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MEASUREMENT OF QUALITY INDEXES IN PROCESS MONITORING: AN ORIGINAL APPROACH TO UNCERTAINTY ANALYSIS

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Abstract − In this paper is presented an original approach to the definition of the metrological characteristics of the measurement systems used in process index monitoring.

The model proposed allows the evaluation of the probability that the process under statistical control shows a different behaviour from that inferred by the measured data, relating to the input conditions. The proposed model is developed in general hypothesis on the statistic characteristics of the process index monitoring and a typical application cases is presented.

Keywords: Systems Qualification, Measurement Uncertainty and Quality Indexes.

1. INTRODUCTION

Often, when a process is controlled by measurements of quality indexes is necessary to compare the measured data with a given reference value. It's obvious, that system measurement uncertainty can play an important rule for the decision-making in the process supported; in fact measurement uncertainty may erroneously show data over or under a warning or a threshold levels [1-5]. For example in the case of measurement station for air quality monitoring an erroneous evaluation of the effects of measurement uncertainty may involve an incorrect decision about overcoming of air pollution limits. In order to solve this problem, an original statistical model has been set up. This model proposed represents a tool that allows formal treating of the examined problem. With the help of this model it's possible to determine the metrological characteristics required for the measurement system, once defined the per cent level of confidence required to the decision-making process. Adopting a formal way to express the features of this model, it's fair to say that it's allows to calculate the erroneous decision probability P_α, that we making in stating that the process index is over the control limits, but it is within these limits, and vice versa to calculate the erroneous decision probability P_β, that we making in stating that the process index is within the control limits, but it is over these limits. Clearly the adopted decision-making criterion assumes that a process index is over control limits when the measured data associated to this index is over control limits, and assumes that the process index is within the control limits when the measured data associated to this index is within the control limits [6-7].

The definition of the proposed model arises from the fact that, once one has found the functional relationship between the erroneous decision probability and *i*) the statistical characteristics of the monitored process index and *ii*) the measurement system uncertainty, it's possible, for a given process, to invert this relationship in order to obtain the determination of the metrological characteristics of the utilized measurement system, with an a priori imposition of the erroneous decision probability. Therefore this relationship may be used both into an analysis phase, in order to obtain the erroneous decision probability, for a given process and a given measurement system, and into a synthesis phase, in order to design metrological system characteristics for a given erroneous decision probability. It's worth noticing that the definition of erroneous decision probability, for a given system, allows to adopt an objective criterion for the understanding of the allowed system measurement uncertainty in correlation with the monitored statistical characteristics of the process.

2. THE PROPOSED MODEL

For a given quality index of a monitored system, the aim of monitoring consists in ensuring that the values of such index are within control limits. It is assumed that: *i*) the considered quality index may be modeled like a stationary stochastic process, whose probability distribution function (PDF) is known; *ii*) the acquired data for such quality index have a known PDF around their expectation; *iii*) this PDF doesn't depend on the particular value of the expectation of the data; *iv*) the expectation value of the acquired data is not biased by a systematic error. In the hypothesis listed above the following variables can be considered: *i*) a continuous random variable (R.V.) x_m associated with the monitored quality index of the process; *ii*) a continuous R.V. x associated with the realizations of measured data around their expectation (the R.V. x represents the randomness of any measurement and its variance is just the standard uncertainty of the measurement); *iii*) a continuous R.V. y associated with the measured data, which is obviously the sum of x_m and x.

With these variables, the specifications on the quality index of the process can be set into membership restrictions to an appropriate interval for the R.V. x_m. In the case in which $x_m \in [-\delta + x_a, x_a + \delta]$ the model is called *two sides bounded*, where x_a is the expected value for the monitored index and δ is the reference displacement limit of the R.V. x_m from x_a; whereas in the case in which $x_m \le \delta$ it is called *one side bounded*.

