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TEACHING THE CORRECT USE OF BASIC MEASURING INSTRUMENTS IN A NONLINEAR CONTEXT: A TRAINING SESSION ON A FLUORESCENT TUBE

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Abstract: In the frame of a basic course on electrical measurements, a training session teaching the correct use of measuring instruments is presented. The context is that of a fluorescent tube, a non-linear device allowing interesting mathematical developments and measuring observations. In this session, the students are asked to evaluate the pertinence of dimming the lamp in terms of luminous efficiency and pollution of the mains.

Keywords: didactic training session, nonlinear measurements, fluorescent tube.

1. INTRODUCTION

Laboratory training practice is the outcome of every course devoted to industrial measurements. The capabilities that the students must master at the end of such a teaching are at the core of a basic training in electrical measurements and instrumentation. Here are a few keywords of these particular skills: loading effect of volt-, am- , wattmeters and their correction, influence of harmonic distortion on the measurement accuracy depending on the functional principle of the meters (which meter is the best choice?), experimental errors (propagation and treatment) ... Emphasize is also brought on the necessary link between these skills and a good mastery of electrical AC circuits.

The classical fluorescent tube (TL tube) lighting e.g. our offices, with its iron ballast, is a remarkable non-linear device allowing both interesting mathematical developments and measuring observations at industrial frequency (50 Hz). A good understanding of the functional principle of the measuring instruments, together with a correct use of them is often critical in this practice.

A message to keep from this training is that it is often questionable to apply in a nonlinear context recipes only valid for linear circuits.

2. METHODOLOGY

This training session is part of a basic course on electrical measurement, compulsory for students in electrical and electromechanical engineering. It is proposed according to a *problem-based learning approach*. Students have to plan an experimental procedure on a TL tube in order to evaluate the pertinence of dimming the lamp (i.e. reducing the applied voltage) from an economical point of view, and simultaneously to estimate the consequences on the pollution of the mains.

The experimental set-up is presented on Fig. 1 (the students have to find it on their own). The tube is a 36-W 1.2-m-long model, and the ballast a conventional ferromagnetic one. All the voltages and current values in this paper are RMS ones obtained on an exemplative set-up. The device is powered by the 230V AC mains through an isolation transformer, for security reasons. One then enters into a cursor autotransformer that allows to set the input voltage between 0 and 230V. The main loop is composed by (a) the primary coil of a current transformer *CT*, (b) the TL tube in parallel with its starter and (c) the ballast. The current transformer is used to upscale the current passing in the lamp in order to cope with the 5A-range of both the ammeter and the wattmeter. The current in the lamp being of the order of 0.5 A, we select an amplifying factor of 10. The secondary coil is connected to the current measuring instruments (ammeter and current circuit of the wattmeter).

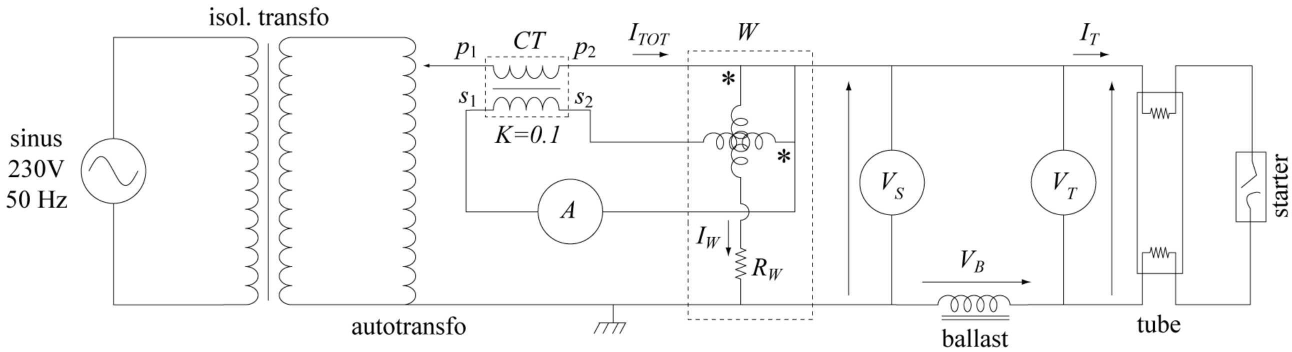

Fig. 1. Experimental set-up.

