XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

CURVE-FITTING-ALGORITHM (CFA) AS POWER QUALITY BASIC ALGORITHM

M. Caciotta[1], *F. Leccese*[2], *A. Trifirò*[3]

[1] Dipartimento di Ingegneria Elettronica, Università degli Studi di Roma "Roma Tre", Rome, Italy, caciotta@uniroma3.it
[2] Dipartimento di Ingegneria Elettronica, Università degli Studi di Roma "Roma Tre", Rome, Italy, leccese@uniroma3.it
[3] Dipartimento di Ingegneria Elettronica, Università degli Studi di Roma "Roma Tre", Rome, Italy, tony.trif@tiscali.it

Abstract: Many computing techniques are used to evaluate Power Quality parameters, each showing specific advantages and disadvantages. This paper presents an improvement of Curve Fitting Algorithm (CFA) having good accuracy in the estimation of the signal's power quality parameters.

Keywords: curve-fitting algorithm (CFA), power quality (P.Q.).

1. INTRODUCTION

The sudden changes in power systems, produced by many reasons as the use of large power reactive loads or by unwelcome struck by lightning, can be the cause of malfunctions and often damages of others electrical systems connected to the electrical network.

The economics problems consequents with these kind of technical problems are often very hard to sustain, so it be born the necessity to quantify the effects that can be produced by use of power electrical systems on others electrical systems linked by the electrical network. This yielded the Power Quality and Electrical System Reliability important topic in the studies of power system areas and leaded the International Electrotechnical Commission (IEC) to the definition of parameters that give the measure of the quality of the energy [1,2,3,4,5].

IEC has defined a series of standards to deal with power quality issues. The two most widely referenced standards or guidelines are the IEC EMC series as indicated with the acronym IEC 61000-x-y (with x:1-6 and y:1-7) and the IEEE 1159[6,7]. These guidelines provide the fundamental principles on EMC issues, describe the various definitions and terminologies used in the standards, describe and classify the characteristics of the environment or surroundings where the equipment will be used, provides guidelines on compatibility levels for various disturbances, define the maximum levels of disturbances caused by equipment or appliances that can be tolerated within the power system, define the immunity limits for equipment sensitive to EMC disturbances, provide indications on the design of equipment for measuring and monitoring power quality disturbances outlining the equipment testing procedures to ensure compliance with other parts of the standards, give hints on the installation techniques to minimise emission as well as to strengthen immunity against EMC disturbances, describe the use of various devices for solving power quality problems, give the specific standards to a certain category of equipment or for certain environments including both emission limits and immunity levels standards.

The IEC's guidelines identify thirteen parameters to quantify the quality of the electrical energy:

1. Power frequency;
2. Magnitude of the supply voltage;
3. Supply voltage variations;
4. Rapid voltage changes;
5. Supply voltage dips;
6. Short interruptions of the supply voltage;
7. Long interruptions of the supply voltage;
8. Temporary power frequency overvoltages between live conductors and earth;
9. Transient overvoltages between live conductors and earth;
10. Supply voltage unbalance;
11. Harmonics voltage;
12. Interarhmonics voltage;
13. Mains signaling voltage of the supply voltage.

Each one of these entities faces and determines a part of the problems that may be found in a context so wide.

We would want that the waveform presents on electrical network should be a perfect sine wave, but the presence of the effects of large sudden changes in the net produce significant deformations in the wave. The thirteen parameters, defined by the IEC, quantify these deformations and allow us to determine the quality of the energy of the electrical signal under control.

Technically speaking, to have these parameters, first of all, it is necessary to measure the signal. A probe able to pick up the electrical signal in real time and with a great level of confidence face this problem. After which it is necessary to study signal's characteristics extracting information from it. For this purpose, and so to valuate the quality of energy, there are many different approaches like FFT, applications of adaptive filters, artificial neural networks, singular value decomposition (SVD), higher order spectra, prony model and min-norm model [8,9,10,11,12,13,14,15,16,17,18]. The aim of all these techniques is to obtain the spectrum of the signal. For examples prony model and min-norm method present high-

