XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

A COMPARATIVE EVALUATION BETWEEN FREQUENCY ESTIMATION ALGORITHMS FOR POWER QUALITY ASSESSMENT IN DSP IMPLEMENTATION

Václav Matz[1], Pedro M. Ramos[2], Nuno B. Brás[2] and A. Cruz Serra[2]

[1] Faculty of Electrical Engineering, Department of Measurement, Czech Technical University,
Technicka 2, 166 27 Prague,Czech Republic
e-mail: matzv@fel.cvut.cz
[2] Instituto de Telecomunicações and Department of Electrical and Computer Engineering,
Instituto Superior Técnico, Technical University of Lisbon, IST,
Av. Rovisco Pais 1, 1049-001 Lisbon, Portugal
Phone: +351-218418485 Fax: +351-218418472 e-mail: Pedro.Ramos@Lx.it.pt

Abstract: Numerous frequency estimation algorithms are available for power quality assessment of frequency. In this paper, a comparative study in terms of accuracy, number of operations and memory cost is presented to select the best solution for implementation in a digital signal processor, in the context of typical power quality frequency disturbances measurements.

Keywords: Frequency estimation, power quality assessment.

1. INTRODUCTION

Frequency estimation algorithms for time-varying frequency signals have been analyzed due to its transversal application in electrical and electronics engineering. In particular, many algorithms were developed for power quality, once frequency is, by itself, an important power quality indicator. In addition, several disturbance detection algorithms are based on the precise knowledge of the signal instantaneous frequency [1].

In this comparative study, published algorithms or relevant innovative techniques are analyzed, tested and compared. They include FFT based algorithms like interpolated FFT (IpDFT) [2], Chirp-Z Transform (CZT) [3], adaptative filters like Kalman filtering based methods [4] and least-squares methods like sine-fitting [5].

Testing is focused on selected algorithms accuracy and frequency change response. Since DSP implementation is the final purpose, evaluation takes into account its performance when implemented. Therefore, algorithms are also evaluated in terms of number of operations, memory cost and overall velocity performance.

A noisy and harmonic signal with fundamental frequency typical from a power system signal is used to test each algorithm. Signal to Noise Ratio (SNR) changes helps to define a threshold for each algorithm that can define its usefulness for implementation in DSPs to analyze power signals.

2. ALGORITHMS

In this section, the different algorithms that will be compared are described and their main advantages and disadvantages are presented.

The input power signal is digitized with an Analog to Digital Converter (ADC), with a total of N samples, acquired at the sampling frequency f_S .

2.1. FFT

The FFT algorithm is one of the simplest ways to estimate a signal frequency. However, when the objective is to estimate time-varying frequency, it has strong limitations because of its spectral resolution. In fact, the spectral resolution Δf is the inverse of the time window size $t_{window} = N / f_S$

$$\Delta f = \frac{f_s}{N} = \frac{1}{t_{window}} . \qquad (1)$$

In frequency changing signals, t_{window} should be as small as possible to maximize time resolution of the frequency estimation. However, with the FFT, this causes poor frequency resolution.

Two techniques can be used to improve frequency resolution: IpDFT and CZT. Both allow improving the spectral resolution without having to increase t_{window} .

2.2. Interpolated Discrete Fourier Transform

IpDFT is a well known technique that allows improving the frequency estimation resolution of a signal with a strong fundamental frequency peak. Convoluting the used window function spectrum with a pure frequency signal, a theoretical function for expected spectrum of the frequency peak is obtained. Interpolating the FFT peak results with this function, improved frequency estimation is obtained.

Even so, there are some algorithm limitations: slight changes on frequency during time window can generate deviations on fundamental frequency peak, reducing algorithm precision – issue addressed in [5]. Also, powerful spurious or harmonic frequencies near the fundamental frequency can distort the resulting interpolation.

2.3. Chirp-Z Transform

The Chirp-Z Transform (CZT) is an algorithm for numerically evaluating the Z-transform of a sequence on N samples. The algorithm is based on the fact that the values of the Z-transform on a circular or spiral contour can be expressed as a discrete convolution.

