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UNCERTAINTY ESTIMATE OF COMBINATION OF VERIFIED WEIGHTS

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Abstract - Weights are frequently used in combinations. In most cases it is assumed that weights of the same set have large covariances. The safest approach is to assume that the correlation coefficient is equal to one. However, this may lead to an overestimation of the combined uncertainty. A model can be constructed, based on the calibration/verification method suggested by the OIML (Organisation Internationale de Métrologie Légale), to avoid the unnecessary overestimation of the combined uncertainty. The model suggests that the uncertainty of combinations of weights with the same nominal value can be easily calculated to reduce the combined uncertainty. It also explains that magnitude of the correlation among weights with different nominal values depends on the accuracy class of the weights.

Keywords: correlation uncertainty weights

1. INTRODUCTION

The OIML R111 International Recommendation on Weights [1] declares that the reference weights in any combination are so strongly correlated that the combined standard uncertainty should be calculated as the sum of the standard uncertainties of the individual reference weights:

$$u(\sum m) = \sum u(m_i) \tag{1}$$

Some laboratories believe that this model is valid only for combinations of weights with the same nominal values, but is not true for weights for different nominal values. In this paper a model for handling this issues is described.

2. BASIC FACTS AND ASSUMPTIONS

The OIML classes of accuracy for weights are: E_1, E_2, F_1, F_2, M_1, M_2 and M_3. E_1 weights are used to ensure traceability between the national mass standard(s) and weights of lower classes. E_1 These weights are used for verification of weights of lower class and/or other calibrations/verifications.

As a result of the realisation of the mass scale by subdivision the masses of the weight set are strongly correlated [2]. The method of subdivision is usually used only for E_1 weights. All other weights are verified using weights of the next higher class of accuracy (Fig. 1).

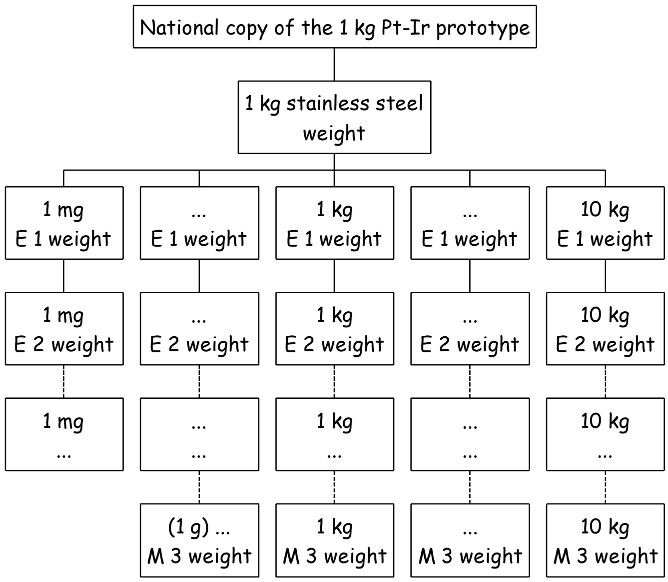


Fig 1. A typical traceability chain for weights.

Weights of different classes of accuracy are normally not used together in combination.

Weights of each class of accuracy have maximum permissible errors (MPE) at least three times larger than those of the next higher class of accuracy.

The uncertainties of verified weights are calculated following the OIML recommendation so the uncertainties do not exceed one third of the relevant maximum permissible error.

A lot of uncertainty components have contribution to the combined uncertainty of verification, but only the uncertainty in the standard causes strong correlation between the standard and the verified weights.