In the first case, the erroneous decision probability, that we making in stating that the process index is beyond the control limit, evaluate as the joint probability of the events $A = \{x_m \in [-\delta + x_a, x_a + \delta]\}$ and $B = \{|x + x_m - x_a| > \delta\}$ is:

$$P_\alpha = P(x_m \in [-\delta + x_a, x_a + \delta] \quad \cap \quad |x + x_m - x_a| > \delta)$$

where the Event A is the condition that the monitored process index is beyond the control limit (CL), and the event B is the condition that the measured data associated to this index is below the control limit, due to measurement uncertainty. It can be shown that, under the assumed hypotheses for the two sides bounded model, P_α is given by:

$$P_\alpha = \int_{-\delta}^{\delta} [1 - F_x(\delta - \tilde{x}') + F_x(-\delta - \tilde{x}')] \cdot f_{\tilde{x}_m}(\tilde{x}') d\tilde{x}' \tag{1}$$

where x' is a variable used in order to express the occurrence of x_m, $f_{\tilde{x}_m}(\cdot)$ is the PDF of the R.V. $\tilde{x}_m$, given by $f_{x_m}(x' - x_a)$ and $\tilde{x}_m$ is a R.V. given by $(x_m - x_a)$.

This Equation allows the evaluation of P_α from the cumulative distribution function (CDF) of the measured data around their expectation $F_x(\cdot)$ and the PDF of the process index $f_{\tilde{x}_m}(\cdot)$.

In a similar way is possible the evaluation of the erroneous decision probability P_β, that we making in stating that the process index is below the control limit, as the joint probability of the events $\overline{A} = \{x_m \in X_m - [-\delta + x_a, x_a + \delta]\}$ and $\overline{B} = \{|x + x_m - x_a| < \delta\}$:

$$P_\beta = P(x_m \in X_m - [-\delta + x_a, x_a + \delta] \quad \cap \quad |x + x_m - x_a| < \delta)$$

then P_β is given by:

$$P_\beta = \int_{\tilde{X}_m - [-\delta, \delta]} [F_x(\delta - \tilde{x}') - F_x(-\delta - \tilde{x}')] \cdot f_{\tilde{x}_m}(\tilde{x}') d\tilde{x}' \tag{2}$$

For the one side bounded model the erroneous decision probability, that we making in stating that the process index is beyond the control limit, is:

$$P_\alpha = P(x_m \le \delta \quad \cap \quad x + x_m > \delta)$$

then:

$$P_\alpha = \int_0^{\delta} [1 - F_x(\delta - x')] \cdot f_{x_m}(x') dx' \tag{3}$$

Moreover, the erroneous decision probability, that we making in stating that the process index is below the control limit, is:

$$P_\beta = P(x_m > \delta \quad \cap \quad x + x_m < \delta)$$

then:

$$P_\beta = \int_{X_m - [0, \delta]} F_x(\delta - x') \cdot f_{x_m}(x') dx' \tag{4}$$

The equations (1) and (2) are the constitutive equations of the bilateral model whereas equations (3) and (4) are the constitutive equations of the unilateral model; in fact these equations express the erroneous decision probabilities in terms of the statistical characteristics of the process and in terms of the statistical distribution of the measured data around their expectation.

It's worth noticing that, since x_m and x are independent R.V., the PDF of y is the convolution of their PDF:

$$f_y(y') = conv(f_{x_m}(\cdot), f_x(\cdot), y') \tag{5}$$

therefore the PDF of x_m can be obtained, making the deconvolution of the PDF of y and x:

$$f_{x_m}(x') = deconv(f_y(\cdot), f_x(\cdot), x') \tag{6}$$

From the precedent reasoning it follows that there are two possibility for the probabilities P_α and P_β determination: a) the PDF of x_m and x are known; b) the PDF of x and y are known; in this case the PDF of x_m is achieved by means of (6).

The case a) is of concern when there are previous information about both the process and the utilized measurement system, whereas the case b) is of concern when there are data only about the utilized measurement system the data about the process are achieved from the measured data.

Moreover, it's worth noticing that for the (6) the probability distribution function of x_m is given by the convolution of the PDF of y and x, so if $\sigma_x / \sigma_y << 1$ then:

$$f_{x_m}(x') = deconv(N(\mu_y; \sigma_y), U(-a; a), x') \approx N(\mu_{x_m}; \sigma_{x_m}).$$