The following notations will be used for the voltages, currents and powers involved (capital letters denote RMS values):

- $v_S(t)$, V_S: the voltage across the whole "tube plus ballast" (after the primary coil of the current transformer);
- $v_T(t)$, V_T: the voltage across the tube alone;
- $v_B(t)$, V_B: the voltage across the ballast alone;
- V_D: the DC voltage measured at the output of a photosensor aiming at the lamp. It is proportional to the luminous flux emitted by the lamp;
- $i_{TOT}(t)$, I_{TOT}: the total current in the experimental setup, equal to one tenth of the current in the secondary coil of the current transformer (if we suppose it error-free for now);
- $i_T(t)$, I_T: the current in the tube and the ballast;
- $i_W(t)$, I_W: the current in the voltage coil of the wattmeter (see below);
- P_{T+B}: the effective electrical power consumption of the whole "tube plus ballast";
- P_T: the effective electrical power consumption of the tube;
- P_B: the effective electrical power consumption of the ballast.

The students have the following instruments at their disposal:

- V_1: a true-RMS voltmeter of accuracy class 0.5 and full scale 250V;
- V_2: a rectifier-voltmeter (sinus RMS scaled) of accuracy class 1, internal resistor 2MΩ, and full scale 450V;
- V_3: a DC digital voltmeter of accuracy class 0.5 and full scale 3V;
- A_1: a true-RMS ammeter of accuracy class 0.5 and full scale 5A;
- A_2: a rectifier-ammeter (sinus RMS scaled) of accuracy class 1 and full scale 5A;
- W: an electrodynamic wattmeter of accuracy class 1.5, internal resistor 26.5kΩ, and full scale 200V-5A.
- a dual-channel digital oscilloscope, with a differential probe to display voltages and an inductive current probe to display currents.

The true RMS instruments are of the moving iron type but any other type (thermocouple, electronic, ...) proposing the appropriate range would be suitable.

Note that regarding those measurements, the students have not only to choose the most adequate instrument among a range of devices at their disposal, but also to compare their results with those obtained with less adequate devices, even if more popular, and justify the difference. The point is that the lamp voltage is strongly non-linear (nearly of rectangular shape) with the consequence to also distort the current. The fact that *true RMS* instruments are generally to be used in this particular context is here clearly evidenced to the students.

The objective of the training session in terms of performances is to keep measurement errors below 2%.

The training session lasts two hours and is prepared from a document composed of twelve questions. Most of them require specific measurements from the students, while some demand analytical calculations. The students work by groups of three and are requested to deliver a grouped report two weeks after the session. The questions asked, together with the expected answers (results, analysis, ...), difficulties (if any) encountered by the students and acquired skills are presented hereafter, as the synthesis of several years of experience with this training programme.

A theoretical analysis of a simplified model is reported in Appendix. The aim is to help the students both to anticipate and confirm their results.

3. QUESTIONS, ANSWERS & ACQUIRED SKILLS

3.1 Signal waveforms

- Question #1: sketch on a same graph the expected curves v_T and i_T versus time considering a purely resistive ballast, for a sinus voltage $v_S(t)$.
- Answer: observing the static DC tube characteristic (see Fig. 2), the demanded graph can be sketched as on Fig. 3. Between t_0 and t_1, the applied voltage v_S is not sufficient to start the discharge in the lamp. Consequently the voltage on the tube v_T is equal to v_S. In t_1, v_S

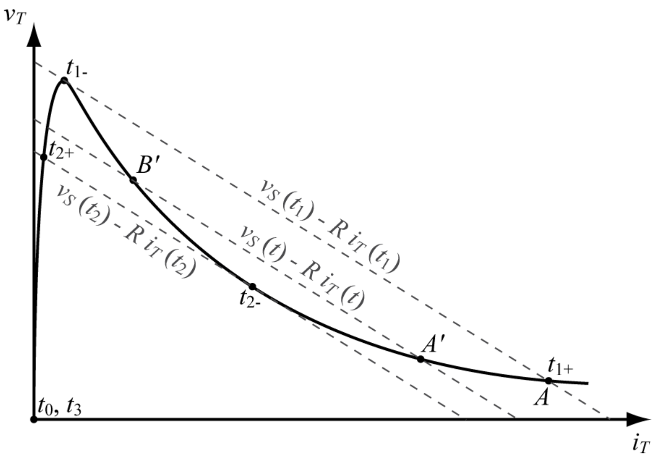

Fig. 2. DC characteristic of the tube, together with the load lines obtained with a resistive ballast.