One of the applications of CZT method is to provide narrowband frequency analysis. In this case the frequency resolution is

$$\Delta f = \frac{f_{window}}{f_s \, t_{window}} \tag{2}$$

where f_{window} is the chosen spectral window where extra resolution is required. This algorithm allows improving frequency resolution without changing the time window.

However, the number of operations for CZT is N^2 whereas for FFT it is $N \log_2 N$. This will lead to poor DSP implementation performance. Problems can also arise from the selection of spectrum window to analyze. Nevertheless, CZT method is a good candidate for power quality frequency estimation since typical fundamental frequency deviation is 10% around 50 Hz, making the selection of frequency spectrum interval simple.

2.4. Kalman filtering

The Kalman filtering technique has been extensively used for signal frequency estimation. Published papers try to improve the maximum frequency slew-rate that can be estimated [4]. Noise and process variance optimization techniques were used for Non-linear Kalman filters (EKF) as well as heuristic algorithms to reset the covariance matrix in the presence of disturbances for the same EKF filters.

The linear Kalman filter proposed in [4], specifically developed for power quality, is the algorithm tested. It is a complex data linear Kalman filter to estimate the phase that can be used to estimate the frequency. Its formulation allows very good stability due to its linear base. It has a good processing velocity by avoiding matrix inversion calculations since it is composed by one single state. The referred model can be expanded to include harmonics and is presented in [4] also for three-phase signals frequency estimation.

2.5. Sine-fitting

Sine-fitting of digitized records, as presented in [6], allows estimation of the frequency, phase, amplitude, and DC value of a input signal

$$y(t) = C + D\cos(2\pi f t + \phi) \tag{3}$$

after digitalization by minimizing the least square error.

According to [6], five periods are required to achieve the best coefficients estimation. During those five periods, the signal is supposed to have a constant frequency. This obviously limits the algorithm time resolution when the power signal frequency suffers quick variations. Nevertheless, the algorithm can still be applied to records containing less than five periods. The algorithm has very good precision even in the presence of harmonics as the sine-fitting algorithm acts as a very narrow band pass filter centered on the signal frequency. However, since it is a iterative algorithm, convergence depends on the initial algorithm estimations [7]. In each iteration a matrix with four columns and N rows needs to be built, multiplied with its transpose and the resulting matrix (4×4) must be inverted and again multiplied with the transposed matrix. Finally the result is multiplied with a vector containing the acquired samples. All these operations and the number of iterations take a toll on the total number of required operations.

2.6. Continuous Wavelet Transform

The wavelet transform is a signal processing tool often used in power quality analysis [8], which provides signal representation corresponding to a time frequency plane. The Continuous Wavelet Transform (CWT) is defined by

$$CWT_x^{\psi}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \cdot \psi^*\left(\frac{t-\tau}{s}\right) dt\,, \tag{4}$$

where ψ is the mother wavelet; s is a scale corresponding to the frequency; τ is translation corresponding to the location of series of wavelets that are derived from the mother wavelet. The scale for each frequency analyzed is

$$a_i = \frac{f_{mother\ wavelet}}{t_{window} \times f_{analyzed}}\,, \tag{5}$$

where $f_{mother\ wavelet}$ is central frequency of the mother wavelet, t_{window} is the sampling period and a_i is the used scale which corresponds to $f_{analyzed}$. To achieve a good frequency resolution, many values of a_i are required and therefore more evaluations of (4) are needed. In this case the CWT is not appropriate for precision frequency estimation. However, it is still considered in this paper as a possible frequency estimation algorithm.

2.7. Short Time Fourier Transform

The Short Time Fourier Transform (STFT), also called windowed Fourier transform, has numerous applications in speech, sonar, and radar processing. The STFT is used for the time and frequency representation of the input signal . This is the advantage for the analyzing of non-stationary signals [9].

The STFT can be defined by

$$STFT(t, \omega) = \int_{-\infty}^{\infty} x(\tau) \cdot w^*(\tau - t) e^{-j\omega\tau} d\tau \tag{6}$$

where $w(t)$ is the window function which is used for the time and frequency resolutions. The use of a wide window will give good frequency resolution but poor time resolution, while the use of a narrow window will give good time resolution but poor frequency resolution. The frequency resolution in this case is

$$\Delta f = \frac{1}{t_{window}} . \tag{7}$$

For the frequency estimation the best frequency resolution is needed. It means that the used window should be the widest. However, this affects the algorithm performance for time-varying frequency signals.