3. COMBINATION OF TWO WEIGHTS OF THE SAME NOMINAL VALUE

The verification of a weight $\{m_1, u(m_1)\}$ is a difference measurement $\{k_1, u(k_1)\}$ from the standard $\{m_0, u(m_0)\}$ which can be expressed:

$$m_1 = m_0 + k_1 \tag{2}$$

Assuming the independence of m_0 and k_1, the covariance between the standard and the verified weights is equal to the variance of m_0 [3]. Assuming the independence of m_0, k_1 and k_2 the covariance of the combination of two verified weights derived from the same standard is also the variance of m_0. Assuming the same uncertainty for the two weights $u(m)$ and that the uncertainty of the verified weights are three times larger than the uncertainty of the standard, the variance of the uncertainty of the combination is:

$$u^2(m_1+m_2) = u^2(m_1) + u^2(m_2) + 2u(m_1,m_2) = \\ = 2u^2(m) + 2u^2(m)/9 = 20/9 \cdot u^2(m) \quad (3)$$

This value is closer to the value of the uncorrelated weights than the fully correlated ones, so the combined uncertainty is significantly smaller than the one suggested by the OIML.

4. COMBINATION OF SEVERAL WEIGHTS WITH THE SAME NOMINAL VALUE

The investigation of this kind of combination of weights is particularly important, because it is frequently used for calibration of weighing instruments for heavier loads.

Fig. 2. Set of weights (20 kg) for calibration of weighing instruments

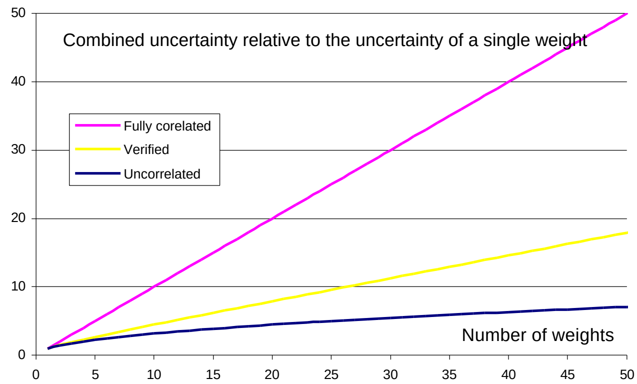
The model can be solved for n weights derived from the same standard. The uncertainty is [4]:

$$u^2(\sum_{i=1}^{n} m_i) = \sum_{i=1}^{n} u^2(m_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} u(m_i,m_j) \quad (4)$$

The formula (4) can be rewritten using $u(m_i) = u(m_j) = u(m)$, $u(m_i,m_j) = [u(m)/3]^2$ and $\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} 1 = n\cdot(n-1)/2$:

$$u^2(n\cdot m) = u^2(m)\left[n + \frac{n\cdot(n-1)}{9}\right] \quad (5)$$

Graph 1 shows the results for different combinations according to this model ("verified"), to the model assuming full correlation $u^2(n\cdot m) = n^2u^2(m)$, and to the model assuming no correlation $u^2(n\cdot m) = nu^2(m)$. The magnitude of the overestimation of the correlation following the OIML recommendation is so significant if a large numbers of weights are used that weights of higher class of accuracy are needed to compensate for this. On the other hand, the uncorrelated model is definitely an underestimate of the uncertainty of the weight combinations.



Graph 1. Uncertainties of weight combinations

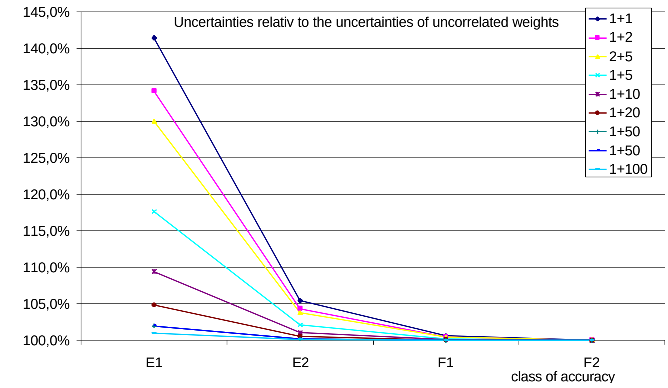
5. COMBINATION OF TWO WEIGHTS WITH DIFFERENT NOMINAL VALUES

Weights with different nominal values are not verified using the same standard, so the correlation will be traced back, via the standards used, to the correlation of the E_1 weights. This means that the value of the correlation is the same between any weights of any classes of accuracy derived from the same E_1 standard and it gets less and less relative to the maximum permissible error with the lowering classes of accuracy.