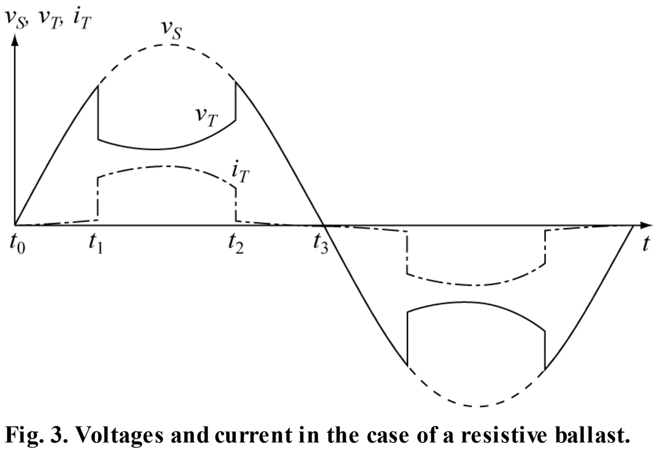

Fig. 3. Voltages and current in the case of a resistive ballast.

is sufficient enough and the tube starts to emit light. v_T suddenly diminishes and the tube reaches the functioning point A on Fig. 2. A current i_T in the tube immediately appears. Between t_1 and t_2, i_T follows the variation of v_S and v_T remains almost constant. It actually slightly varies in the opposite sense compared to v_S, because of the negative slope in the surroundings of point A. In t_2, v_S becomes too low to maintain the discharge. The tube thus lights off, i_T passes to zero, and v_T is again equal to v_S. In t_3 the whole sequence starts again. The nonlinear shape of v_T and, in a lesser degree, of i_T obviously appears. It is to be noted that the negatively sloped part of the DC characteristic offers two functioning point to the circuit, one of them being unstable (point B on Fig. 2). The main result of this process is that the tube practically acts as a voltage stabilizer (v_T is almost independent from v_S amplitude—sign not included).

Acquired skills: reflection on a current-voltage curve, familiarization with the phenomenon of discharge.

Difficulties: the students do generally not answer correctly to the question, because of a lack of rigour while interpreting the tube current-voltage curve, and a too superficial understanding of the discharge phenomenon. The functioning point resulting from the intersection of the lamp curve with the load line translating proportionally to v_S is in particular misunderstood.

– Question #2: sketch v_S, v_T, v_B and i_T versus time observed with the oscilloscope. Compare with the results of Question #1. Compare also with the simplified theory given in Appendix, supposing a purely inductive ballast.

– Answer: the students note that v_S is sinusoidal, v_T is almost square (as in the case of a resistive ballast) but with a sensible exceeding at the beginning of each half period, and i_T is a sinus with some harmonic distortion (see Fig. 4, where $-v_B$ is represented, for sake of readability). They also observe that v_B, i.e. the difference between v_S and v_T, is a complex signal. The role of the inductive ballast is similar to that of the resistive one in Question #1. The particularity here is a smoothing and a phase shift in the current. The obtained curves are similar to those predicted in Appendix.

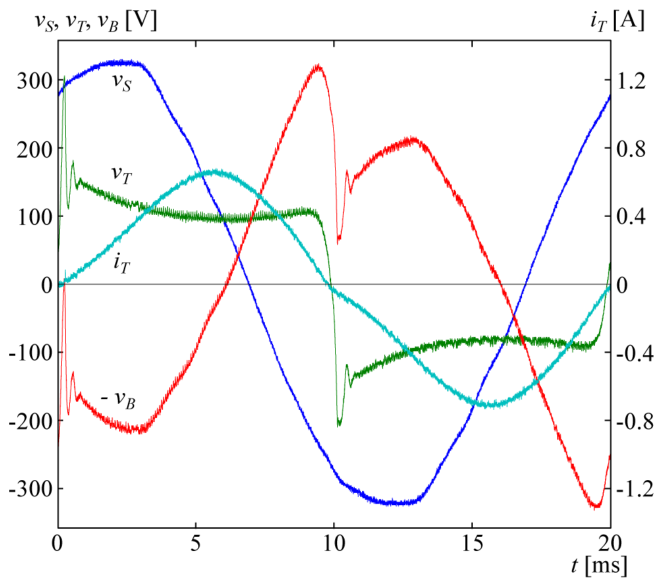

Fig. 4. Signals obtained on the experimental set-up.

Acquired skills: use of an oscilloscope in a floating context, visualization of nonlinear signals, better understanding of the discharge phenomenon.

Difficulties: the shapes of the signals are generally respected on the sketches but, curiously, the scales, units and phase differences are almost never well represented. The students are obviously lacking of rigor.

3.2 Impact of signal distortion on accuracy

– Question #3: knowing that the harmonic distortion of i_T is mainly due to 8% of third harmonic (see Appendix), do you think that ammeter A_2 is adequate to measure the RMS value of i_T? Justify and verify experimentally by comparing with the measurement provided by ammeter A_1.

