum dB-error per km of the transfer function when only 1 term in the series expansion is used

diameter a	ADSL	ADSL+	VDSL
0.4mm	0.006	0.01	0.03
0.5mm	0.01	0.02	0.06
0.6mm	0.02	0.03	0.09
0.8mm	0.04	0.07	0.18
1.0mm	0.07	0.11	0.28

consists of 2 equal cylindrical conductors as shown in figure 2. Both conductors are separated with an insulator. A conducting screen can be present, e.g. in the case of a screened pair.

Workable models are obtained by truncating the series and solving the remaining system analytically [8]. The order of the remaining series depends on the required accuracy. A simulation shows that when only 1 term in the series expansion is used, the model accuracy is sufficient. Consider hereto, an unscreened ($b = \infty$) copper 2-wire line with wire diameter a, wire separation $D = 2(a + t)$, insulation thickness $t = 0.5mm$ and the relative dielectric permittivity of the insulator $\epsilon_r = 2.26$. The transfer function H is calculated for $l = 1km$ for 4 terms in the series expansion, which can be considered as the asymptotic solution in practice, and 1 term of the same series. The deviation between the 2 transfer functions is calculated and shown in table 1. Three frequency bands were used: ADSL (4,3125kHz-1,104MHz), ADSL+ (4,3125kHz-2,208MHz), VDSL (4,3125kHz-12MHz).

Generally, the errors are very small and they increase if the frequency increases. If more accuracy is needed, then the 2-term series can be applied and the maximum error found will drop to $0.02dB$.

After a re-parametrization [8] one can find the following expression for the total series impedance of a line with length l (a perfectly conducting screen was considered):

$$Z_s(\omega)l = a_4 s + a_1\sqrt{-s}\frac{J_0(a_3\sqrt{-s})}{J_1(a_3\sqrt{-s})} + a_1 a_2 a_3 s\psi \qquad (2)$$

$$\psi = \frac{S_1(u)}{a_2(1 + S_2(u)) + \frac{J_0(a_3\sqrt{-s})}{J_2(a_3\sqrt{-s})}} \qquad (3)$$

$$S_1(u) = 1 - \frac{4u}{1 - u^2} \qquad (4)$$

$$S_2(u) = \frac{8u(1 + u^2)}{(1 - u)^2(1 + u)^2} \qquad (5)$$

with

$$\begin{aligned} a_1 &= R_0 \cdot a\sqrt{\mu\sigma} \cdot l & (6)\\ a_2 &= (\frac{a}{D})^2 & (7)\\ a_3 &= a\sqrt{\mu\sigma} & (8)\\ a_4 &= L \cdot l & (9)\\ u &= (\frac{D}{2b})^2 & (10) \end{aligned}$$

The used symbols have the following meaning: R_0 is the DC-resistance of both wires and $J_i(x)$ stands for the complex Bessel function of the first kind of order i. In case a screen is absent, set $b = \infty$ and as a consequence $u = 0$, $S_1 = 1$ and $S_2 = 0$. The external self-inductance L at DC, attributed to the field between the conductors, which are separated by distance D, is given by:

$$L = \frac{\mu}{\pi}(ln(\frac{D}{a}) - ln(\frac{1 + u}{1 - u})) \qquad (11)$$

Remark that the screening of a pair will decrease L. If the conductors are parallel, which means that the line is uniform, then parameter a_4 is not truly an independent parameter. However, an extra degree of freedom was introduced by considering a_4 as an independent parameter because the wire twist, which is not incorporated into the model, will increase the inductance. This unmodelled dynamic in (2) will introduce a modelling error. This modelling error can be reduced if a_4 is considered as an independent variable. If the proximity effect function $\psi = 0$, then VUB model becomes the Kelvin model.