resolution estimated spectrum but their accuracy are strongly dependent by signal distortion, the sampling window and the number of samples taken into the estimation process, moreover the computational complexity is more than FFT algorithm that is the point of reference and comparison for the other techniques because it has been used in Power as well as most other engineering fields and, therefore, it is the most common technique for system spectrum estimation [1,8,9,14]. Others techniques, like applications of adaptive filters and artificial neural networks, operate adequately only in the narrow range and at moderate noise levels moreover present bad resolution [12]. Others, like singular value decomposition (SVD), are clearly superiors than FFT but the computational effort is so hard that are suitable only for offline analysis of recorded waveforms absolutely unsuitable to satisfy the necessity to monitor the signal in real time [13]. FFT is the algorithm most widely spread throughout the power system field and permits a convenient assessment of magnitude and phase information. Nevertheless all system spectrum estimations based on Fourier transform request special care due to possible problems with aliasing, spectral leakage and picket-fence that could involve an incorrect version of the spectrum [5]. These performance limitations are particular troublesome when analysing short data records, which occur frequently in practice, because many measured process are brief. The use of windowing techniques alleviates these problems trying of minimise sidelobe's levels, and consequently making as narrow as possible the mainlobe width of the spectrum frequency of the window function used. In this way it includes only the spectral of interest, with minimal sidelobes levels to reduce the contribution from interfering spectral components. In fact, if the magnitude of the window function is reduced towards zero at the boundaries, any discontinuity in the original waveform is weighted to a very small value and thus the signal is effectively continuous at the boundaries. This implies a more periodic waveform which has more discrete frequency spectrum [5]. Anyway it is easy to assert that FFT techniques present high computational cost and don't permit to have measurements in real time, aspect fundamental in our application.
Least squares fitting, or its variant absolute value, can be applied to extract harmonic information without the drawbacks necessary for FFT algorithms and them particularly suitable for real time application.

1. CFA

Curve-Fitting Algorithm is another techniques to verify power quality voltage parameters [5,19,20,21,22,23,24]. It uses least square error estimation to find the magnitude and phase of the signal frequencies. Curve fitting selects the best fit of a curve to a waveform and measures the discrete residual values between the waveform and the fitted curve. In the least squares method, the size of these residuals is measured by the sum of their squared values. This is then minimised to obtain the least squared error, and the amplitude and phase of the best fitted curve calculated. Least squares curve fitting has both computational and theoretical advantages over Fourier processing. A least squares curve-fitted approach is most useful periodicity clearly exists in the data like in our case. Also of particular advantage is that in a curve-fitted approach, it is not necessary to truncate data exactly every period as with Fourier Transform[5].

The aim of this work is to present an improvement of curve-fitting algorithm approaches that allow to describe power quality in real time and with more accuracy.

2.1. Modified CFA

In scientific literature Curve-Fitting Algorithm assumes a fixed sampled window, since 0 until T, and operates minimizing the integral Θ with respect to A and B:

$$\Theta = \int_0^T \left[y(t) - A\sin(\omega t) - B\cos(\omega t)\right]^2 dt \tag{1}$$

where y(t) is the sampled function, Θ is the summation of square differences from a sinusoidal signal positioned with a phase $\varphi = a\tan A/B$ fixed in the 0-2π range with respect to the window. With the methodology of Lagrange multipliers, it is possible to implement the well known best fit procedure, considering the frequency f related to T by the $\omega = 2\pi f = 2\pi / T$.
The first statement to face is the real value of frequency that must be equal to the settled one. It can be calculated developing the equation 1, we obtain that ω is:

$$\omega_{calculated} = \frac{1}{2}\frac{\left[\int_0^T y(t)\cos(\omega t)dt\right]^2 - \left[\int_0^T y(t)\sin(\omega t)dt\right]^2}{\left[\int_0^T y(t)\cdot t\cdot\cos(\omega t)dt\right]\cdot\left[\int_0^T y(t)\sin(\omega t)dt\right] - \left[\int_0^T y(t)\cdot t\cdot\sin(\omega t)dt\right]\cdot\left[\int_0^T y(t)\cos(\omega t)dt\right]} \tag{2}$$