2.8. Hilbert Transform

The Hilbert transform technique [10] is useful for analyzing the signal frequency as a function of time. The basic idea in the Hilbert transform analysis is to obtain the envelope and instant phase of the input signal. With $x(t)$ as the input signal, the Hilbert transform is

$$y(t) = H\{x(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{\tau - t}d\tau . \tag{8}$$

With, $x(t)$ and $y(t)$, a complex signal $z(t)$ can be defined as,

$$z(t) = x(t) + jy(t) . \tag{9}$$

The phase of $z(t)$ is

$$\psi(t) = \tan^{-1}\frac{y(t)}{x(t)} , \tag{10}$$

while the instantaneous frequency is

$$\omega(t) = -\frac{d}{dt}\psi(t) = -\frac{d}{dt}\left(\tan^{-1}\frac{y(t)}{x(t)}\right). \tag{11}$$

The Hilbert transformed series has the same amplitude and frequency content as the original real data and includes phase information that depends on the phase of the original data. The Hilbert transform is useful in calculating instantaneous attributes of signals, especially the amplitude and frequency. The instantaneous amplitude is the amplitude of the complex Hilbert transform and the instantaneous frequency is the time rate of change of the instantaneous phase angle.

3. NUMERICAL SIMULATIONS

The proposed algorithms were used for the evaluation of the frequency estimation considering three types of signals: pure signal, signal with harmonics and noisy signal. In both situations, the fundamental frequency is 50 Hz – Fig. 1.

The noisy signal was simulated adding Gaussian white noise

$$x_n(t) = A\cos(\omega t + \varphi) + n(t). \tag{12}$$

The noise level was modified to change the signal to noise ratio (SNR) and test the algorithms effectiveness to different situations.

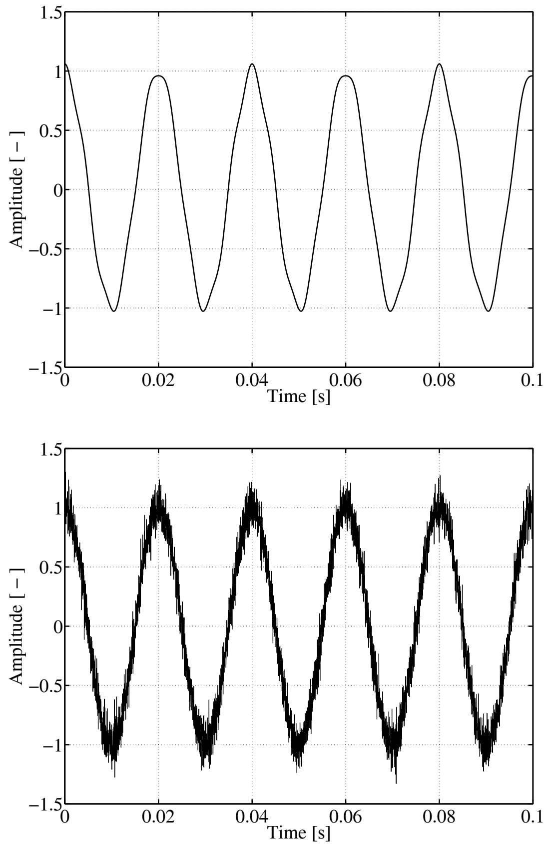

Fig. 1. Simulated harmonics (top) and noisy (bottom) signals

The signal with harmonics contains the 5th and 9th harmonic with 1 % and 5 % level of 1st harmonic. The harmonic signal is

$$x_h(t) = A\cos(\omega t + \varphi) + \frac{A}{100}\cos(5\omega t) + \frac{A}{20}\cos(9\omega t) . \tag{13}$$

For the CZT, the frequency range corresponding to the analyzed window was chosen centered at 50 Hz with a range of ± 5 %. The frequency resolution is $\Delta f = 0.001$ Hz .

In the case of CWT an appropriate mother wavelet is proposed. The mother wavelet is the sine function with sampling frequency 50 kHz. By carefully selecting the correct scales, the frequency resolution is 0.0416 Hz for 50 different scales. The frequency estimation is determined as the frequency/scale where the minimum value of the maximum absolute value of the coefficient is obtained (Fig. 2).