Answer: a sinus-RMS-scaled rectifier-ammeter is sensitive to the mean value of the rectified current., with its scale graduated so that the measured value is multiplied by 1.111, i.e. the form factor of a sinus. This type of instrument thus indicates the correct RMS value only in the case of a perfectly sinusoidal signal. In the present case, i_T can be expressed as:

$$i_T(t)=\sqrt{2}\,I_f\left(\sin\omega t+0.08\sin(3\omega t+\varphi)\right) \qquad (1)$$

where I_f is the fundamental's RMS amplitude, ω the angular frequency and φ the phase difference between the two components. The worst case in terms of error is when φ is either 0° or 180°. In these cases, the mean value of i_T is:

$$I_{T,mean}=\sqrt{2}\,I_f\frac{\omega}{\pi}\int_0^{\pi/\omega}\left(\sin\omega t\pm 0.08\sin 3\omega t\right)dt \qquad (2)$$

and ammeter A_2 will indicate 1.111 $I_{T,mean}$. The RMS value of the current $I_{T,RMS}$ can be expressed from the RMS value of the fundamental I_f as:

$$I_{T,RMS}=\sqrt{1+0.08^2}\,I_f \qquad (3)$$

Therefore the relative error on the current ρ_{IT} committed by ammeter A_2 is simply:

$$\rho_{I_T}=\frac{1.111\cdot I_{T,mean}-I_{T,RMS}}{I_{T,RMS}} \qquad (4)$$

giving in the two worst cases mentioned above either -2.4% or +3% of error. The objective being to keep measurement errors below 2%, the conclusion is that ammeter A_2 cannot be used to measure I_T.

The experimental verification confirms the error calculation, as students generally obtain an error ranging from 3 to 4%.

Acquired skills: error calculation, confrontation of theory with practice, critical comparison of the performances reached by instruments using different measuring principles.

Difficulties: the students almost systematically make the mistake of "forgetting" the unknown phase difference between the two components of the current. This leads to an incomplete error calculation.

- Question #4: to measure the RMS value of v_T, one has to use a true-RMS voltmeter obligatorily, because of the high distortion of the signal. Supposing that v_T is square, what would be the error committed by voltmeter V_2? Verify experimentally.

Answer: similarly to ammeter A_2 (see Question #3), the rectifier-voltmeter V_2 is measuring 1.111 $V_{rect,mean}$. In the case of a square signal, the peak amplitude, the mean value and the RMS value are equal. Voltmeter V_2 will thus produce an error of +11.1% on V_T. This is verified experimentally, even though the observed error is generally smaller. This is due to the fact that v_T is not perfectly square (see Question #2) but partially trapezoidal.

Acquired skills: error calculation, further familiarization with the notions of peak amplitude, mean value and RMS value, making the students aware that a modern and popular digital voltmeter is not necessarily suitable in particular situations.

3.3 Vectorial calculations

- Question #5: to evaluate I_T, one has to subtract the currents derived by the voltage circuit of the wattmeter W and the voltmeter V_S from the current measured by ammeter A. To do that, it is legitimate to suppose that all the signals are sinusoidal (approximation to the fundamental). The correction is thus vectorial since the currents involved have not the same phase.

- Answer: the current consumed by V_S can be neglected because of the high value of its internal resistor (compared to that of the wattmeter). One can thus only consider I_W, i.e. the current in the voltage coil of the wattmeter, simply equal to V_S / R_W. The quantities involved are represented on Fig. 5, where I_{TOT} is equal to one tenth of the current measured by A (because of the current transformer). Graphically, one easily finds:

$$I_T^2 = I_{TOT}^2 + I_W^2 - 2 I_{TOT} I_W \cos\varphi_{TOT} \quad (5)$$

where φ_{TOT} is the phase difference between V_S and I_{TOT}, expressed as:

$$\cos\varphi_{TOT} = \frac{P_{TOT}}{I_{TOT} V_S} \quad (6)$$

where P_{TOT} is the total power consumed by the tube, the ballast and the voltage circuits. Experimentally, for $V_S = 230$V, one measures $I_{TOT} = 0.41$A and $P_{TOT} = 46.8$W. Applying (5) and (6) gives $I_T = 0.406$A, i.e. a difference of 1% with I_{TOT}, which is nonnegligible.

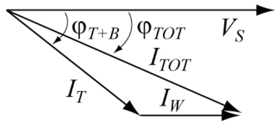

Fig. 5. Quantities involved for the current calculation.

Acquired skill: vectorial calculation.

- Question #6: calculate the capacitor to place in parallel of the whole "tube plus ballast" to obtain a power factor as close to 1 as possible. To do that, we will approximate the signals to their fundamental component. Verify experimentally.

Answer: to obtain $\cos\varphi_{T+B} = 1$, one would need a capacitor inducing a current I_C equal to:

$$I_C = I_T \sin\varphi_{T+B} = 0.406 \sin\arccos 0.48 = 0.36[A] \quad (7)$$

so that the total current $\vec{I}_{TOT} = \vec{I}_C + \vec{I}_T$ is aligned with V_S.