In the (3) equation $\omega_{calculated}$ must be equal to ω, so this value will be modified till obtain the equality: this justifies the *modified* adjective in CFA
This procedure may be obtained using a cyclic procedure sweeping the frequency, that is heavy to compute.
This work present a method that avoid the cyclic calculation and to obtain directly the correct estimation of ω.
To obtain it, we can rewrite eq. 3 in the generic relation:

$$\omega_{calculated} = \frac{Num(\omega,T)}{2\cdot Den(\omega,T)} \tag{3}$$

where obviously:

$$\omega_{calculated} \neq \omega \;\; ; \;\; \omega = \frac{2\pi}{T}$$

So, the difference among the real frequency and calculated one is indicated by Δω and developing by means of Taylor series numerator and denominator of (3) we can write the next relation:

$$\omega_{cal} + \Delta\omega = \frac{Num + \frac{\partial Num}{\partial\omega}(\omega_{cal} - \omega + \Delta\omega) + \frac{\partial^2 Num}{\partial\omega^2}\frac{(\omega_{cal} - \omega + \Delta\omega)^2}{2} + \frac{\partial^3 Num}{\partial\omega^3}\frac{(\omega_{cal} - \omega + \Delta\omega)^3}{6}\cdots}{2\left[Den + \frac{\partial Den}{\partial\omega}(\omega_{cal} - \omega + \Delta\omega) + \frac{\partial^2 Den}{\partial\omega^2}\frac{(\omega_{cal} - \omega + \Delta\omega)^2}{2} + \frac{\partial^3 Den}{\partial\omega^3}\frac{(\omega_{cal} - \omega + \Delta\omega)^3}{6}\cdots\right]} = \frac{Num}{2\cdot Den} = \omega_{rea} \tag{4}$$

Developing the previous relation, we obtain the follow g-grade general equation:

$$\sum_{i=0}^{g} a_i (\delta\omega)^i = 0 \tag{5}$$

where is possible to define the "a_i" coefficients like:

$$a_i = \frac{2\omega}{i!}\frac{\partial^i Den}{\partial \omega^i} + \frac{2}{(i-1)!}\frac{\partial^{n-1} Den}{\partial \omega^{n-1}} - \frac{1}{i!}\frac{\partial^i Num}{\partial \omega^i} \qquad (6)$$

In the appendixes 1 and 2 there are the determination of the $\frac{\partial^i Den}{\partial \omega^i}$ and the $\frac{\partial^i Num}{\partial \omega^i}$.

The g value is a choice linked to the accuracy required by CEI EN 50160, that in this case must be chosen equal to 3, that means the resolution of a third grade algebraic equation.

When work frequency is fixed, it's simple to calculate system's harmonics replacing δω with ω and it is possible to demonstrate that system's interarhmonics are estimable using following relations:

$$\alpha(T,\omega_2,\omega_1) = \frac{\mathrm{H}(T,\omega_2,\omega_1)\cdot \mathrm{A}(T,\omega_2,\omega_1) - \Gamma(T,\omega_2,\omega_1)\cdot \mathrm{B}(T,\omega_2,\omega_1)}{\mathrm{H}(T,\omega_2,\omega_1)^2 - \Gamma(T,\omega_2,\omega_1)^2} \qquad (7)$$

$$\beta(T,\omega_2,\omega_1) = \frac{\mathrm{H}(T,\omega_2,\omega_1)\cdot B(T,\omega_2,\omega_1) - \Gamma(T,\omega_2,\omega_1)\cdot \mathrm{A}(T,\omega_2,\omega_1)}{\mathrm{H}(T,\omega_2,\omega_1)^2 - \Gamma(T,\omega_2,\omega_1)^2} \qquad (8)$$

$$A = \sqrt{\alpha^2 + \beta^2} \qquad (9)$$

where A is the means square root of interarhmonics fields and H(T, ω_2 , ω_1), Γ(T, ω_2 , ω_1), A(T, ω_2 , ω_1) and B(T, ω_2 , ω_1) are respectively:

$$\mathrm{H}(T,\omega_2,\omega_1) = \frac{1}{2(\omega_2-\omega_1)^2}\left\{ \begin{aligned} &\frac{\sin(2\omega_2 T) + \sin(2\omega_2 T) - 2\sin[(\omega_1+\omega_2)T]}{T} + \ln\frac{(\omega_1+\omega_2)^{2(\omega_1+\omega_2)}}{(2\omega_1)^{2\omega_1}(2\omega_2)^{2\omega_2}} + \\ &+\frac{T^2}{4}\left[(2\omega_1)^3 + (2\omega_2)^3 - 2(\omega_1+\omega_2)^3\right] - \frac{T^4}{4}\left[(2\omega_1)^5 + (2\omega_2)^5 - 2(\omega_1+\omega_2)^5\right] \end{aligned} \right\} \qquad (10)$$