For dielectrics that show very few losses, such as polyethylene, the following model is used for the shunt admittance Y_p of the per-unit-length distributed circuit: $Y_p = G + sC$ with constants G and C the per-unit-length conductance and the per-unit-length capacitance. If higher loss dielectrics are used, such as polyvinylchloride, more advanced models are necessary. Often the following model is used: $Y_p = tan(\delta)\omega C + sC$, with δ the loss angle. However, this model is non-causal! The authors recommend to remove this model from the standards.

7 RESULTS

Four different polyethylene insulated cables have been used:

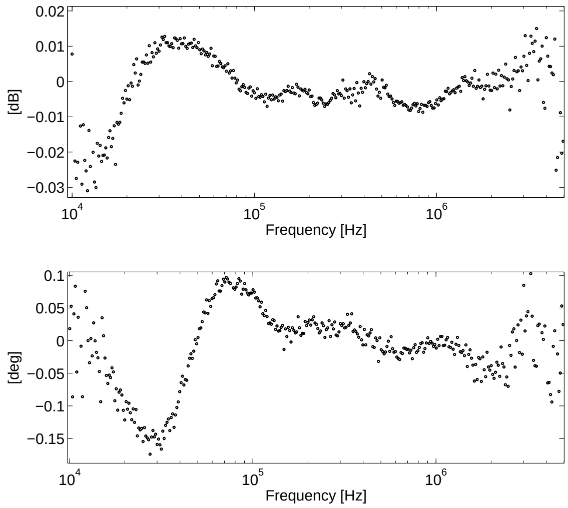

Figure 3: The approximation error of the transfer function of a 400m long 0.4mm France Telecom cable with a 7 by 8 rational function in the s-domain.

- 20 pair 200 m long Belgacom with 0.5 mm wires
- 20 pair 400 m long British Telecom with 0.5 mm wires
- 8 pair 400 m long France Telecom with 0.4 mm wires
- 8 pair 400 m long France Telecom with 0.6 mm wires

The transfer function H_{meas} and characteristic impedance $Z_{c,meas}$ of all pairs in a cable were measured using a network analyzer and applying the 'open/short' technique. The noise on the measurements was measured too. Next, estimators were used to identify the model parameters of the considered models.

When identifying the rational functions a minimax method was used, implemented as an iterative re-weighted weighted least-squares estimator. For high polynomial orders in the rational function, it becomes more difficult to calculate a valid update for the parameters in the iterative minimization procedure. For example, with polynomial orders larger than 10, classic iterative minimizers such as Newton-Gauss, suffer from numerical instabilities. Instead, minimizers that use optimal conditioning should be used. The stability of a rational function can be imposed (except for real poles in the Warburg-domain) using pole reflection or mirroring during minimization, however, the risk of getting stuck in a local minima increases. When the stability is imposed, the errors cannot be brought within the 95%-confidence interval derived from the measurement noise. This is due to small systematic measurement errors, cable non-homogeneities, connector influences, temperature effects etc. If stability is not an issue, then even better approximations are possible. A good set of initial values to start-up the weighted least-squares estimator, is important for not ending up in a local minimum. In particular, the starting value for the delay must be close enough to the true value.

Table 2 shows the maximum approximation errors for the rational functions in the z-, s- and w-domains respectively. The rational functions in the 3 domains have roughly the same performance. All rational functions were stabilized. The transfer functions can be modelled very accurately as can be seen in figure 3. The used numerator/denominator orders amounted to 7/8. The approximation of the characteristic impedance is less good (see figure 4) but this may be completely attributed to non-homogeneities of the cable. Due to the irregular impedance changes, it was decided to keep the orders of the rational function low (3/3).