This effect can be demonstrated for weightshavier than 50 g, because, for those weights, the maximum permissible error and the uncertainty of the weights are linear functions of the nominal values. For two weights ($k\cdot m$, and $l\cdot m$) the correlation is:

$$u(m_k,m_l) = k\cdot l\cdot u^2(m)/3^{2j} \quad (6)$$

Where $u(m)$ is the uncertainty of m mass and j is the number of subsequent verifications from the E_1 standard (e.g. for F_2 weights j is three). Graph 2 shows possible combinations of two weights with different nominal value as a function of the class of accuracy. The uncertainty contribution caused by the correlation quickly vanishes with the increasing number of subsequent verifications.



Graph 2. Uncertainties of weight combinations relative to the uncorrelated weights

6. COMBINATION OF WEIGHTS FROM THE SAME SET OF WEIGHTS

A set of weights is usually arranged to allow any load, from the minimum value to the combination of all the weights in the set, to be produced in increments equal to the smallest weight in the set. The sequence of the set is composed one of the following combinations: (1;2;5) x 10^n kg, (1;1;1;2;5) x 10^n kg, (1;2;2;5) x 10^n kg, (1;1;2;2;5) x 10^n kg, where n represents a positive or negative whole number or zero.

Fig. 3. A set of weights (1mg-2 kg)

This regulation means that a set of weights may contain a maximum of three weights with the same nominal value. The weights with the same nominal value are usually verified using the same standard. (Fig. 4, case 1)

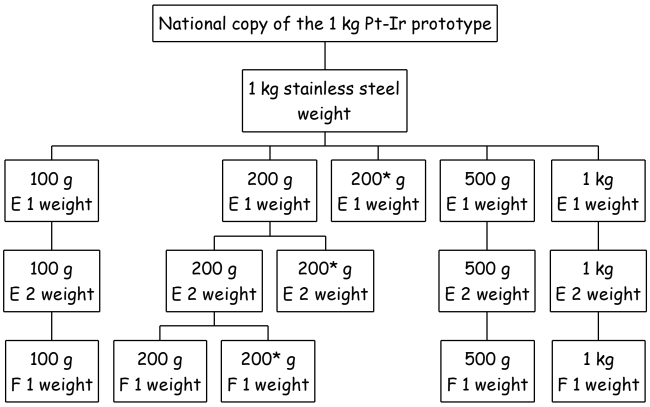
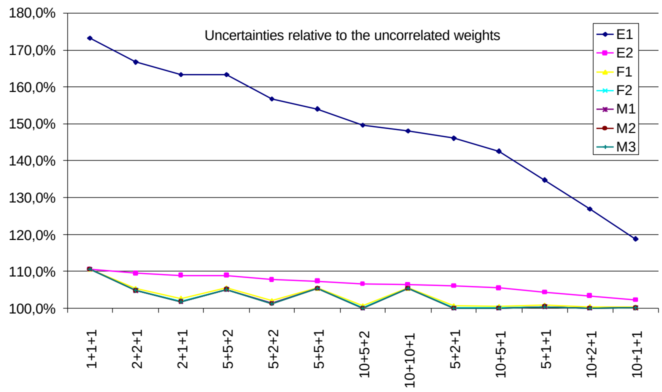


Fig. 4. A possible traceability chain of F_1 weights, case 1

The uncertainties of the possible combinations of three weights in a decade are shown on graph 3. The data for class E_1 additionally illustrates the combinations of fully correlated weights.