$$\Gamma(T,\omega_2,\omega_1) = \frac{1}{(\omega_2-\omega_1)^2}\left\{ \begin{aligned} &\frac{2\cos(\omega_1 T)\cos(\omega_2 T) - 1 - \cos(2\omega_1 T) - \cos(2\omega_2 T)}{T} + \\ &+(3\omega_1 - \omega_2)T + \frac{(3\omega_1^3 + \omega_2^3 + 6\omega_1\omega_2)T^3}{18} + \cdots \end{aligned} \right\} \qquad (11)$$

$$\mathrm{A}(T,\omega_2,\omega_1) = \frac{\omega_2}{(\omega_2-\omega_1)}\int_0^T y(t)\left[\frac{1-\cos(\omega_2 t)}{\omega_2 t}\right]dt - \frac{\omega_1}{(\omega_2-\omega_1)}\int_0^T y(t)\left[\frac{1-\cos(\omega_1 t)}{\omega_1 t}\right]dt \qquad (12)$$

$$\mathrm{B}(T,\omega_2,\omega_1) = \frac{\omega_2}{(\omega_2-\omega_1)}\int_0^T y(t)\frac{\sin(\omega_2 t)}{\omega_2 t}dt - \frac{\omega_1}{(\omega_2-\omega_1)}\int_0^T y(t)\frac{\sin(\omega_1 t)}{\omega_1 t}dt \qquad (13)$$

If interarhmonics fields are n, the calculus of the last two integral is hard because it is necessary to repeat it 2n times, but, if interarhmonics fields are contiguous, the calculus is more simple because it is repeated only n+1 times.

2.2. Uncertainty in Modified CFA

The uncertainty linked to the cutting off of the series at the third terms is about 0.2% [25].

To compute the accuracy for each of previous integrals in A and B terms, we must consider the uncertainty linked to sampling and that linked with numerical integration.

In the integral calculation, uncertainty linked to samples is half a quantum or ½ q.

With n independent samples by the Central Limit Theorem we must have: $\sigma = \frac{1}{2}q\sqrt{n}$ derived by the Gauss Error Theory that assures $\sigma = \eta\sqrt{n}$.

We consider a pound in our integrals varying by the integrand: "*sin c α*", "*α sin c α*", "*α^2 sin c α*" and "*α^3 sin c α*". The definite integral extended to a whole period furnishes

$$p_{s_i} = \frac{1}{T}\int_0^T t^i\left|\sin(\omega t)\right|dt \;\; ; \;\; p_{c_i} = \frac{1}{T}\int_0^T t^i\left|\cos(\omega t)\right|dt \qquad (14)$$

Generically is possible to write:

$$p_{S_i} = \frac{1}{2\pi}\frac{1}{\omega^{i+1}}\left(\int_0^{\pi}\alpha^i \sin\alpha d\alpha - \int_{\pi}^{2\pi}\alpha^i \sin\alpha d\alpha\right) \qquad (15)$$

$$p_{C_i} = \frac{1}{2\pi}\frac{1}{\omega^{i+1}}\left(\int_0^{\frac{\pi}{2}}\alpha^i \cos\alpha d\alpha - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\alpha^i \cos\alpha d\alpha + \int_{\frac{3\pi}{2}}^{2\pi}\alpha^i \cos\alpha d\alpha\right) \qquad (16)$$

Developing calculations, uncertainty linked to quantization errors are given by:

$$\Delta\left(\int_0^{2\pi} t^i \sin(\omega t)dt\right) = \frac{A2^{-(n+1)}\sqrt{n}k_i}{\omega^{i+1}} \qquad (17)$$

with

$k_0 = 2/\pi$, $k_1 = 2$,

$k_2 = (3\pi^2 - 4)/\pi \approx 8.65$, $k_3 = (5\pi^2 - 12) \approx 38$

$$\Delta\left(\int_0^{2\pi} t^i \cos(\omega t)dt\right) = \frac{A2^{-(n+1)}\sqrt{n}y_i}{\omega^{i+1}} \qquad (18)$$

with $y_0 = 2/\pi$, $y_1 = 2$,

$y_2 = (5\pi^2 + 4\pi - 8)/2\pi \approx 9.1$, $y_3 = (7\pi^2 + 6\pi + 24)/2 \approx 64.5$.

