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WHICH PHYSICAL QUANTITY OF TURBULENT STRUCTURES IS MEASURED IN CROSS CORRELATION FLOWMETERS ?

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Abstract – Measurements of flow velocity with cross correlation functions of ultrasonic signals show that the travelling time of structures determined by the peak of the functions deviates from the average flow velocity. This difference usually is explained by the difference between line integral of measurement and area integral of the average flow velocity.

A comparison of the frequency distribution of single velocity components in the fluid determined by particle image velocimetry with the cross correlation measuring method shows that the most frequent components in the fluid are in accordance with the travelling time of structures measured by cross correlation. The physical explanation can be given by means of the impulse response.

Keywords: flowmeters, cross correlation, ultrasound

1. INTRODUCTION

Measurements of velocities by cross correlation methods (ccf) base on the determination of travelling time of a pattern between two barriers. The maximum of ccf as measure of highest similarity indicates the travelling time. The ccf represents the auto correlation function (acf) shifted by the travelling time.

Mathematically the ccf results from the convolution of the acf with the impulse response. So long as the pattern between the two barriers is constant the theory is well explained. But in case of flow measurement the pattern of dissipating structures changes between the two barriers. Which physical quantity is measured in this case?

Stochastic processes are stationary if the statistical characteristics are independent of the local position. Therefore the probability density functions (PDF) of stationary processes are independent of the local position, too. In stationary processes the probability density of two sets are independent of two local positions x and y but only dependent of their difference. The ergodicity of a stochastic process requires its stationarity. Local ergodic processes admit the application of local average values instead of time average values. For this reason correlation functions can be applied to such processes.

2. EXPERIMENTS

The flow velocity of a gaseous fluid in a pipe of 100 mm diameter has been determined by the measurement of cross correlation functions of the signals of two parallel ultrasonic beams, figure 1.

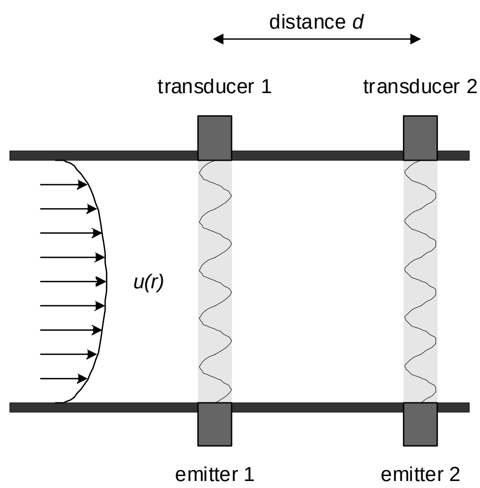

Fig. 1: Principle of transit time measurement

The ultrasonic signals with a carrier frequency of 220 kHz are modulated by turbulent structures of fluid with a velocity dependent modulation frequency from some Hertz to about 3 kHz. The ultrasonic signals are modulated in amplitude as well as in phase. The demodulation can be done by undersampling the carrier frequency and by Hilbert transform of the side bands [1, 4].

Experiments have shown that the evaluation of the phase leads to more stable results. The phase signals are cross correlated. The peak of the function marks the travelling time of the modulating structures. This travelling time deviates from the average flow velocity time and must be calibrated.

There are two models for the explanation of causal connections between flow measurement by cross correlation and real flow velocity by calibrated devices. The first one proceeds on the assumption that the velocity measured by ccf corresponds to the integrated velocity along the measuring path. The ratio of the cross section integral to the line integral is applied for correction of systematic errors completed by calibration methods as function of Reynolds number [1 – 4].

The average velocity is defined by the area integral

$$\bar{u}_{def} = \frac{1}{A}\int_A u(A)\, dA \tag{1}$$

whereas the real measurement only uses the ultrasonic beam path which is given by

$$\bar{u}_{us} = \frac{1}{R}\int_R u(r)\, dr \ . \tag{2}$$

The flow profile is described by the simplified power law for turbulent pipe flow

$$u(r) = \hat{u}\left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \tag{3}$$

where R is the radius of the pipe and n represents the velocity dependent deformation of the profile

$$\frac{1}{n} = 0{,}25 - 0{,}023\,\lg(\mathrm{Re}) \tag{4}$$

in the range of $4\cdot 10^3 \le \mathrm{Re} \le 3{,}2\cdot 10^6$ with Re = Reynolds number [5].

