XVII IMEKO World Congress
Metrology in the 3rd Millennium
June 22–27, 2003, Dubrovnik, Croatia

EXPERIMENTAL VERIFICATION OF CALCULATED FREQUENCY CHARACTERISTICS OF OCTOFILAR RESISTORS

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Abstract – With an intention to reveal possible inaccuracies of mathematical models used in calculations of frequency characteristics of resistors of octofilar design, comparisons of a 12 906 Ω and a 1 Ω octofilar resistor against monofilar and quadrifilar calculable resistors of the same nominal values have been performed. A possibility to carry out these precision comparisons by means of a commercially available LCR meter has been investigated.

Keywords: calculable resistor, octofilar design, LCR meter

1. INTRODUCTION

Measurements of frequency dependences of both conventional and quantum resistance standards can conveniently be made by comparison with calculable resistors, i.e. with resistors constructed in such a way that frequency dependences of their values can be calculated, with a sufficient accuracy, from the knowledge of their constructional parameters. Of all available versions of calculable resistors, coaxial monofilar resistors are of the simplest design and calculation of their frequency characteristics can be made with the lowest uncertainties. On the other side, coaxial resistors of higher resistance values, as well as coaxial resistors for higher current loads, can often be of unreasonable length (e.g. the length of a 12 906 Ω coaxial resistor for characterization tests of quantum Hall devices would be approximately 3 m even if a Nikrothal wire of a diameter of only 20 µm were used as its resistive element). However, this difficulty can be overcome by appropriately folding the resistive element.

For example, one can arrive at a so-called quadrifilar reversed resistor [1] by folding the resistive element in such a way that it forms a double loop arranged so that the current flows in opposite directions in its two halves. With such an arrangement, magnetic fluxes produced by the two halves of the loop tend to cancel. At the Department of Measurements (DM), Faculty of Electrical Engineering, Czech Technical University, a 12 906 Ω quadrifilar resistor having the resistive double loop made from a 20 µm Nikrothal wire has been prepared. The time constant of this resistor is 15,2 ns, and the relative change of the parallel equivalent resistance from its DC value is less than 5 parts in 10^7 for frequencies up to 5 kHz.

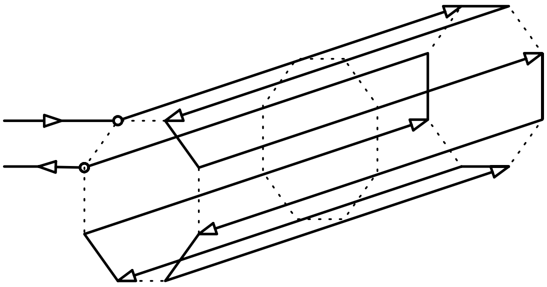

Fig. 1. Resistive element of an octofilar resistor

To arrive at an even shorter resistive element and to make thermoregulation less difficult, a 12 906 Ω octofilar resistor has been fabricated recently [2], the resistive element of which is formed by four long loops connected in series (Fig. 1). This element is again made from the 20 µm Nikrothal wire and its length is one half of that of the 12 906 Ω quadrifilar resistor. The time constant of the octofilar resistor is 9,4 ns and, in a frequency range from 500 Hz to 5 kHz, the frequency dependence of its parallel equivalent resistance is more than seven times smaller than that of the quadrifilar resistor.

In calculating the frequency preformance of an octofilar resistor, changes in resistance arising from parasitic inductances and capacitances, as well as from eddy currents induced both in the wire itself and in the cylindrical shield of the resistive element, have to be evaluated.

As for the effect of parasitic inductances, calculations based on a uniform transmission line model of the octofilar resistor have shown that the corresponding relative change of the parallel equivalent resistance from the DC value is [2]:

$$r_{\mathrm{L,M}} = \frac{\omega^2}{R^2}\left(L - 16M_1 + 16M_2 - 16M_3 + 8M_4\right)^2 \quad (1)$$

where ω is angular frequency, R is AC resistance of the resistive element, L is self-inductance of the total length of the resistive element and M_i , $i = 1, \ldots, 4$, is mutual inductance between one of the parallel parts of the resistive element and the *i*-th adjacent part.