For the STFT algorithm, the appropriate window function with proportional size must be selected. The Hann window function with the widest length is used with a frequency resolution of 0.01 Hz.

For the other algorithms, there is no need to set extended properties because the frequency estimation is derived directly from the principle of each algorithm.

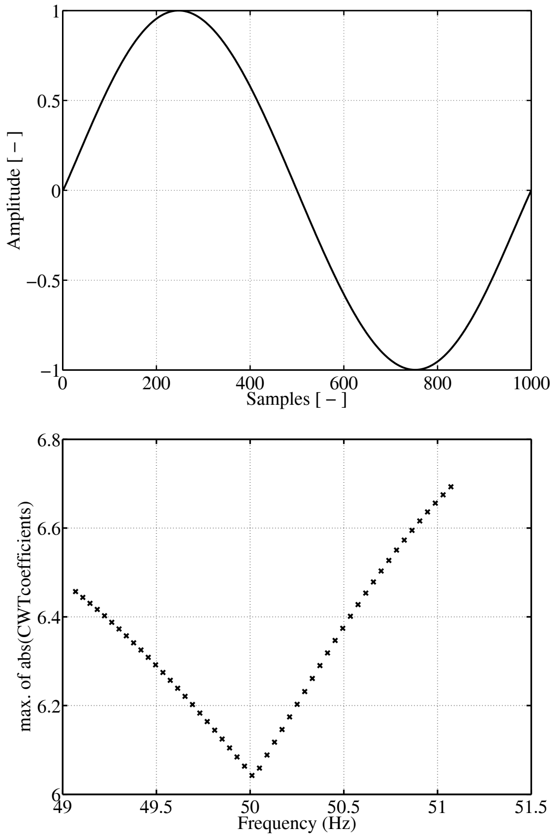

Fig. 2. Mother wavelet (top) and example of frequency detection with the CWT and the proposed mother wavelet (bottom)

These algorithms are tested for frequency estimation. Each algorithm needs different settings for good performance of frequency estimation. The frequency deviation from nominal value, time of performance and code lines are some of parameters tested for each algorithm. In the case of noisy signals the *SNR* is changed from 10 dB up to 40 dB.

In Fig. 3, the evaluation of frequency estimation algorithms based on *SNR* level of white noise is presented. All algorithms have stable frequency estimation for $SNR > 20\,\mathrm{dB}$. Up to $SNR = 20\,\mathrm{dB}$ the frequency estimation is not stable because of the relatively high level of noise. The IEEE standard [11] defines that the level of noise in case of power systems is approximately 1 % of typical voltage magnitude that corresponds to $SNR = 40\,\mathrm{dB}$.

For the determination of the best frequency algorithm the accuracy, stability, number of code lines and overall velocity performance must be evaluated. In Table 1, some of these properties are described for the different algorithms. From these results it is clear that the best algorithm is the IpDFT with the lowest time for evaluation. On the other hand the time for the CWT algorithm is the worst because number of scales is very high to achieve good frequency resolution.

In Fig. 4, the IpDFT is tested with a simulated time-varying signal. The signal frequency changes from 50 Hz up to 52.5 Hz with steps of 0.1 Hz each taking 0.4 s. The relative frequency error (also shown in Fig. 4) shows that the algorithm error never exceeds 0.05 %.

