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UNCERTAINTY EVALUATION IN SMALL ANGLE CALIBRATION USING ISO GUM APPROACH AND MONTE CARLO METHOD

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Abstract - This work investigates the determination of measurement uncertainty in calibration of small angle measurement instruments. After attentive study of the sources of variation, an expression was developed to determine the measurement uncertainty. Two approaches were used to determine the measurement uncertainty: ISO GUM and Monte Carlo simulation method. An example is presented and the calibration of an Electronic level was carried out using a sine table at the Metrology Laboratory, in the University of Brasília, Brazil. The expanded uncertainty results showed good agreement of both techniques and Monte Carlo method proved to simplify analysis when uncertainty involves expressions with some degree of complexity.

Key words: uncertainty, Monte Carlo method, small angle

1. INTRODUCTION

Nowadays, the search for quality has been promoting an increasingly effort in the enterprises that are looking for ISO 9000 standard certification. As a consequence, the demand for calibration services has been growing up. One of the requirements to attain ISO 9000 certification is that calibration results must be expressed in conjunction with the measurement uncertainty.

The measurement uncertainty determination is addressed by an ISO publication since 1993 whose well known title is Guide to the Expression of Uncertainty in Measurement (GUM) [1]. This document is largely used all around the world and it was translated to several different languages. Some difficulties related to its application comes from that cases where complex formulae relating input and output quantities is developed. A revision of this standard is taking place in order to address this and other key aspects of the GUM.

The calibration of small angle measurement instruments like the Spirit level and the Electronic level used in mechanical industry is a necessary effort to assure quality of the measurements [2]. This may be accomplished using a sine bar and gauge blocks to establish standard angles and than comparing to the measured angles. The Measurement uncertainty may be determined according to ISO GUM [1] but it may be pointed out that trigonometric relationship among variables brings some difficult when deriving the expression to obtain the measurement uncertainty formulae.

An approach that is growing in acceptance by researchers is the Monte Carlo simulation and its application is performed generating the variability according to expected probability distributions of each variable [3, 4]. In this work the calibration of an Electronic Level is carried out to compare these two approaches used to determine the measurement uncertainty.

2. CALIBRATION AND UNCERTAINTY

Since the plane angle is defined in terms of the full circle and there is no primary standard artifact for the angle, angle measurement is better performed when the round angle is divided as equal as possible. Thus, the calibration of angle measurement instruments may be carried out by using a measuring table or by using a sine bar. The option by the sine table may be done when dealing with calibration of small angle measuring instruments. In this case, the angle determined is related to the length of the gauge blocks and its uncertainty is closely related to the gauge blocks uncertainties.

An experimental assembly scheme to measure an inclination θ using an Electronic Level placed over a sine table is showed in Fig. (1). If this Electronic Level has a bias, the systematic error may be determined comparing standard angle θ_s obtained on sine table with angle θ measured using the Electronic Level. The calibration procedure involves gradually increasing the standard angle θ_s and recording the correspondent indication θ to draw a calibration curve.

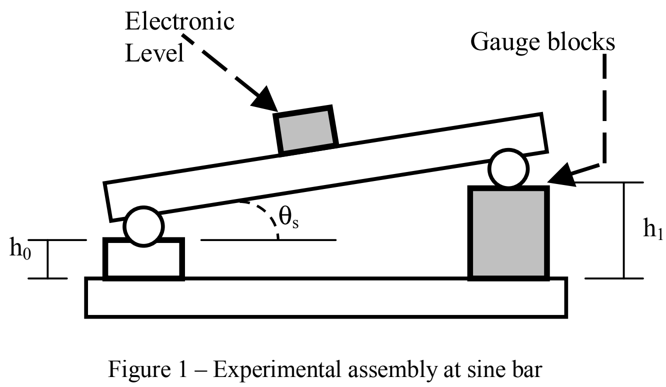

Figure 1 – Experimental assembly at sine bar

2.1 – ISO GUM approach

According to ISO Guide [1], determination of measurement uncertainty begins setting out a mathematical model that holds all variables influencing the measurand. The first step is the investigation of the variables involved in the measurement using the Electronic Level. It was considered that measured angle θ depends on standard angle at sine bar (θ_s), instrument bias ($\Delta\theta_s$), bias associated to instrument resolution (ΔR), bias associated to temperature variation in relation to reference temperature 20° C (ΔT_{20}), bias associated to temperature difference between instrument and gauge used (ΔT_{dif}) and roundness of sine bar cylinder (ΔC). A mathematical model that represents the effect of these variables is showed in Eq. (1). The effect of drift was not considered.

$$\theta = \theta_s + \Delta\theta_s + \Delta R + \Delta T_{20} + \Delta T_{dif} + \Delta C \qquad (1)$$