Since $\mu_y = E[y] = E[x_m + x] = E[x_m] = \mu_{x_m} = \mu$ and $\sigma_y^2 = \sigma_{x_m}^2 + \sigma_x^2$ with $\sigma_x = a/\sqrt{3}$ it is:

$$\sigma_{x_m} = \sigma_y \cdot \sqrt{1 - \left(\frac{\sigma_x}{\sigma_y}\right)^2} = \sigma_y \cdot \sqrt{1 - \left(\frac{a}{\sigma_y \sqrt{3}}\right)^2} \tag{7}$$

and:

$$f_{x_m}(x') \approx N\left(\mu \; ; \sigma_y \cdot \sqrt{1 - \left(\frac{a}{\sigma_y \sqrt{3}}\right)^2}\right) \tag{8}$$

3. A TYPICAL APPLICATION CASE

The project of a new measurement system, whose performance assures prearranged values of erroneous decision probabilities P_α and P_β, has to be made as follows: for the adopted measurement system is assumed a suitable PDF for the distribution of the data around their expectation (i.e.: if measurement uncertainty is due to a quantization process we can assume that *x* has uniform PDF, but in general an estimate of the PDF of *x* may be obtained with the use of well known standard statistical methods); the PDF of the index process, to be monitored, is known in the case a) or it can be obtained the from measured data with the use of (6) in the case b). Once the PDF of *x* and x_m, are known, the (1) and (2) (for the bilateral case) or the (3) and (4) (for the unilateral case) can be applied in order to obtain P_α and P_β.

In many cases it's easy to get the functional relationship between the probabilities P_α and P_β and a) the measurement standard uncertainty of the utilized measurement system and b) the control limit value δ. Furthermore this relationship may be expressed into normalized diagrams in which the measurement standard uncertainty and the control limit value are normalized with respect to the standard deviation of the measured data, which accounts both for the standard deviation of the process and for the standard uncertainty of the utilized measurement system.

In the following is analyzed the two sides bounded case in which the R.V. x_m is normal with $x_a = \mu_{x_m}$ and the R.V. x is uniform in $[-a,a]$, with $a < 2\delta$. normal with zero mean and standard deviation σ_x. In this case, it is possible to demonstrate that P_α and P_β can be expressed as in equation (9), where $G(\cdot)$ is the CDF of the gaussian distribution. It's worth noticing that (9) can be written as: $P_\alpha = P_\alpha(\delta/\sigma_y, a/\sigma_y)$ and $P_\beta = P_\beta(\delta/\sigma_y, a/\sigma_y)$ which are two functions of the parameters δ/σ_y and a/σ_y. These two functional relationships make feasible to plot normalized diagrams in which P_α e P_β are functions of a/σ_y,

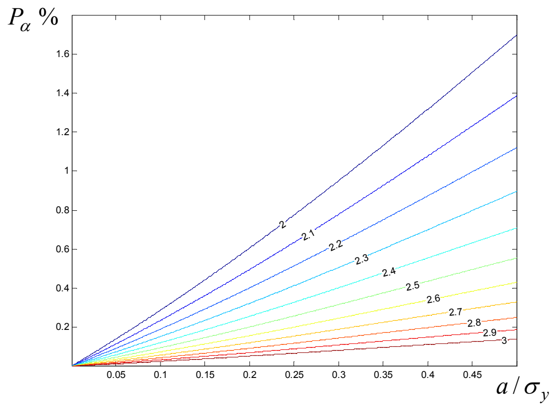

Figure 1. Normalized diagram of P_α % versus a/σ_y with δ/σ_y as parameter.

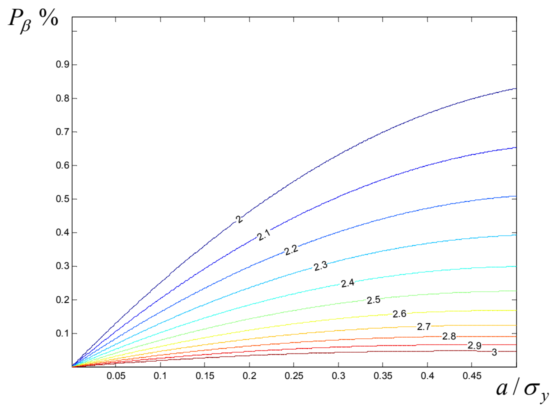

Figure 2. Normalized diagram of P_β % versus a/σ_y, with δ/σ_y as parameter.

provided that δ/σ_y has been fixed.