Putting (3) into (1) the average velocity is

$$\bar{u}_{def} = \frac{1}{\pi\, r^2}\int_0^R\int_0^{2\pi} \hat{u}\left(1 - \frac{r}{R}\right)^{\frac{1}{n}} r\, d\varphi\, dr \ , \tag{5}$$

$$\bar{u}_{def} = \frac{2\hat{u}\, n^2}{(n+1)(2n+1)} \ . \tag{6}$$

The line integral is

$$\bar{u}_{us} = \frac{1}{l}\int_{-R}^{+R} \hat{u}\left(1 - \frac{r}{R}\right)^{\frac{1}{n}} dr \ , \tag{7}$$

$$\bar{u}_{us} = \frac{n\,\hat{u}}{n+1} \ . \tag{8}$$

The relationship between (6) and (8) is

$$r_{dev} = \frac{u_{us}}{\bar{u}_{def}} = \frac{2n+1}{2n} = 1 + \frac{1}{2n} \ , \tag{9}$$

As n is a function of Reynolds number the deviation is decreasing with increasing flow velocity and is in the range between 1,08 to 1,05. That means that the measured velocity by cross correlation is higher than the real average flow velocity.

Only a few authors until now [6 - 9] interpret the measuring device that means the section between the two barriers including sensors as a linear time invariant system. The cross correlation function (ccf) $\phi_{12}(t)$ is given by the convolution of impulse response $h(t)$ and auto correlation function (acf) $\phi_{11}(t)$

$$\phi_{12}(t) = h(t) * \phi_{11}(t) \ . \tag{10}$$

The transit time of structures between two ultrasonic barriers bases on the determination of the maximum of the cross correlation function. This maximum results from the skewed shape of the impulse response and the symmetric auto correlation function. An example of impulse functions is given in figure 2.

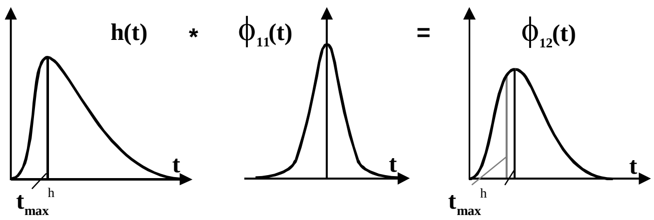

Fig. 2: Influence of asymmetry of the impulse response on the peak of cross correlation function

The width of acf is in inverse proportion to the bandwidth of the signal modulated by the structures in the fluid and therefore it is dependent on the flow velocity. The higher the velocity the wider is the bandwidth and the narrower is the width of acf, figure 3. That means that the deviation between the maximum of ccf and impulse response is the bigger the lower the flow velocity is.

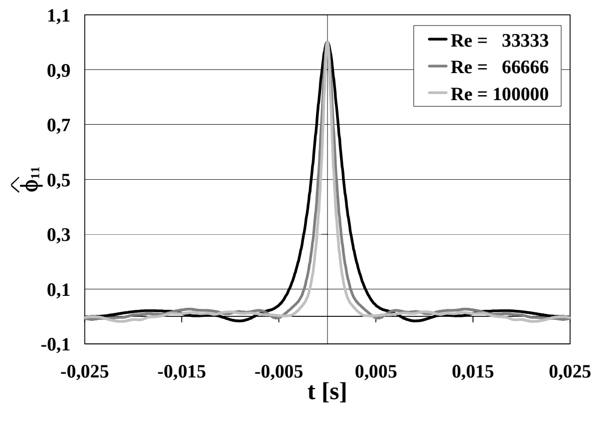

Fig. 3. Auto correlation function of phase modulated signal with parameter Reynolds number Re

The structures in the fluid are transported with different velocities on account of the velocity profile and dependent on the position in the pipe. This transportation process influences the impulse response.

A second effect is the diffusion process of structures in the fluid between the two barriers influencing the impulse response. An exact description is given in [9].

The velocity distribution $f(u)$ can be determined by the law of transformation of probabilities from the density distribution of markings (structures) in the pipe cross section or in the volume of detection which is specified by the sensor, respectively. It is given by

$$|f(u)| = \left| f(r) \frac{1}{du/dr} \right| . \tag{11}$$

$f(r)$ is the density distribution of structures across the pipe cross section. Applying the power law (3) for the flow profile and considering the limited area of the sensor beam the velocity distribution is

$$|f(u)| = \frac{2 \cdot n}{\hat{u}} \left[\left(\frac{u}{\hat{u}} \right)^{n-1} - \left(\frac{u}{\hat{u}} \right)^{2n-1} \right] . \tag{12}$$

The distribution of the concentration of structures considering both effects results from the spatial convolution of the velocity profile with the Gaussian local distribution of fluctuations. The maximum of the impulse response is shifted to higher time values by the turbulent diffusion, figure 4.