To determine the uncertainty on numerical integration, considered as a "rectangular series" we have:

$$\varepsilon_a \le \frac{b-a}{2}Mh \qquad (19)$$

being b-a the integration limits equivalent to T, h integration interval linked each other by the T/n, where M is the maximum value of the first derivative in the integration interval:

$$M \ge \left|f'(x)\right| \text{ in b-a} \qquad (20)$$

$$\varepsilon_a \le \frac{T^2}{2n}M \qquad (21)$$

To evaluate M we consider conditions:

$$S_i(t) = \int_0^t y(\tau)\tau^i \sin(\omega\tau)d\tau \;\; ; \;\; C_i(t) = \int_0^t y(\tau)\tau^i \cos(\omega\tau)d\tau \qquad (22)$$

The integrand, derivative is :

$$f_S'(t) = y'(t)S_i(t) + y(t)S_i'(t) \; ; \; f_C'(t) = y'(t)C_i(t) + y(t)C_i'(t) \qquad (23)$$

By general consideration of behavior, the value of the maximum of derivatives S'(t^*) and C'(t^*) will be that allows S''(t^*) = 0 and C''(t^*) = 0.

Supposing we have y(t) = A/2 sin (ωt + α), is possible to obtain t^* that furnishes the maximum, as an example, of functions $f_S'(t)$, obtaining

$$f_S'(t) = \frac{A}{2}\omega\cos(\omega t^* + \alpha)S_i(t^*) + \frac{A}{2}\sin(\omega i^* + \alpha)S_i'(t^*) \qquad (24)$$

and

$$\frac{\partial f_S'(t)}{\partial \alpha} = 0 = -\frac{A}{2}\omega\sin(\omega t^* + \alpha)S_i(t^*) + \frac{A}{2}S_i'(t^*)\cos(\omega c^* + \alpha) \qquad (25)$$

that furnishes:

$$\sin(\omega i^{*}+\alpha)=-\frac{S_i^{'}(t^{*})}{\sqrt{\omega^{2}S_i^{2}(t^{*})+S_i^{'2}(t*)}} \tag{26}$$

and in similar way for $f_C^{'}(t)$, we have:

$$\cos(\omega o^{*}+\alpha)=\frac{\omega S_i(t^{*})}{\sqrt{\omega^{2}S_i^{2}(t^{*})+S_i^{'2}(t*)}} \tag{27}$$

It must result for sin function :

$$f_S^{'}(t^{*})=\frac{A}{2}\frac{\omega^{2}S_i^{2}(t^{*})-S_i^{'2}(t*)}{\sqrt{\omega^{2}S_i^{2}(t^{*})+S_i^{'2}(t*)}} \tag{28}$$

while for cosine function:

$$f_C^{'}(t^{*})=\frac{A}{2}\frac{\omega^{2}C_i^{2}(t^{*})-C_i^{'2}(t*)}{\sqrt{\omega^{2}C_i^{2}(t^{*})+C_i^{'2}(t*)}} \tag{29}$$

Simple considerations allows to demonstrate that t^{*} can be determined, in the sin case as:

$$\mathrm{tg}\left(\omega\, t^{*}\right)=\frac{2\, i\, \omega t^{*}}{(\omega\, t^{*})^{2}-i(i-1)} \tag{30}$$

while, in the cosine case, the relation became:

$$\mathrm{tg}\left(\omega\, t^{*}\right)=\frac{i(i-1)-(\omega\, t^{*})^{2}}{2\, i\, \omega t^{*}(\omega\, t^{*})^{2}} \tag{31}$$

The integral error became:

$$\varepsilon_{(s,c)_i}\le\frac{T^{2}}{2n}f_{(s,c)_i}(t*) \tag{32}$$

with $f_{(S,i)} = S_i$; $f_{(C,i)} = C_i$;

$$\varepsilon_{(s,c)_i}\le\frac{T^{2}}{2n}\frac{A}{2}\frac{\omega^{2}f_{(s,c)_i}(t^{*}_{(s,c)_i})-f^{'2}_{(s,c)_i}(t^{*}_{(s,c)_i})}{\sqrt{\omega^{2}f^{2}_{(s,c)_i}(t^{*}_{(s,c)_i})-f^{'2}_{(s,c)_i}(t^{*}_{(s,c)_i})}} \tag{33}$$