The measurement uncertainty model is obtained applying error propagation on Eq. (1) and thus we have Eq. (2). As showed, the combined standard uncertainty (u_θ) of measured angle is a function of the standard uncertainties associated to the variability of measured angles ($u_{\Delta\theta s}$), the standard angle established using the sine bar ($u_{\theta s}$), the instrument resolution ($u_{\Delta R}$), the temperature variation in relation to standard reference ($u_{\Delta T20}$), the temperature difference between standard and the instrument ($u_{\Delta Tdif}$) and the roundness of sine bar cylinder ($u_{\Delta C}$).

$$u_\theta^2 = u_{\theta_s}^2 + u_{\Delta\theta_s}^2 + u_{\Delta R}^2 + u_{\Delta T_{20}}^2 + u_{\Delta T_{dif}}^2 + u_{\Delta C}^2 \qquad (2)$$

The standard angle depends on trigonometric relation established when gauge blocks are positioned over the sine table to perform measurement. Thereby the standard angle θ_s is calculated by arcsine function of the ratio between the height of gauge blocks (h_1) decreasing the initial height on sine table (h_0) and the distance between cylinders of sine table (L). Eq. (3) shows this expression.

$$\theta_s = arcsin\left(\frac{h_1 - h_0}{L}\right) \qquad (3)$$

Since the standard angle θ_s determined by Eq. (3) is a function of the height of gauge blocks, the uncertainty $u_{\theta s}$ is related to the gauge blocks standard uncertainties u_{h0} and u_{h1} and the sine bar length standard uncertainty u_L. After the application of error propagation, the standard uncertainty $u_{\theta s}$ is determined according to Eq. (4).

$$u_{\theta_s}^2 = \left(\frac{\partial\theta}{\partial h_0}\right)^2 \cdot u_{h_0}^2 + \left(\frac{\partial\theta}{\partial h_1}\right)^2 \cdot u_{h_1}^2 + \left(\frac{\partial\theta}{\partial L}\right)^2 \cdot u_L^2 \qquad (4)$$

The sensitivity coefficients related to the first, second and third terms in Eq.(4) are determined by partial derivatives of angle θ in respect to L, h_0 and h_1 respectively. Since there are two parameters in these expressions, it must be considered the variable transformation $u = (h_1 - h_0) / L$. Thus, these coefficients are calculated by Eq. (5), (6) and (7).

$$a_1 = \frac{\partial\theta}{\partial h_0} = \frac{\partial\theta}{\partial u}\cdot\frac{\partial u}{\partial h_0} = \frac{-1}{L}\cdot\frac{1}{\sqrt{1-((h_1-h_0)/L)^2}} \qquad (5)$$

$$a_2 = \frac{\partial\theta}{\partial h_1} = \frac{\partial\theta}{\partial u}\cdot\frac{\partial u}{\partial h_1} = \frac{1}{L}\cdot\frac{1}{\sqrt{1-((h_1-h_0)/L)^2}} \qquad (6)$$

$$a_3 = \frac{\partial\theta}{\partial u}\cdot\frac{\partial u}{\partial L} = -\frac{(h_1-h_0)}{L^2}\cdot\frac{1}{\sqrt{1-\left(\frac{(h_1-h_0)}{L}\right)^2}} \qquad (7)$$

The standard uncertainty of length L on sine table (u_L) may be classified as type B uncertainty source and a rectangular probability distribution is admitted. The uncertainty of height h_0 on sine table (u_{h0}) may be classified as type B uncertainty source and a rectangular probability distribution is considered.

The standard uncertainty of gauge blocks length (u_{h1}) may be classified as type B uncertainty source having a normal probability distribution and it is determined using gauge block calibration results. Since the uncertainty of each individual gauge block (m) may be expressed by $u_m = A + B \cdot L_m$, where A and B are constants and L_m is the length of an individual gauge block, the uncertainty of m gauge blocks stacked may be determined using Eq. (8).

$$u_{h_1} = \sqrt{m \cdot A^2 + A \cdot B \cdot L_m + B^2 \cdot \sum_{i=1}^{m} L_i^2} \qquad (8)$$

The standard uncertainty of variability of measured angle was determined as type A uncertainty source having normal probability distribution, measuring three times the standard angle with the Electronic Level. The standard uncertainty of the Electronic Level resolution was determined as type B uncertainty source having rectangular probability distribution. The standard uncertainties of the temperature variation in relation to standard reference ($u_{\Delta T20}$), the temperature difference between standard and the instrument ($u_{\Delta Tdif}$) and the roundness of the sine table cylinder ($u_{\Delta C}$) were determined as type B uncertainty sources having rectangular probability distribution.