Graph 3. Uncertainties of possible combinations of three weights, case 1 (graphs for F_1… M_3 are almost the same)

It can be clearly seen that the combinations containing weights with the same nominal value have larger uncertainties. These uncertainties do not get smaller relative to the maximum permissible error with the lowering class of accuracy, because the cause of the correlation is the same standard, used for the verification of the weights with the same nominal value, not the correlation of the E_1 weights.

In the case of the traceability seen in Fig. 5 (case 2) there is no standard used for verification of more than one of the weights in the set. This means that any combination of weights in the set are only correlated only due to the correlation of the E_1 set, so the correlation and thus the combined uncertainty are smaller, Graph. 4.

For weights belong to classes of accuracy E_1 and E_2 the values of uncertainty are the same in both cases, but from F_1 the difference is significant. In the second case from F_1 in practice the uncorrelated model is valid.

Usually, there is no information about which case was chosen for verification of the weights, so the worst case (case 1) should be assumed.

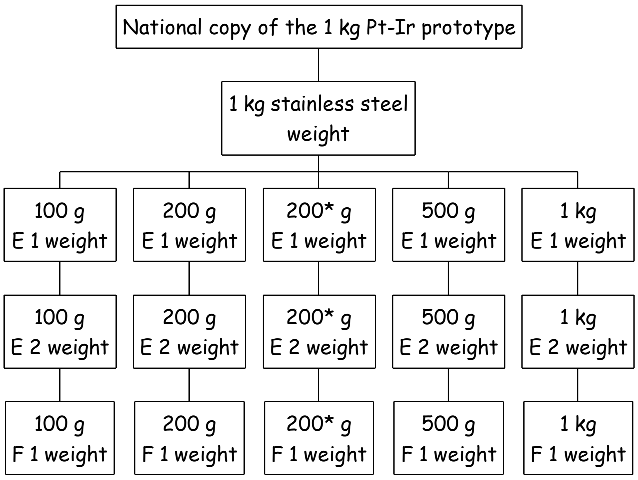
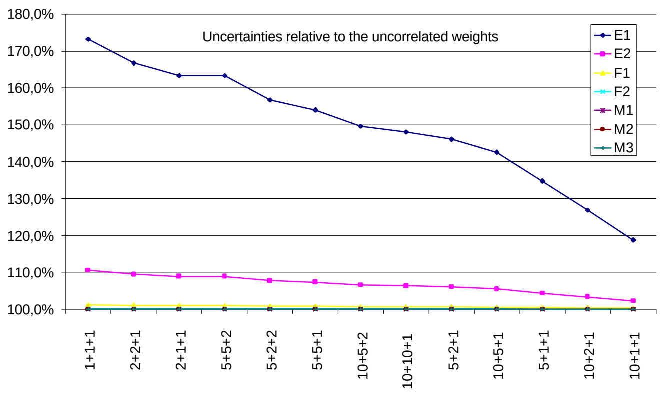


Fig. 5. Another traceability chain of F_1 weights, case 2



Graph 4. Uncertainties of combinations of three weights

7. CONCLUSIONS

Using the model, described in this work, based on the suggested OIML verification method, the estimated uncertainty of the weight combinations can be significantly reduced. In case of combination of weights with same nominal values can easily be calculated and at the same time leads to a smaller combined uncertainty that makes this model useful for calibration laboratories.

A bit more theoretical result is that the construction of the traceability chain (not using the same standard to verify more than one weight from the set) effects the correlation among the weights in the set, so the uncertainty of combinations of weights will be smaller.

REFERENCES

[1] OIML R 111:1994 International Recommendations, "Weights of classes E_1, E_2, F_1, F_2, M_1, M_2, M_3"

[2] R. Schwartz, "Guide to mass determination with high accuracy" *PTB-Bericht*, MA-40, 1995

[3] E. Gáti, "Handling correlated quantities in metrology", *OIML bulletin* Volume XL, 1999

[4] „Guide to the Expression of Uncertainty in Measurement"; BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML; 1995

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