By the underlining of ω, we can obtain:

$$\frac{\varepsilon_{(s,c)_i}}{A}\le\frac{(2\pi)^{i+1}}{4n\omega^{i+1}}H_{S,C_i} \tag{34}$$

being $H_{S,0}$ = 1,000; $H_{S,1}$= 1,135; $H_{S,2}$= 0,700; $H_{S,3}$= 0,104; $H_{C,0}$= 1,000; $H_{C,1}$= 0,192; $H_{C,2}$= 0,903; $H_{C,3}$= 1.000.
General expression of uncertainty related to the computing of each integral concerning our problem can be expressed as

$$\Delta\begin{pmatrix}S_i\\C_i\end{pmatrix}=(\omega)^{i+1}\left[2^{-(n+1)}\sqrt{n}K_{(S,C)_i}+\frac{(2\pi)^{i}}{4n}H_{(S,C)_i}\right] \tag{35}$$

If we synthesize the expression (3) of ω as:

$$\omega=\frac{1}{2}\frac{C_o^{2}-S_0^{2}}{(S_0C_1-C_0S_1)}=\frac{Num}{Den} \tag{36}$$

it is possible to synthesize the uncertainty of ω as:

$$\Delta\omega=\frac{(2C_0\Delta C_0+2S_0\Delta S_0)}{Den}-\frac{Num(C_1\Delta S_0+S_0\Delta C_1+C_0\Delta S_1+S_1\Delta C_0)}{Den^{2}} \tag{37}$$

In terms of relative uncertainty:

$$\frac{\Delta\omega}{\omega}=\Delta C_0\left(\frac{2C_0}{Num}+\frac{S_1}{Den}\right)+\Delta S_0\left(\frac{2S_0}{Num}+\frac{C_1}{Den}\right)+\Delta C_1\frac{S_0}{Den}+\Delta S_1\frac{C_0}{Den} \tag{38}$$

In many calculations this value doesn't exceed tens of ppm.

3. FIRSTS EXPERIMENTAL RESULTS

The firsts experimental results to valuate the sensibility and the accuracy of CFA have been obtained by a new instrument able to determine in real time the thirteen quality parameters. This instrument has been developed "ad hoc" for this purpose because in commerce there isn't an instrument able to satisfy all the characteristics reached. It is composed by a PC with an acquisition card that pick up an electrical signal directly on the electrical network by means of opportune transducers. The acquisition frequency is 128 KHz corresponding to 2560 samples that define a buffer of 0.02 s. Each buffer is then analyzed by CFA program that is able to extract the information.

Table 1 shows the value of the fundamental for each 0.02 s buffer with own uncertainty.

Tab. 1: Frequency variation obtained with CFA algorithm

Time Buffer (ms)	Value (HZ)	Uncertainty
0-20	49,98052	0.0212%
20-40	49,99997	0.0132%
40-60	49,98052	0.0070%
60-80	49,98052	0.0117%
80-100	49,98052	0.0001%
100-120	49,99997	0.0002%

Figure 1 shows the behavior of the fundamental frequency for a 0.12 s time window.

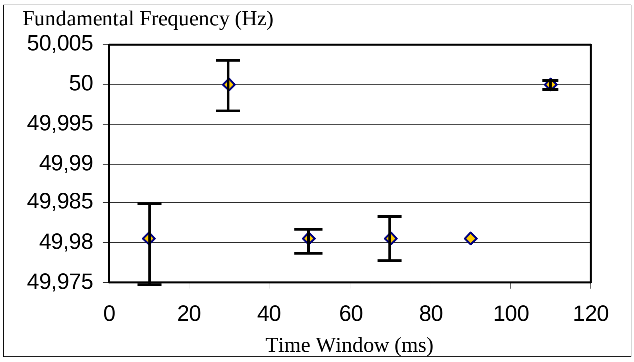

Fig. 1: Frequency variation obtained with CFA algorithm

As it is possible to see, the instrument characterizes little variations in the fundamental frequency, anyway close with 50Hz.

To obtain a comparable accuracy by FFT algorithm, it should use a zero-padding technique with a number of zeroes equal to 500 times the number of samples used. With only 2560 samples we should have a sensibility not greater to 50Hz.

Another advantage, that will be faced in other works, is that this form of CFA formula allows to define the interarhmonics fields in an easy way and according to CEI

6000-4-7 (where the fields are 240). Moreover CFA formula give us information about flicker in amplitude modulation field of the fundamental frequency according to CEI EN 50160. A first experiment on real collected values allow us to say that the frequency calculus accuracy is similar to fundamental amplitude and it is equal to 0.2% for g=3. Evaluation of all parameters can be obtained in real time using a PC with only 1000 signal points sampled at 1000 Hz frequency.