2. 2 – Monte Carlo approach

Other approach used to determine measurement uncertainty is the Monte Carlo simulation, that is recommended when dealing with complex measurement processes in dimensional metrology [3, 4]. This method involves the determination of the probability distribution of the measurand by simulating the values of all variables involved in measurement. Fig. (2) shows a general scheme of the simulation procedure.

In the present investigation, the main function that deals with measurement process is represented by Eq. (1), in which standard angle θ_s is determined according to Eq. (3). In this expression, each variable have a characteristic distribution of its values that may be represented by a probability density function. Thereby, it is possible to simulate its values by generating random numbers according to the expected probability distribution and according to the specified range of variation. The theoretical background on this subject may be found in the literature [5].

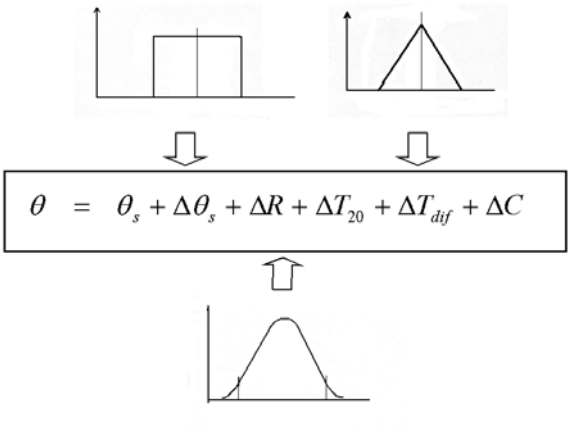

Figure 2 – General scheme of the simulation procedure

Random numbers were generated according to normal and rectangular distributions using Excel software and it was simulated 10000 trials admitting the probability distribution of each variable as pointed out to uncertainties determined by ISO GUM approach. The standard deviation of the measured angle was considered as the combined standard uncertainty and its value was compared with the values estimated using ISO GUM approach.

In these two approaches investigated, the expanded uncertainty was determined multiplying u_θ by the coverage factor k=2 adopting normal probability distribution with 95% probability.

3. RESULTS

The Electronic level was calibrated in 12 different angles on sine bar, starting at 0^o and using $0{,}5^o$ steps as angle interval to increase and decrease the angle.

The combined standard uncertainty was calculated according to ISO GUM to standard angle of 306,9927 min and the results are presented in Tab. (1). In this table, the standard uncertainties were determined considering: the circularity error of sine table cylinder as 0,2 µm, the constants of the gauge blocks calibration as A=0,045 and B=0,15, the experimental standard deviation of measured angle as 0 µm, the resolution of the Electronic Level as 1 minute, the temperature variation during measurement of 20 ± $0{,}1^o$C, the temperature difference of $0{,}4^o$C, the range of variation of the length h_0 on sine table as 12 µm and the range of variation of length L on sine table as 16 µm.

As observed in Tab. (1), the resolution of the Electronic Level was the most significant effect influencing the combined standard uncertainty of the Electronic Level. It was shown that expanded uncertainty is 0,578 min for a coverage probability of 95%.

The Results using Monte Carlo simulation method showed similar values of expanded uncertainty for the same coverage probability of 95% and it is 0,586 min.

Table 1 – Uncertainty of Electronic level – ISO GUM approach (θ_S = 306.9927 min)

Uncertainty source (u)	*Symbol*	*Type*	*Prob. Distr.*	*DF*	*Sens. Coeff.*	*Results u*
Sine bar height	h_0	B	Rectangular	∞	$-4*10^{-6}$ 1/µm	1,7321 µm
Gauge blocks height	h_1	B	Normal	∞	$-4*10^{-6}$ 1/µm	0,1124 µm
Sine bar Length	L	B	Rectangular	∞	$4*10^{-7}$ 1/µm	2,3094 µm
Variability of angle measured	$u_{\Delta\theta s}$	A	Normal	2	1	0,0000 min
Electronic level resolution	$u_{\Delta R}$	B	Rectangular	∞	1	0,2886 min
Temperature variation	$u_{\Delta T20}$	B	Rectangular	∞	1	0,0000 min
Differential temperature	$u_{\Delta Tdif}$	B	Rectangular	∞	1	0,0000 min
Roundness	$u_{\Delta C}$	B	Rectangular	∞	1	0,0008 min
Combined standard uncertainty					*u = 0,289 min*	
Expanded uncertainty 95%			*k=2*		*U = 0,578 min*	

4. CONCLUSIONS

A comparison of ISO GUM approach and Monte Carlo simulation method was carried out using Electronic Level calibration results and it was observed good agreement between these techniques. Monte Carlo simulation reduced time spent in analysis and is suitable when elaborate mathematical expressions are developed to model the measurement.

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