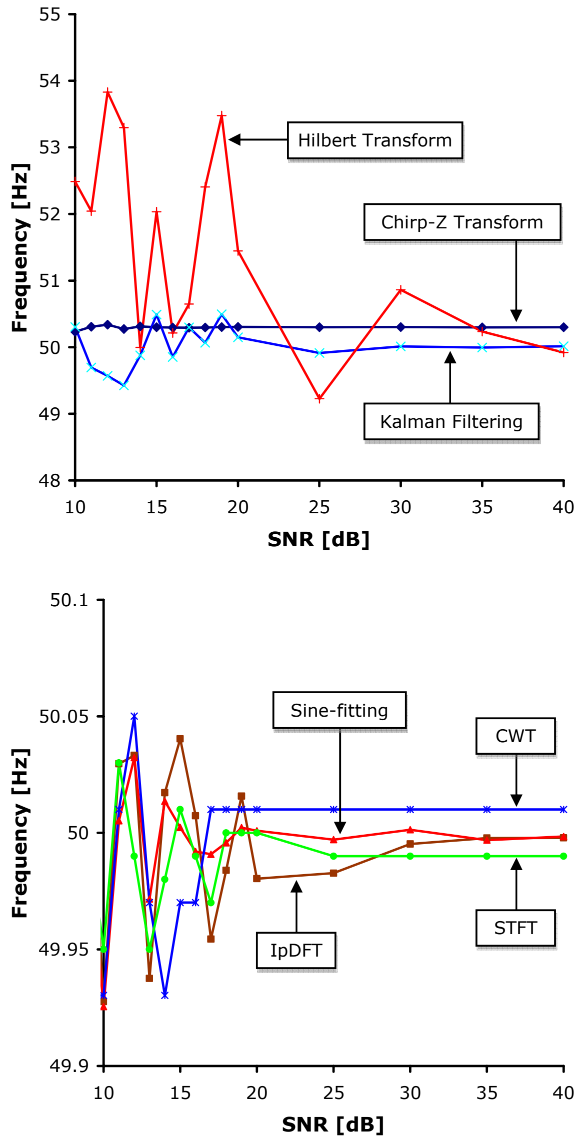

Fig. 3. Frequency estimation of noisy signals

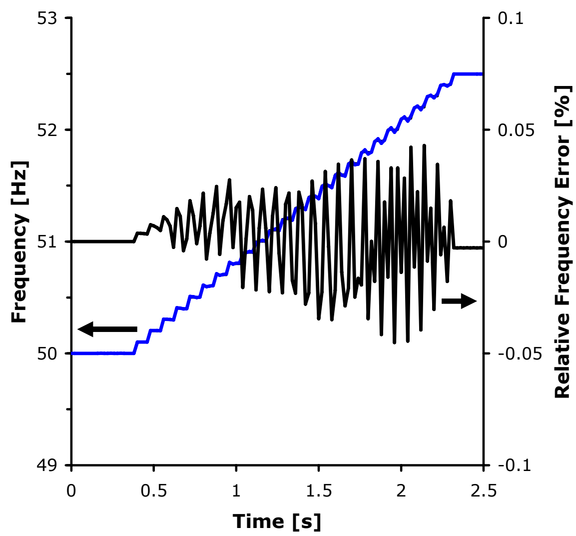

Fig. 4. Frequency estimation of simulated signal with time frequency change.

Table. 1. Evaluation of frequency estimation algorithms for a sampling rate of 50 kS/s and 5000 samples.

Signal type	*Parameter*	Chirp-Z transform	IpDFT	Kalman filtering	Sine-fitting	CWT	STFT	Hilbert transform
Clear	Time [s]	0.1373	0.0131	0.1354	0.0921	28.2526	6.8186	0.0494
	Frequency [Hz]	50.3010	50.0000	49.9996	49.9993	50.010	49.9900	50.0000
	Deviation [Hz]	0.3010	0	0.0004	0.0007	0.01	0.01	0
	Deviation [%]	0.5984	0	0.0008	0.0013	0.0201	0.0201	0
Noisy	Time [s]	0.1373	0.0131	0.1354	0.0921	28.2526	6.8186	0.0494
	Deviation [Hz]	0.34	0.07	0.58	0.07	0.07	0.05	3.83
	SNR [dB] (>10)	12	10	13	10	10	10	12
Harmonics	Time [s]	0.1152	0.0359	0.1527	0.0889	27.6555	6.6516	0.0517
	Frequency [Hz]	50.3010	49.9998	50.0082	49.9988	50.0100	49.9900	50.7851
	Deviation [Hz]	0.3010	0.0002	0.0082	0.0012	0.0100	0.0100	0.7851
	Deviation [%]	0.5984	0.0004	0.0164	0.0024	0.0201	0.0200	1.5459

4. EXPERIMENTAL RESULTS

The IpDFT was used to monitor the algorithm of a real power signal. The system setup used was presented and calibrated in [12] and uses a closed loop Hall effect sensor from LEM [13] and a 16-bit National Instruments DAQ USB-9215B operated at 50 kS/s for the sensor data acquisition. The sensor output time signal was measured during 7 hours. The frequency estimation during this interval is shown in Fig. 5.