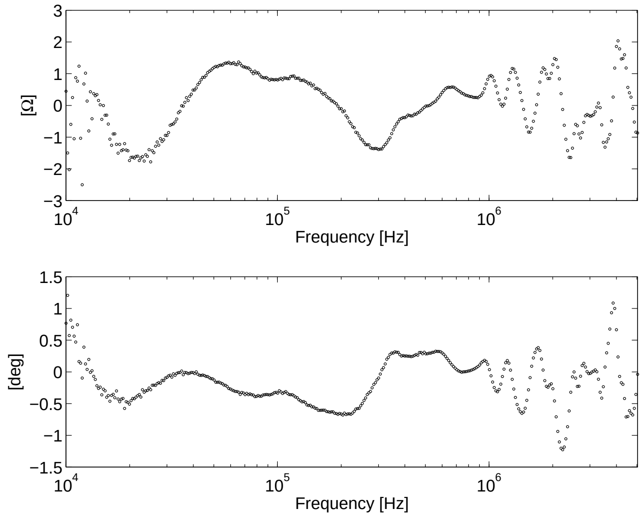

Figure 4: The approximation error of the characteristic impedance of a 400m long 0.4mm France Telecom cable with a 3 by 3 rational function in the s-domain.

Table 2: Rational functions performances

plain	cable	$H_{meas} - H_{model}$ error [dB]	$H_{meas} - H_{model}$ error [degrees]	$Z_{c,meas} - Z_{c,model}$ error [Ω]	$Z_{c,meas} - Z_{c,model}$ error [degrees]
z	Bel 0.5mm	0.01	0.1	2	1
	FT 0.4mm	0.02	0.3	5	3
	FT 0.6mm	0.02	0.2	3	1.5
	BT 0.5mm	0.02	0.2	5	3
s	Bel 0.5mm	0.02	0.1	5	3
	FT 0.4mm	0.03	0.2	2	1
	FT 0.6mm	0.01	0.1	3	1.5
	BT 0.5mm	0.02	0.2	5	3
w	Bel 0.5mm	0.04	0.3	5	2
	FT 0.4mm	0.02	0.2	6	2
	FT 0.6mm	0.01	0.2	2	1.5
	BT 0.5mm	0.03	0.3	5	3

Bel: Belgacom; FT: France Telecom; BT: British Telecom

All (semi-) physical models try to approximate the true behavior as good as possible but modelling errors will exist. In general, these errors appear to be a lot bigger than the errors caused by the noise on the measurements. A fre-

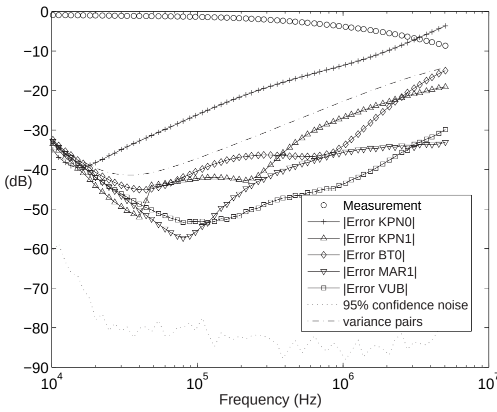

Figure 5: The approximation of the transfer function of a Belgacom 0.5mm cable (200m long) with the physical and semi-empiric models.

quency domain weighted least-squares estimator was applied to identify the parameters of the considered models. The approximation errors of the semi-empiric and the true physical models are larger than the errors obtained when using the rational functions. This could be expected because the number of parameters is limited and fixed, and the quality of the physical models, semi-empiric or not, depends on the build-in behavior of a closed-form analytical expression. Another constatation was that the variance of the measured transfer functions of the pairs that are incorporated in one cable can be larger than the variance of the modelling errors. This is especially true for the 20-pair cables. In practice, bigger cables are installed in the access network (e.g. 200-pair cables). As a consequence, it may be expected that the pair variance (spread) is even larger than what has been noticed in this study. This means that whichever model will be used, it is probably accurate enough to describe the mean multi-pair cable behavior. The approximation quality is shown in figures 5, 6, 7 and 8. In general, the KPN0 model is not flexible enough and cannot be used as such. The more advanced KPN1 model (used with 8 free parameters) only provided results for the Belgacom cable. For the other cables no good approximation was found due to the difficulty to create a good set of starting values. The BT0 model (with 11 parameters) performs well but often manual adjustments were necessary on its starting values. The MAR1 (7 parameters) and VUB (5 parameters) models are both usable models however the starting values are easier to obtain for the VUB model using (6-10).