In purely sinusoidal circuits, the current in a capacitor is expressed as $I_C = \omega C V_S$, inferring $C = 4.8\mu$F. Experimentally, with such a capacitor one obtains, for $V_S = 230$V: $I_{TOT} = 0.24$A and $P_{T+B} = 44.8$W. We thus have in that case $\cos\varphi_{T+B} = 0.81$. The improvement is obvious, even though we are still far from having a power factor equal to 1. This is of course due to the fact that the approximation to the fundamental is not completely appropriate.

Acquired skills: reflection on the pollution of the mains by tube loads, vectorial calculation.

Difficulties: the students have the reflex of calculating the current as if the signals were perfectly sinusoidal. This is indeed what they have to do in the present case, but the fact that they generally do not realize that it is actually an hypothesis, only roughly verified, induces that they don't analyze correctly the reason why experimentally they do not obtain $\cos\varphi_{T+B} = 1$. Their explanation is generally that the capacitor must be defective or has not the correct value!

- Question #7: determine approximatively the R-L series model of the ballast. Verify if the value obtained for L is comparable to that obtained with the simplified theory. We here again make the approximation to the fundamental.

Answer: to determine this model, one needs to measure the RMS value of V_B, i.e. the voltage on the ballast. Using voltmeter V_1, we obtain $V_B = 168$V. The power consumed by the ballast P_B is simply the difference between P_{T+B} and P_T, i.e. 10.3W. The power factor of the ballast is thus:

$$\cos\varphi_B = P_B / V_B I_T = 0.15 \quad (8)$$

It is then easy to determine the two terms R and L of the series model:

$$R = \frac{V_B}{I_T}\cos\varphi_B = 62.5\,\Omega \;\; ; \;\; L = \frac{V_B}{\omega I_T}\sin\varphi_B = 1.3\text{H} \quad (9)$$

From the simplified theory, one has:

$$L = \frac{2\sqrt{2} V_S V_T}{\pi \omega P_T}\sin\varphi_0 = 1.4\text{H}, \; with \cos\varphi_0 = \frac{\pi V_T}{2\sqrt{2} V_S} \quad (10)$$

i.e. a difference of about 8% with the experimental result. This is satisfactory seen that contrarily to what supposed in our calculations, the ballast is not loss-free, and the voltage across it far from being sinusoidal (see Fig. 4).

Acquired skills: impedance modeling, confrontation of theory and practice.

Difficulties: again, the reason why there is such a difference between the experimental results and the simplified theory is generally misunderstood. It seems that supposing sinusoidal signals is so common that the students do not think further.

– Question #8: the current transformer was supposed error-free until now. It is actually of class 0.2, i.e. the current and phase errors ε and δ are typically lower than 0.2%. Evaluate the influence of these errors on the measurements.

Answer: by definition, one has (phasor notation):

$$K\,\bar{I}_{sec}=\bar{I}_{prim}\left(1+\epsilon+j\,\delta\right) \tag{11}$$

where I_{prim} and I_{sec} are the currents respectively in the primary and in the secondary coil of the transformer and K is the rated transformation ratio, i.e. 0.1 in our case. The relative error on the RMS value of the current is ε, thus negligible. Concerning the error on the power P_{T+B}, one has:

$$\begin{aligned} P_{T+B} &= K\,V_S\,I_{sec}(1+\epsilon)\cos(\varphi_{T+B}-\delta) \\ &\approx K\,P_{meas}(1+\epsilon+\delta\,\tan\varphi_{T+B}) \end{aligned} \tag{12}$$

The error is thus $\varepsilon + \delta \tan \varphi_{T+B} \approx 0.6\%$, which deserves consideration.

Acquired skills: mastery of current transformer's complex errors, mainly in a power measurement context.

– Difficulties: few students correctly evaluate the incidence of φ_{T+B} on the power error.

3.4 Power and power factor measurements

– Question #9: evaluate the power consumption of the whole "tube plus ballast" P_{T+B}.

Answer: the power consumed by the voltmeters can again be neglected, for the same reason as in Question #5. P_{T+B} is thus simply:

$$P_{T+B}=P_{TOT}-\frac{V_S^2}{R_W} \tag{13}$$

Experimentally, one finds $P_{T+B} = 44.8$W. The wattmeter thus consumes about 4% of the total power, which is nonnegligible.

Acquired skills: correct use of a wattmeter, power correction.

Difficulties: about one third of the students does not realize that the measured power is not that of the tube with its ballast, but includes the power consumption of the wattmeter and voltmeter V_1.