4. CONCLUSION

The Curve Fitting Algorithm allows to determine with a little number of samples with respect other algorithms, and so saving much computational time, the fundamental frequency of the signal under observation with an acceptable level of accuracy. This represents the major problem for every program that face a real time analysis of a signal.

CFA permits to determine easily the harmonics and their phase in the observation window after the individuation of fundamental.

An extension of Curve Fitting Algorithm concept, allows the determination of interharmonics fields according with definitions presents in CEI EN 50160.

The possibility to monitor with sliding window the signal under observation allow us to identify very short voltage interruptions too.

A first experimental prototype has been realized already operative, that will be used to determine inter-harmonic presence modifying CFA.

APPENDIX 1

Being S_i and C_i defined as:

$$S_i = \int_0^T y(t)\cdot t^i \cdot \sin(\omega t)dt \qquad ; \qquad C_i = \int_0^T y(t)\cdot t^i \cdot \cos(\omega t)dt$$

$$Num = C_0^2 - S_0^2$$

$$\frac{\partial Num}{\partial \omega} = -2C_o S_1 - 2S_0 C_1 - \frac{4\pi}{\omega^2} y(T) C_o$$

$$\frac{\partial^2 Num}{\partial \omega^2} = 2\left\{ S_1^2 + \frac{2\pi}{\omega^2} y(T) S_1 - C_0 C_2 - C_1^2 + S_0 S_2 + \frac{2\pi}{\omega^2} y(T) T S_0 + \frac{4\pi}{\omega^3} y(T) C_o + \frac{2\pi}{\omega^2} y(T) S_1 + \frac{4\pi^2}{\omega^4} y^2(T) \right\} =$$
$$= 2\left\{ S_1^2 - C_0 C_2 - C_1^2 + S_0 S_2 + \frac{2\pi}{\omega^2} y(T) \left[2S_1 + TS_0 + \frac{2}{\omega} C_o + \frac{2\pi}{\omega^2} y(T) \right] \right\}$$

$$\frac{\partial^3 Num}{\partial \omega^3} = 2\begin{bmatrix} 2S_1C_2 + S_1C_2 + S_3C_0 + C_1S_2 + S_0C_3 + \frac{2\pi}{\omega^2} y(T)\left(2C_2 + TC_1 + C_2 + C_0T^2\right) \\ -\frac{4\pi}{\omega^3} y(T)(3S_1 + TS_0) - \frac{12\pi}{\omega^4} y(T) C_o - \frac{24\pi^2}{\omega^5} y^2(T) \end{bmatrix}$$

APPENDIX 2

$$Den = C_1 S_0 - S_1 C_0$$

$$\frac{\partial Den}{\partial \omega} = -S_2 S_0 - C_2 C_0 + C_1^2 + S_1^2 - \frac{2\pi}{\omega^2} y(T)[TS_0 + S_1]$$

$$\frac{\partial^2 Den}{\partial \omega^2} = -C_3 S_0 - 3S_2 C_1 + S_3 C_0 + S_1 C_2 + \frac{2\pi}{\omega^2} y(T)\left[C_0 T^2 + 2C_2 - 3C_1 T\right] + \frac{4\pi}{\omega^3} y(T)(TS_0 + S_1)$$

$$\frac{\partial^3 Den}{\partial \omega^3} = S_4 S_0 - 4C_3 C_1 + 3S_2^2 + C_4 C_0 - 2S_3 S_1 + C_2^2 + \frac{2\pi}{\omega^2} y(T) *$$
$$* \left[S_0 T^3 + 6S_2 T - 3S_3 - 2S_1 T^2\right] - \frac{4\pi}{\omega^3} y(T)\left[C_0 T^2 + 2C_2 - 3C_1 T\right] + \frac{24\pi^2}{\omega^4} y^2(T) T^2$$

REFERENCES

[1] J. Arrillaga, D Bradley, P.S. Bodger "Power System Harmonics", John Wiley & Sons, 1985.

[2] "Electrical Power Quality", Stars in a Circle Publications, 1991.

[3] "Electrical Power Systems Quality", McGraw-Hill, 1997.

[4] J. Arrillaga, B.C. Smith, N.R. Watson, A.R. Wood "Power System Harmonic Analysis", John Wiley & Sons, 1997.

[5] J. Arrillaga, N.R. Watson, S. Chen: "Power System Quality Assessment", John Wiley & Sons, 2000.