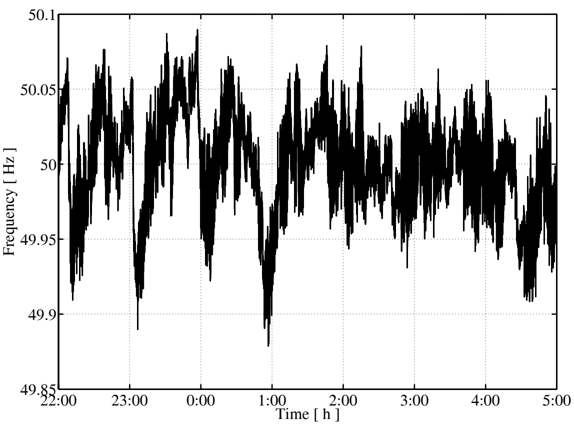

Fig. 5. Frequency estimation of real power signal.

5. CONCLUSIONS

A comparative evaluation between different algorithms to estimate frequency for power quality assessment has been presented. The conclusions point to the IpDFT algorithm as the most precise, accurate and fastest algorithm.

ACKNOWLEDGMENTS

Work sponsored by the Portuguese national research project reference POSC/EEA-ESE/57708/2004 entitled "Fast and accurate power quality measurements using analog to digital converters and digital signal processing techniques".

The work developed by Václav Matz in Portugal was sponsored by a ERASMUS scholarship.

REFERENCES

[1] D. Castaldo, D. Gallo, C. Landi and A. Testa, "A digital instrument for nonstationary disturbance analysis in power lines", IEEE Trans. on Instr. and Meas. vol. 53, n. 5, pp. 1353-1361, Oct. 2004.

[2] J. Schoukens, R. Pintelon and H. Van hamme,'The Interpolated Fast Fourier Transform: A Comparative Study,' IEEE Trans. on Instr. and Meas., vol. 41, n. 2, pp. 226-232, April 1992.

[3] L. R. Rabiner, "Theory and application of digital signal processing." Englewood Cliffs, NJ: Prentice-Hall, 1975.

[4] A. K. Pradhan, A. Routray and D. Sethi, "Voltage phasor estimation using complex linear kalman filter", Eighth IEE International Conference on Developments in Power System Protection, vol. 1, pp. 24-27, April 2004.

[5] D. Agrez , "Frequency estimation of the non-stationary signals using interpolated DFT", IMTC/2002, vol. 2, pp. 925-930, May 2002

[6] "IEEE Std. 1057-1994, Standard for digitizing waveform records", The Institute of Electrical and electronics Engineers, Inc., New York, December 1994.

[7] M. Fonseca da Silva, Pedro M. Ramos and A. Cruz Serra, "A new four parameter sine fitting technique", Measurement, vol. 35, n. 2, pp. 131-137, March 2004.

[8] Tao Lin, Mineo Tsuji, Eiji Yamada, "A Wavelet Approach to Real Time Estimation of Power System Frequency", Proceeding of 40th SICE Annual Conference, Nagoya, Japan, July, 2001.

[9] Z. Leonowicz, T. Lobos, J. Rezmer, "Spectrum Estimation of Non-stationary Signals in Power Systems", International Conference on Power System Transients, New Orleans, USA, Oct. 2003.

[10] David. G. Long, "Comments on Hilbert transform based signal analysis", BYU Microwave Remote Sensing (MERS) Laboratory, MERS Tech. Rep. # MERS 04-001, Brigham Young University, Provo, UT, 2004.

[11] "IEEE Std. 1159-1995, IEEE Recommended Practice for Monitoring Electric Power Quality", The Institute of Electrical and electronics Engineers, Inc., New York, December 1995.

[12] Pedro M. Ramos, Nuno B. Brás, A. Cruz Serra, "A new calibration method for current and voltage sensors used in power quality measurements", IMTC 2006 –, Sorrento, Italy, pp. 2283-2288, April 2006.

[13] Datasheet, "Voltage Transducer LV 25-P", LEM Corporation, available at http://www.lem.com.