– Question #10: connect appropriately the wattmeter in order to measure the power consumed by the tube itself. If necessary, correct the measured value taking into account the power consumption of the voltage circuit of the apparatus.

Answer: since the tube acts as a voltage stabilizer (see Fig. 3 and 4), for a defined v_S, the current in the ballast is also defined, whatever is connected in parallel to the tube. Consequently, when the voltage circuit of the wattmeter is in parallel with the tube, the measured power (i.e. that of the tube and of the internal resistor of the wattmeter voltage circuit) is equal to that consumed by the tube itself when the wattmeter is not connected. This is experimentally confirmed by adding a resistor in parallel to the tube: the measurement remains unchanged despite the added consumption. We have thus here a very unusual situation where it is appropriate to bring no correction due to the wattmeter consumption!

Acquired skills: reflection on a nonlinear circuit and on the correct use of a wattmeter.

Difficulties: this phenomenon is generally very badly understood by the students. Only about 10% of them correctly analyze the situation. This is partly explained by the fact that they are tempted to apply the same principle than in Question #5, but mostly this is simply because the phenomenon involved here is rather complex. The message is that it is not adequate to apply in a nonlinear context recipes only valid for linear circuits.

– Question #11: verify that the experimental power factors of the tube itself, and of the tube and its ballast, are close to those obtained by the simplified theory given in Appendix.

Answer: the simplified theory announces a power factor ($PF = P / VI$) of 0.9 for the tube itself. It was more or less verified experimentally: with $V_S = 230$V, we obtained $V_T = 108$V, $I_T = 0.406$A and $P_T = 34.5$W. We thus obtain a power factor of 0.79, i.e. a difference of 14% with the simplified theory. The power factor $(PF)_{T+B}$ of the tube and its ballast is theoretically equal to:

$$(PF)_{T+B}=0.9\frac{V_T}{V_S}=0.42 \tag{14}$$

which is rather close to what we measured experimentally: with $V_S = 230$V, $I_T = 0.406$ and $P_{T+B} = 44.8$W, we obtain a value of 0.48, i.e. a difference of 13% with the simplified theory. In both cases, the difference between theory and practice is due to the restrictive hypothesizes made to calculate the power factors. Namely, cos φ_{T+B} is better than predicted by the theory due to the losses in the ballast.

Acquired skills: confrontation of theory and practice.

3.5 Dimming

– Question #12: evaluate the pertinence of dimming the light in terms of luminous efficiency and power factor.

Table 1. Incidence of dimming, a.o. on the power factors and on the luminous efficiency (measured data are in straight characters and computed data in italic).

V_S [V]	V_T [V]	I_T [mA]	P_{T+B} [W]	P_T [W]	V_T / V_S [%]	$(PT)_{T+B}$	$(PF)_T$	$F_L / F_{L,230V}$ [%]	$F_L P_{T,230V} / F_{L,230V} P_T$ [%]	$\eta_L / \eta_{L,230V}$ [%]
230	108	0.406	44.8	34.5	*47.0*	*0.48*	*0.79*	100	*100*	*100*
220	110	0.366	42.2	32.3	*50.0*	*0.52*	*0.80*	94.8	*101.3*	*101*
210	112	0.331	36.3	29.6	*53.3*	*0.52*	*0.80*	85.1	*99.2*	*105*
200	115	0.291	32.3	27.0	*57.5*	*0.55*	*0.81*	78.8	*100.7*	*109*
190	118	0.256	29.6	23.5	*62.1*	*0.61*	*0.78*	67.7	*99.4*	*110*

Answer: starting from 230V, we will diminish V_S by steps of 10V until the tube lights off. At each step, a series of quantities are measured and/or calculated. Those are shown on Table 1. Note that the DC voltage V_D at the output of the photosensor (not mentioned on Table 1) is proportional to the luminous flux F_L emitted by the tube. It only allows to make relative flux measurements, but it is sufficient in our case. We define the luminous efficiency $\eta_L = F_L / P_{T+B}$ (lm/W). From Table 1, we can make the following observations at decreasing V_S:

(a) I_T, P_{T+B} and P_T diminish. V_T increases, due to the negative slope in the DC characteristic of Fig. 2.
(b) $(PF)_{T+B}$ increases, in the same proportion as the ratio V_T / V_S, as predicted by (A.15). In terms of pollution of the mains, it is thus advantageous to dim the light.
(c) The power factor of the tube $(PF)_T$ is steady and rather close to 0.9, as predicted by the simplified theory.
(d) The luminous flux obviously diminishes, while the ratio $(F_L / P_T) / (F_{L,230V} / P_{T,230V})$ remains constant. It proves that F_L is only due to the power consumed by the tube alone.
(e) The luminous efficiency, normalized regarding that at 230V, increases. It is thus also advantageous to dim the tube in terms of luminous efficiency.