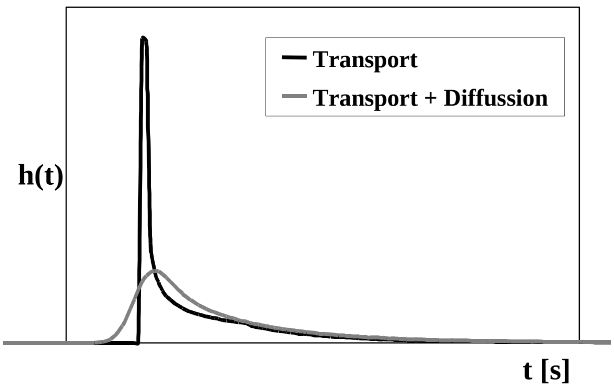

Fig. 4. Impulse responses for transport and diffusion processs

3. EXPERIMENTAL RESULTS

Measured cross correlation functions are shown in figure 5.

The width of ccf is decreasing with increasing flow velocity coincident with increasing correlation coefficient. This effect results from the increasing bandwidth of signals with increasing velocity. Wide band signals cause narrow auto correlation functions. In the velocity domain the width of correlation functions is increasing with increasing velocity.

The frequency distribution of velocity components has been determined by the evaluation of particle image velocimetry pictures [4, 9].

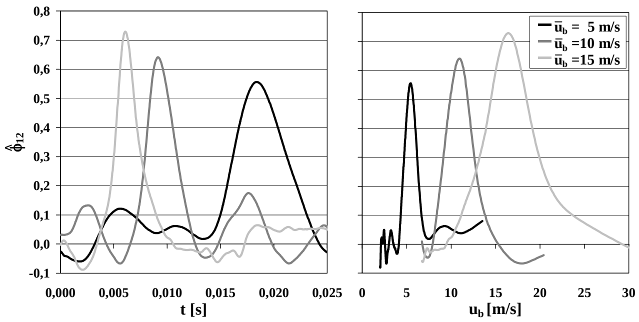

Fig. 5. Cross correlation functions for different Reynold numbers in time (a) and velocity (b) domain

Figure 6 shows the profile and frequency distribution of flow velocity components. The skew frequency distribution shows a distinct peak at 22,9 m/s. The skew distribution results from the impulse response.

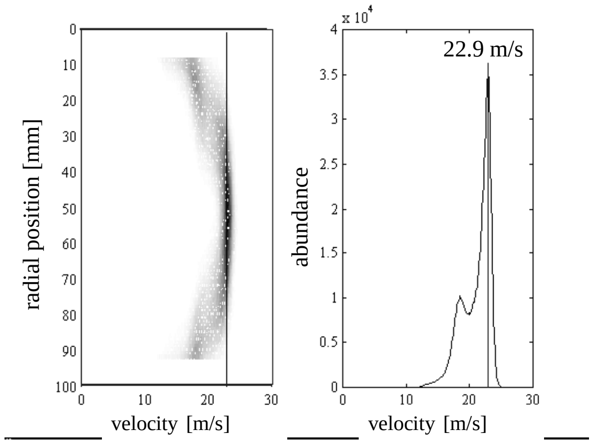

Fig. 6. Flow profile (left) and frequency distribution of flow velocity components

Figure 7 shows the result of simulation and real measurements.

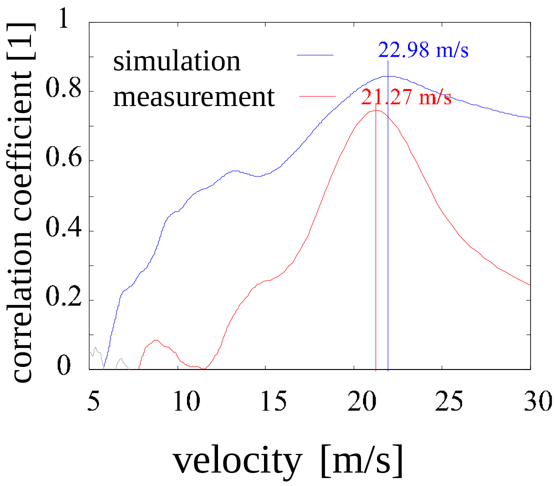

Fig. 7. Result of simulation compared with real measurements

A comparison of both figures shows a good conformity of velocities measured by the most frequent value of the skew probability density distribution and the cross correlation function.

Physically that means that the most frequent velocity components in the fluid are measured by the cross correlation method.

4. CONCLUSIONS

It could be shown that the cross correlation function results from the convolution of the auto correlation function with the impulse response. The impulse response corresponds to a skew distribution of the probability density of the velocity components in the fluid.

The most frequent components are responsible for the peak in the cross correlation function at the determination of flow velocity.

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