[6] IEC 61000-x-y (with x:1-6 and y:1-7) Electromagnetic Compatibility "Guide",1984-1995.

[7] IEEE 1159: Recommended Practice on Monitoring Electric Power Quality, 1995.

[8] P. Mattavelli, L. Fellin, P. Bordignon, M. Perna: "Analysis of interarhmonics in DC arc furnace installation", Proceedings of 8th International Conference on Harmonics and Quality of Power, Athens, Greece, 1998, Vol. 2, pp. 1092-1099.

[9] A. Girgis, W.B. Chang, E.B. Makram: "A digital recursive measurement scheme for on-line tracking of power system harmonics", IEEE Trans. Power Deliv. 1992, 7, (1), pp. 402-409.

[10] S. Osowski: "Neural network for estimation of harmonics components in a power system": IEE Proc. Gener. Transm. Distrib., 1992, 139, (2), pp. 129-135.

[11] A. Cichocki, T. Lobos : "Artificial neural networks for real time estimations of basic waveforms of voltages and currents", IEEE Trans. Power Syst., 1994, 9, (2), pp. 612-618.

[12] M. Caciotta, V. Orsolini Cencielli: "Neural networks in time domain analysis of signals", Proceedings of the XIII IMEKO World Congress, Torino, Italy, September, 5-9, 1994, vol. 2, pp. 972-977

[13] T. Lobos, T. Kozina, S. Osowski: "Detection of remote harmonics using SVD", 8th Int. Conference on Harmonics and Quality of Power, Athens, Greece, 1998, pp. 1136-1140.

[14] S.Bakamidis, M. Dendrinos, G. Carayannis: "SVD analysis by synthesis of harmonic signals", IEEE Trans. Signal Process., 1991, 39, (2) pp.472-477.

[15] Z. Leonowicz, T. Lobos, P. Ruczewski, J. Szymanda: "Application of higher-order spectra for signal processing in electrical power engineering", Int. Journal for Computation and Mathematics in Electronic Engineering COMPEL, 1998, vol.17, no. 5/6, pp. 602-611.

[16] T. Lobos, J. Rezmer: "Real time determination of power system frequency", IEEE Trans. Instrum. Meas., 1991, 39, (2), pp. 472-477.

[17] M. Meunier, F. Brouage: "Fourier transform, wavelet. Prony analysis: tool for harmonics and quality of power", 8th Int. Conference on Harmonics and Quality of Power, Athens, Greece, 1998, pp. 71-76.

[18] T. Lobos, Z. Leonowicz, J. Rezmer, H.J. Koglin: "Advanced signal processing methods of harmonics and

interharmonics estimation", Proceedings of IEE Conference on Developments in Power System Protection, 2001, no. 479.
[19] S.A. Soliman, A.M. Al-Kandari, K. El-Ngar, M.E. El-Hawary: "New dynamic Filter Based on absolute value algorithm for on-line tracking of power system harmonics", IEE Proceedings of the Part. C, 1995, 142(1), pp. 37-44.
[20] C.S. Moo, Y.N. Chang, P.P. Mok: "A digital measurement scheme for time varying transient harmonics", IEEE/PES, 1994, Summer Meeting Paper, 490-493, PWRD.
[21] M.S. Sachdev, M. Nagpal: "A recursive least error squares algorithm for power system relaying and measurement applications", IEEE Transactions on Power Delivery, 1991, 6(3), pp. 1008-1015.
[22] T. S. Sidhu, M.S. Sachdev: "An iterative technique for fast and accurate measurement of power system frequency", IEEE Transactions on Power Delivery, 1998, 13(1), pp. 109-115.
[23] V.V. Terzija, M.B. Djuric, B.D. Kovacevic: "Voltage phasor and local system frequency estimation using Newton-type algorithm", IEEE Transactions on Power Delivery, 1994, 9(3), pp. 1368-1374.
[24] M. Caciotta, F. Leccese, A. Trifirò: "From Power Quality to Perceived Power Quality", Proceedings of the IASTED International Conference on Energy and Power Systems EPS 2006, Chiang Mai, Thailand, 28-31 March 2006, 526-119.
[25] M. Caciotta, F. Leccese, A. Trifirò: "Frequency Valuation in Curve Fitting Algorithm", Proceedings of the XVII IMEKO WORLD CONGRESS, Metrology for a Sustainable Progress, Rio de Janeiro, Brazil, 17-22 September 2006.