Note that other aspects should be considered to completely evaluate the pertinence of dimming the tube, like the harmonic content of I_T, the tube's lifetime or the arc's stability at low voltage. These aspects are beyond the scope of this training session.

Acquired skills: notion of luminous efficiency, critical analysis of dimming versus pollution of the mains and luminous efficiency.

Difficulties: observations (b) and (e) are generally correctly made by the students, but the others are most of the time ignored.

4. CONCLUSIONS

An original training session was proposed in the frame of a basic course in industrial electrical measurement, as a part of the programme of the diploma in electrical and electromechanical engineering. The aim is to exercise the students to a critical mastery in the use of AC volt-, am-, and wattmeters in a nonlinear context. A deep reflection on measurement corrections and the incidence of nonsinusoidal waveforms is systematically encouraged. A comparison with analytical developments on a simplified model enriches this reflection.

APPENDIX: SIMPLIFIED THEORY

The following analytical developments are, as far as we know, original and reveal interesting conclusions. The aim is to give some preliminary results orienting the students for what regards the expected observations they will do.

We suppose the ideal situation where the tube is a pure voltage stabilizer, i.e. presenting a constant voltage U_T of same polarity as $i_T(t)$, and the ballast is purely inductive, linear and loss-free (modeled by constant L). The circuit is represented on Fig. A1, where:

$$v_S(t)=\sqrt{2}\,V_S\sin(\omega t+\varphi_0) \\ v_T(t)=U_T\,sign\,i_T(t) \qquad (A.1)$$

Note that U_T is equal to the RMS value V_T. The time origin is chosen at the current zero crossing with positive slope.

A.1 Current computation

All the quantities are periodic and both alternations are identical (sign excepted). The system is linear and permanent if we consider one alternation only, since there is inversion of U_T at the current zero crossing. We will thus consider henceforth $i_T(t) \geq 0$ $(0 \leq \omega t < \pi)$.

Applying the superposition principle for the source $u_S(t)$ and the constant "source" U_T, and considering $i_T(t) = 0$ for $t = 0$ and $t = \pi / \omega$, we obtain:

$$i_T(t)=\frac{-\sqrt{2}\,V_S}{\omega L}\cos(\omega t+\varphi_0)-\frac{U_T}{L}\left(t-\frac{\pi}{2\omega}\right) \qquad (A.2)$$

with:

$$\cos\varphi_0=\frac{\pi U_T}{2\sqrt{2}\,V_S} \qquad (A.3)$$

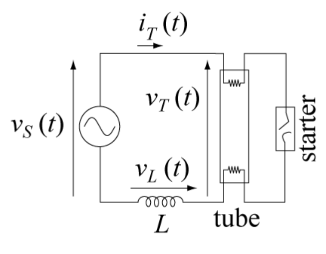

Fig. A1. Circuit used for the simplified theory.

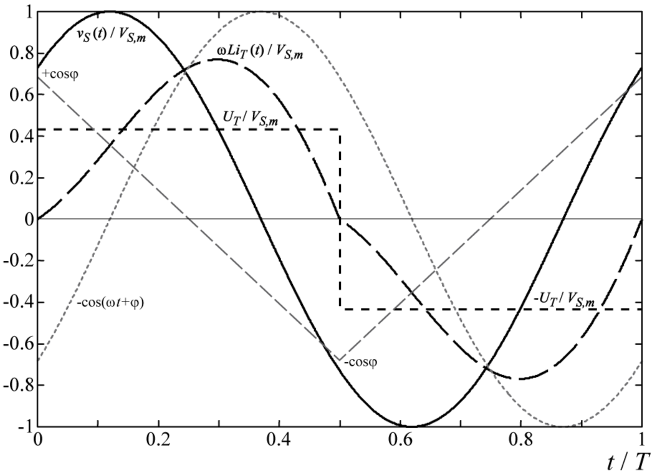

Fig. A2. Signals obtained with the simplified theory.

Current $i_T(t)$ is thus composed of a sinusoidal term lagging $v_S(t)$ by 90°, and a linear one of slope $-U_T/L$. For given V_S, ω and U_T (i.e. given supply and tube), $i_T(t)$ is proportional to $1/L$. Also note that φ_0 is independent from L.

If the resistance of the ballast was taken into account, this linear term would become an exponential of time constant L/R, with the same initial slope. For the 36-W tube with ferromagnetic ballast used here, $L/R \approx 20$ ms, i.e. twice the time of an alternation. The exponential would shift from the ideal straight line of only 20% at the end of one alternation, which brings pertinence to the hypothesis of a loss-free ballast.