Similarly, relative change r_C of the parallel equivalent resistance due to parasitic capacitances is

$$r_C = \frac{\omega^2 R^2}{737280}\Big[1920\,C_0^2 - 4\left(C_0 + 32C_1 + 32C_3\right)^2 - \\ - 60\left(C_0 + 16C_1 + 32C_2 + 16C_3\right)^2 - \\ - 15\left(C_0 + 16C_1 + 16C_2 + 16C_3 + 16C_4\right)^2\Big] \qquad (2)$$

where C_0 is total capacitance between the resistive element and the cylindrical shield and C_i , i = 1, …, 4 is capacitance between one of the parallel parts of the resistive element and the i-th adjacent part.

In case of octofilar resistors of higher resistance values, the change r_C , which is proportional to R^2 , normally predominates over the change $r_{L,M}$. As an illustration, both these changes calculated from constructional parameters of the above 12 906 Ω resistor are plotted as functions of frequency in Fig. 2 (note that $10^2 \times r_{L,M}$ rather than $r_{L,M}$ is plotted). On the other hand, the change $r_{M,L}$, which is proportional to $1/R^2$, usually prevails in case of resistors of low resistance values, such as the 1 Ω resistor described in section 2.2 is (for the corresponding characteristics see Fig. 3) .

Mainly these facts were borne in mind when decision was made to prepare octofilar resitors of substantially different resistance values (12 906 Ω and 1 Ω) and to carry out their comparisons with calculable resistors of other designs in order to verify experimentally the correctness of calculating parasitic inductances and capacitances, as well as to prove the adequacy of the uniform transmission line model used in evaluating frequency characteristics of octofilar resistors.

2. COMPARISONS OF CALCULABLE RESISTORS

2.1. International comparisons in the framework of EUROMET No. 432 project [3]

To make comparisons of 12 906 Ω calculable resistors of different designs possible, a four terminal-pair thermostatted travelling resistance standard with nominal value of 12 906 Ω has been fabricated and provided as transfer standard by the DM acting as the pilot laboratory. A Vishay S 102 K resistor forms resistive element of this standard and a 100 Ω platinum resistance thermometer makes it possible to monitor the inner temperature. Laboratories participating in the comparison have been requested to measure relative changes of the parallel equivalent resistance (AC-DC differences) and time constant of the travelling standard by comparing it with resistors having calculable frequency performance. Monofilar, quadrifilar and octofilar resistors were available and all measurements were made by means of coaxial four terminal-pair bridges. Results obtained by comparing the travelling standard and the 12 906 Ω octofilar resistor at the DM have been in good agreement with arithmetic means of results reported by the other laboratories (this agreement has been better than 15 parts in 10^8 for the AC-DC difference and frequencies up to 5 kHz, and better than 0,6 ns for the time constant). As an example of data obtained by the participants, Figs. 4 and 5 show AC-DC differences r_{LAB} and time constants reported for the 12 906 Ω travelling resistor and a frequency of 1 000 Hz.

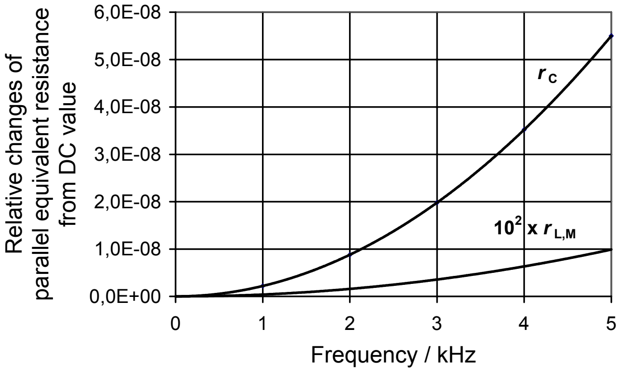

Fig. 2 Changes of parallel equivalent resistance of the 12 906 Ω octofilar resistor as functions of frequency

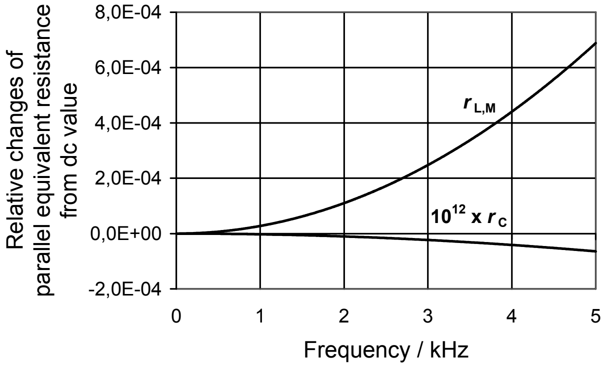

Fig. 3 Changes of parallel equivalent resistance of the 1 Ω octofilar resistor as functions of frequency