It should be noted that δ/σ_y is in the range $[2,3]$; in fact for a gaussian PDF the control level $\delta = 2\cdot\sigma_y$ is usually considered as a warning level, whereas the control level $\delta = 3\cdot\sigma_y$ is usually considered as the threshold limit, be-

$$P_\alpha = \left(1-\frac{\delta}{a}\right)\cdot\left[G\left(\frac{\delta}{\sigma_y\cdot\sqrt{1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2}}\right)-G\left(\frac{\delta-a}{\sigma_y\cdot\sqrt{1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2}}\right)\right]+\frac{\sigma_y\cdot\sqrt{1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2}}{a\sqrt{2\pi}}\cdot\left[e^{\frac{-(a-\delta)^2}{2\cdot\sigma_y^2\cdot\left(1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2\right)}}-e^{\frac{-\delta^2}{2\cdot\sigma_y^2\cdot\left(1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2\right)}}\right]$$

$$P_\beta = \left(1+\frac{\delta}{a}\right)\cdot\left[G\left(\frac{a+\delta}{\sigma_y\cdot\sqrt{1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2}}\right)-G\left(\frac{\delta}{\sigma_y\cdot\sqrt{1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2}}\right)\right]-\frac{\sigma_y\cdot\sqrt{1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2}}{a\sqrt{2\pi}}\cdot\left[e^{\frac{-\delta^2}{2\sigma_y^2\cdot\left(1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2\right)}}-e^{\frac{-(a+\delta)^2}{2\sigma_y^2\cdot\left(1-\left(\frac{a}{\sigma_y\sqrt{3}}\right)^2\right)}}\right] \tag{9}$$

cause in absence of special causes the measured data have 99.9% of probability to be within the interval $\left[-3\cdot\sigma_y+\mu_y,\mu_y+3\cdot\sigma_y\right]$. It's worth noticing moreover that a/σ_y is in the range $\left]0{,}0.5\right]$, in fact greater values for the parameter a/σ_y are not compatible with the hypothesis of gaussian PDF for the distribution of the data, because they could imply a statistical distribution of the data which should reflect the statistical characteristics of the utilized measurement system instead of the monitored index of the process. In the figures 1) to 4) P_α assumes values in the range $\left]0{,}1.7/100\right]$ whereas P_β assumes values in $\left]0{,}0.82/100\right]$; greater values of a/σ_y would imply greater values of the erroneous decision probability, in particular for P_α, and this circumstance may not be allowed for an industrial productive process.

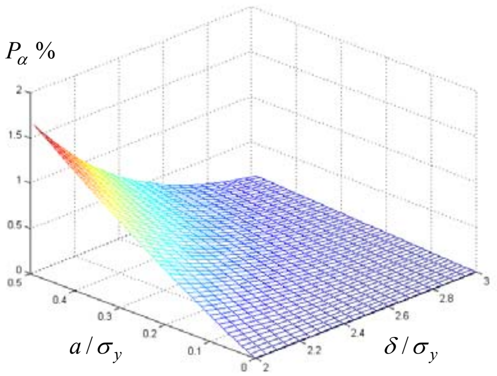

Figure 3. P_α % versus a/σ_y and δ/σ_y .

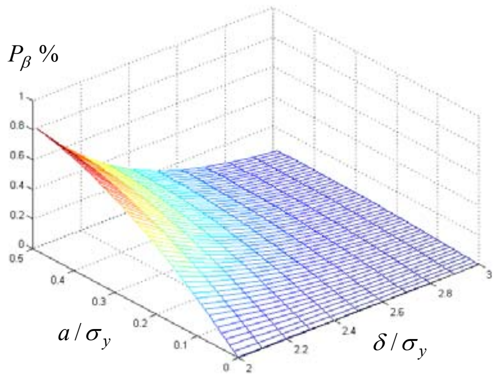

Figure 4. P_β % versus a/σ_y and δ/σ_y .

4. CONCLUSIONS

An original statistical model for the choice of the maximum allowed measurement uncertainty in systems monitoring has been proposed; this model allows the evaluation of the impact of the measurement uncertainty onto the decision-making process supported by quality indexes. With the use of this model one is able to carry out information both on the performance that a measurement system should have, in order to assure prefixed performance in systems monitoring, and on the level of confidence for the measured data in order to correctly estimate the trend of monitored parameters.

An applications of the proposed method for the qualification of measurement systems in process monitoring has been presented.

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