Fig. A2 presents the curves $v_S(t)$, $v_T(t)$ and $i_T(t)$ for a typical value of $U_T = 0.615\, V_S$. These curves were normalized by dividing the voltages by $\sqrt{2}V_S$ and multiplying the current by $\omega L / \sqrt{2}V_S$. The abscissa is the time normalized to the period T.

For the same numerical situation, a Fourier analysis of the current gives a fundamental amplitude of 0.74 and harmonics 3 and 5 of amplitude versus the fundamental of respectively 8.3% and 2.9%. The RMS value of the current is therefore increased of only 0.4% with regard to the fundamental, which authorizes later to fuse both quantities.

The fundamental of $i_T(t)$ (still normalized) is:

$$\frac{\omega L}{\sqrt{2}V_S} i_{T,1}(t) = a_1 \cos\omega t + b_1 \sin\omega t \qquad \text{(A.4)}$$

where constants a_1 and b_1 are determined by the theory of Fourier analysis. All computing made, we obtain:

$$a_1 = -\cos\varphi_0\left(1 - \frac{8}{\pi^2}\right) \quad ; \quad b_1 = \sin\varphi_0 \qquad \text{(A.5)}$$

yielding the fundamental's normalized amplitude:

$$\frac{\omega L}{\sqrt{2}V_S} I_{T,1,m} = \sqrt{a_1^2 + b_1^2} = \sqrt{1 - 0.964\cos^2\varphi_0} \qquad \text{(A.6)}$$

This confirms the value of 0.74 found hereabove for $U_T = 0.615\, V_S$. It also allows us to use the approximation (nonlacunary regimes – see hereafter):

$$\sin\varphi_0 \approx \frac{\omega L}{\sqrt{2}V_S} I_{T,1,m} \approx \frac{\omega L}{V_S} I_T \qquad \text{(A.7)}$$

Note that a permanent conduction of the tube was supposed hereabove. This requires that at zero crossing for $\omega t = 0$, $i_T(t)$ has a positive slope. The limit case is therefore:

$$\left(\frac{di}{dt}\right)_{t=0} = 0 \Leftrightarrow \pi\sin\varphi_0 = 2\cos\varphi_0 \Leftrightarrow \varphi_0 = 32°5 \qquad \text{(A.9)}$$

For $\varphi_0 < 32°5$, i.e. by (A.3) for $U_T > 0.76\, V_S$, current $i_T(t)$ is lacunary (dead zones where the current is nil). This situation is not recommended since it provokes the arc extinction in real cases.

A.2. Power computation

– Effective power: by definition, the effective power of the tube is:

$$P_T = \left[v_T(t)\, i_T(t)\right]_{mean} = U_T \left[i_T(t)\right]_{mean,\,1\ altern} \qquad \text{(A.10)}$$

i.e. by (A.2):

$$P_T = \frac{2\sqrt{2}U_T V_S}{\pi\omega L}\sin\varphi_0 \qquad \text{(A.11)}$$

As for the current, for given V_S, ω and U_T, P_T is proportional to $1/L$ (φ_0 being independent from L).

Supposing that $i_T(t)$ is sinusoidal (this approximation is valid when dealing with RMS value, as shown in Section A.1), one has:

$$P_T \approx 0.9\, U_T I_T \qquad \text{(A.12)}$$

For our idealized situation where the ballast is supposed loss-free, this power is identical to the total power (ballast included). In practice, the losses are not negligible and we actually have $P_{T+B} = P_T + P_B$.

– Power factors: for nonsinusoidal signals, the power factor PF is by definition the ratio between the effective power and the product of the RMS values of the current and the voltage.

By (A.12), the power factor of the tube $(PF)_{tube} \approx 0.9$. This is true for any regime (specific U_S, ω and L).

v_S being sinusoidal, the total power can be written as:

$$P_{T+B} = V_S I_{T,1} \cos\varphi_1 \qquad \text{(A.13)}$$

where $\cos\varphi_1$ is the phase shift between $v_S(t)$ and the fundamental of $i_T(t)$. As $I_T \approx I_{T,1}$, (A.13) is also:

$$P_{T+B} \approx V_S I_T \cos\varphi_1 \approx V_S I_T (PF)_{T+B} \qquad \text{(A.14)}$$

The latter expression, knowing that $P_T = P_{T+B}$ (lossfree ballast) and using (A.3), (A.7) and (A.11), leads to:

$$(PF)_{T+B} \approx 0.9\frac{U_T}{V_S} \qquad \text{(A.15)}$$

The power factor of the whole "tube plus ballast" thus only depends on the ratio U_T / V_S in the ideal case. The value of L controls the power but not the power factor.