Half-length of the uncertainty bar of each reported value corresponds to its expanded uncertainty for a coverage factor k = 2. By means of broken lines, the levels $r_{mean} \pm U(r_{mean})$ and $\tau_{mean} \pm U(\tau_{mean})$ are marked, where r_{mean} and τ_{mean} are unweighted means of the reported AC-DC differences and time constants, and $U(r_{mean})$ and $U(\tau_{mean})$ are expanded uncertainties of these means (k = 2 again).

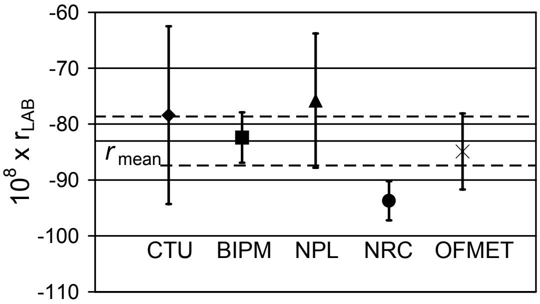

Fig. 4 AC-DC differences reported for the 12 906 Ω travelling resistor and a frequency of 1000 Hz.
Participating institutions:
CTU - Czech Technical University, Prague
BIPM - Bureau International des Poids et Mesures, Sèvres
NPL – National Physical Laboratory , Teddington
NRC – National Research Council, Ottawa
OFMET – Swiss Federal Office of Metrology, Wabern

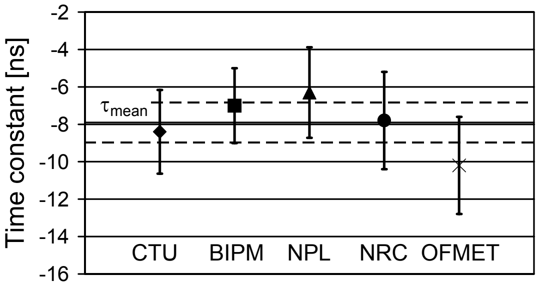

Fig. 5. Time constants reported for the 12 906 Ω travelling resistor and a frequency of 1 000 Hz

TABLE I. Constructional parameters and time constant of the 1 Ω octofilar resistor

Wire radius	0,523 mm
Wire resistivity	$4{,}27 \times 10^{-7}$ Ωm
Distance between adjacent parts of the resistive element	5 mm
Distance between wire parts and axis of the cylindrical shield	6,53 mm
Folded length of the resistive element	252 mm
Inside shield radius	39 mm
Shield thickness	1 mm
Shield resistivity	$2{,}78 \times 10^{-8}$ Ωm
Time constant	$8{,}35 \times 10^{-7}$ s

TABLE II. Constructional parameters and time constant of the 1 Ω coaxial resistor

Wire radius	0,176 mm
Wire resistivity	$4{,}27 \times 10^{-7}$ Ωm
Length of the resistive element	192 mm
Inside shield radius	19,9 mm
Shield thickness	0,055 mm
Shield resistivity	$1{,}74 \times 10^{-8}$ Ωm
Time constant	$1{,}91 \times 10^{-7}$ s

2.2. Comparison of 1Ω calculable resistors

The above comparisons proved correctness of the calculation of frequency characteristics of resistors of a relatively high resistance value, frequency dependence of which is mainly due to the effect of parasitic capacitances. With an intention to prepare a similar comparison of calculable resistors of lower values, frequency dependence of which results mainly from the effect of parasitic inductances, a 1 Ω resistor of the octofilar design and a 1 Ω coaxial resistor have been realized. All relevant constructional parametrs of these resistors and their time constants are summarized in Tables I and II.