In table 3 the model properties are summarized. The most accurate models are the rational functions (RF), which can make the approximation error very small, e.g. magnitude errors of less than 0.05dB/km are possible (see fig. 3). The choice of the domain really depends on the application. For example, one will chose the z-domain if line models are needed for a real-time digital line emulator or the s-domain to model the characteristic impedance in

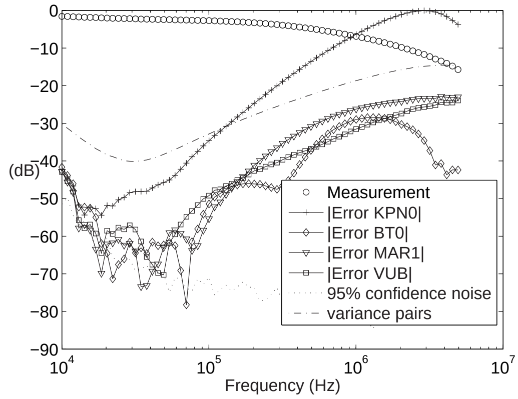

Figure 6: The approximation of the transfer function of a British Telecom 0.5mm cable (400m long) with the physical and semi-empiric models.

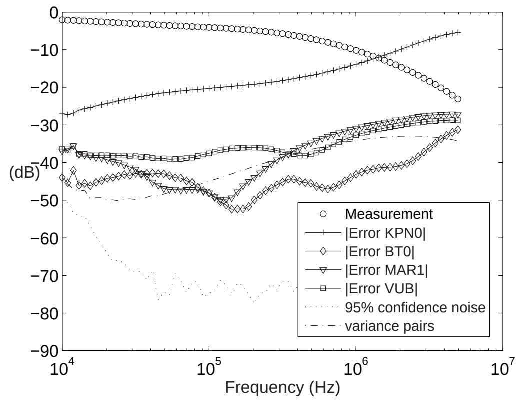

Figure 7: The approximation of the transfer function of a France Telecom 0.4mm cable (400m long) with the physical and semi-empiric models.

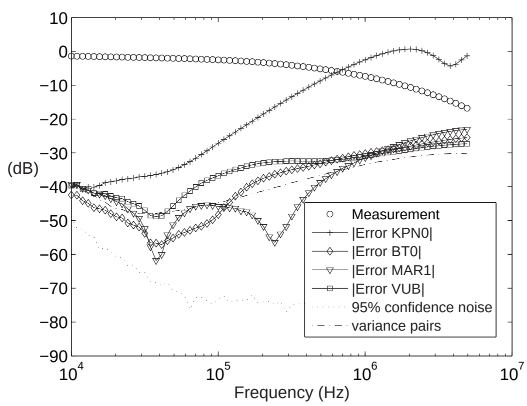

Figure 8: The approximation of the transfer function of a France Telecom 0.6mm cable (400m long) with the physical and semi-empiric models.

case a passive matching circuit has to be designed.

The choice of the semi-empiric or true physical model depends on its use. If it is sufficient to approximate the frequency domain curves, the BT0 and KPN1 models will do the job but these models may not be used to calculate the time domain impulse responses. In figure 9 a number of impulse responses are shown. Apart from the small delay differences (due to the estimation errors), the BT0 model has an unrealistic impulse behavior because the impulse starts too early. The VUB, MAR1 and in this particular case also KPN0 have realistic impulse response shapes. From an estimation point of view, BT0 and KPN1 are difficult to estimate, especially when a good set of initial parameters to start-up the iterative estimator is missing. Generally, a good choice is to use a causal model with a minimal number of parameters such as the VUB or MAR1 model. They are efficient, easy to estimate and are both causal.