TABLE III. AC-DC differences of ratio of parallel equivalent resistances of 1 Ω resistors

f / kHz	d_{calc}	d_{ext}	$d_{\text{ext}} - d_{\text{calc}}$
0,5	$6{,}558 \times 10^{-6}$	$6{,}625 \times 10^{-6}$	$6{,}7 \times 10^{-8}$
1,0	$2{,}624 \times 10^{-5}$	$2{,}650 \times 10^{-5}$	$2{,}6 \times 10^{-7}$
1,5	$5{,}902 \times 10^{-5}$	$5{,}963 \times 10^{-5}$	$6{,}1 \times 10^{-7}$
2,0	$1{,}049 \times 10^{-4}$	$1{,}060 \times 10^{-4}$	$1{,}1 \times 10^{-6}$
2,5	$1{,}640 \times 10^{-4}$	$1{,}656 \times 10^{-4}$	$1{,}6 \times 10^{-6}$
3,0	$2{,}361 \times 10^{-4}$	$2{,}385 \times 10^{-4}$	$2{,}4 \times 10^{-6}$
3,5	$3{,}213 \times 10^{-4}$	$3{,}246 \times 10^{-4}$	$3{,}3 \times 10^{-6}$
4,0	$4{,}197 \times 10^{-4}$	$4{,}240 \times 10^{-4}$	$4{,}3 \times 10^{-6}$
4,5	$5{,}312 \times 10^{-4}$	$5{,}366 \times 10^{-4}$	$5{,}4 \times 10^{-6}$
5,0	$6{,}558 \times 10^{-4}$	$6{,}625 \times 10^{-4}$	$6{,}7 \times 10^{-6}$
5,5	$7{,}935 \times 10^{-4}$	$8{,}016 \times 10^{-4}$	$8{,}1 \times 10^{-6}$
6,0	$9{,}444 \times 10^{-4}$	$9{,}540 \times 10^{-4}$	$9{,}7 \times 10^{-6}$
6,5	$1{,}108 \times 10^{-3}$	$1{,}120 \times 10^{-3}$	$1{,}2 \times 10^{-5}$
7,0	$1{,}285 \times 10^{-3}$	$1{,}299 \times 10^{-3}$	$1{,}3 \times 10^{-5}$
7,5	$1{,}476 \times 10^{-3}$	$1{,}491 \times 10^{-3}$	$1{,}5 \times 10^{-5}$
8,0	$1{,}679 \times 10^{-3}$	$1{,}696 \times 10^{-3}$	$1{,}7 \times 10^{-5}$
8,5	$1{,}896 \times 10^{-3}$	$1{,}915 \times 10^{-3}$	$1{,}9 \times 10^{-5}$
9,0	$2{,}125 \times 10^{-3}$	$2{,}147 \times 10^{-3}$	$2{,}1 \times 10^{-5}$
9,5	$2{,}368 \times 10^{-3}$	$2{,}392 \times 10^{-3}$	$2{,}4 \times 10^{-5}$
10,0	$2{,}624 \times 10^{-3}$	$2{,}650 \times 10^{-3}$	$2{,}6 \times 10^{-5}$

The second column of Table III shows relative AC-DC differences, d_{calc} , of the ratio of parallel equivalent resistances of the octofilar and the coaxial resistor, calculated for frequencies up to 10 kHz from the constructional parameters. Differences d_{ext} shown in the next column have been deduced from the results of a 1:1 comparison of the resistors, executed by means of a commercially available precision LCR meter (Agilent 4284A). With regard to the limited resolution of the meter and considering the fact that AC-DC differences of both resistors are so small that changes in resistance due to changes in frequency are considerably masked by changes due to various disturbances for frequencies below 10 kHz, decision was made to measure the AC-DC differences in a higher frequency range (10 kHz – 100 kHz), where resistance changes due to changes in frequency strongly predominate, and to evaluate AC-DC differences for lower frequencies by extrapolation. The last column of Table III documents a surprisingly good agreement of d_{calc} and d_{ext}. Also the agreement of the calculated and the measured difference of time constants (647 ns and 644 ns respectively) is satisfactory.

3. CONCLUSIONS

Correctness of mathematical models used in calculating frequency characteristics of octofilar resistors has been proved experimentally. A procedure enabling application of commercially available LCR meters to precision comparisons of calculable resistors has been proposed. At the DM, comparison with a calculable resistors by means of the LCR meter is routinely used also in the measurements of frequency characteristics of conventional resistance standards.

ACKNOWLEDGMENTS

This work was supported by the Czech Ministry of Education, Youth and Sports in the framework of the J04/98:210000015 research programme *Investigation of new methods for measurement of physical quantities and their application in instrumentation*.

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