Table 3: The cable model properties

model	param.	acc.	conv.	causal	stable	phys.
RF-s	v	vg	s	y	i	n
RF-z	v	vg	s	y	i	n
RF-w	v	vg	s	y	i	n
KPN0	4	g	f	n	y	se
KPN1	8	g	f	n	y	se
BT0	11	g	f	n	y	se
MAR1	7	g	f	y	y	se
Kelvin	4	fg	f	y	y	y
VUB	5	g	f	y	y	y

vg: very good; g: good; fg: fairly good;
s: slow; f: fast; y: yes; n: no
i: imposed; se: semi-empiric; v: variable

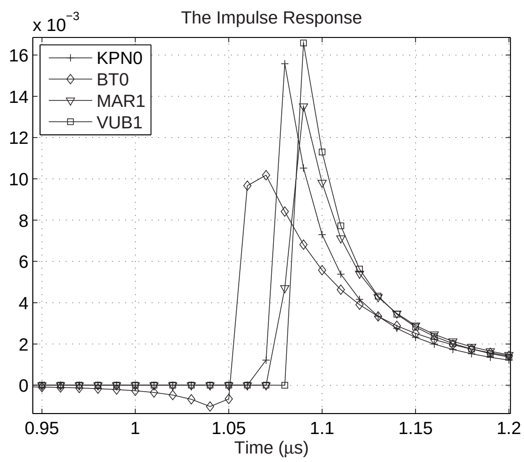

Figure 9: The impulse response of semi-empirical and physical models.

8 CONCLUSION

Transmission line models that are able to characterize the cables of the access network of a telecom operator have been analyzed and classified into three groups. Concerning the physical models, the 5 parameter VUB model is accurate, causal and recommended to describe the average effect of a cable. If a single pair needs to be characterized, then the VUB and Marconi models are well suited or if high accuracy in the frequency domain is needed a rational function is appropriate. Rational functions perform well if enough coefficients are used and they can be stabilized at the expense of an increased, but still small, approximation error.

References

[1] T. Bostoen, P. Boets, M. Zekri, L. Van Biesen, T. Pollet, and D. Rabijns, "Estimation of the Transfer Function of the Access Network by means of 1-Port Scattering Parameter Measurements at the Central Office", *IEEE Journal of Selected Areas in Communication-Twisted Pair Transmission*, Vol. 20, No. 5, june 2002, pp936-948

[2] ANSI T1.413 Issue2 1998, Asymmetric Digital Subscriber Line (ADSL) Metallic Interface

[3] ETSI TS 101 270-1 V1.2.1 (1999-10), Transmission and Multiplexing (TM); Access transmission systems on metallic access cables; Very high speed Digital Subscriber Line (VDSL); Part 1: Functional requirements

[4] R.F.M. Van Den Brink, Cable Reference Models for Simulating Metallic Acces Networks, ETSI STC Permanent Document TM6(97) 02, June 1998

[5] R. Pintelon and L. Van Biesen, Identification of Transfer Functions with Time Delay and its Application to Cable Fault Location, IEEE Transactions on Instrumentation and Measurement, Vol. IM-39, No. 3, pp. 479484, June, 1990.

[6] Heylen, L., J. Musson, Cable models predict phyically impossible behaviour in time domain, ETSI TM6 TD53 Amsterdam, Netherlands, (994t53a0), Nov. 1999.

[7] V. Belevitch, Theory of the Proximity Effect in Multiwire Cables, Part I, Philips Research Reports, Vol. 32, No. 1, pp 16-43, September 1977

[8] P. Boets, "Frequency Identification of Transmission Lines from Time Domain Measurements", *Ph.D. Thesis*, june 1997, Free University of Brussels, Brussels, Belgium

[9] Tomas Nordstrm, "A model for an Austrian PE04 cable", ETSI STC TM6 , TD09 033t09, Sophia-Antipolis, France , 29-3 September, 2003