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PROCESSING FUNCTION OF EDDY-CURRENT INDUCTIVE TRANSDUCER IN AN IMMERSION VERSION FOR THE CONDUCTIVITY MEASUREMENTS OF CHEMICAL MEDIA

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Abstract − This paper contains the processing formula of an inductive transducer in an immersion version for contactless measurements of conductivity of conductive media. In this version, a winding has been wound round an insulating stick in the form of a short coil. Such a construction enables us to easily place the transducer in the investigated medium. Dependencies determining the influence of the electric and magnetic properties of the walls of the vessel, which contains the investigated medium upon the transfer function of the transducer have also been formulated. These dependencies can also be used to designing the transducer in a shield with aforementioned properties. Moreover, experiments have been carried out which consisted in the measurement of the conductivity of the H_2SO_4 electrolyte solutions of known concentrations in a constant temperature by means of the transducer designed and using the theoretical dependencies derived. The results obtained demonstrate that the measurements – by means of the proposed methodology – can be carried out with a sufficient accuracy.

Keywords: eddy-current inductive transducer, conductivity measurements.

1. INTRODUCTION

In the field of measurements carried out with the use of parametric transducers, an essential issue is the knowledge of their processing characteristics. The inductive transducers belong to the group of parametric transducers. For the inductive transducers, the processing function is expressed as a dependence between – on the one hand – the changes of impedance observed at terminals of the coil and – on the other hand – conductivity of the investigated medium, geometrical structure of the transducer and frequency of current supplying the coil winding.

In the literature concerning contacless measurements of conductivity of different media with the use of inductive transducers [1, 2, 3, 4] one can find either formulas derived under too big simplifications (e.g., for the long coil [1]) or formulas derived for a specific geometrical structure of the transducer [2, 3, 4], which cannot be used for metrological analysis of other structures. Solutions presented in [3, 4] and [2] concern, respectively, the construction of the transducer in a flow version and the construction in an immersion version without carcass and insulating shield.

In this paper, an inductive transducer with a winding wound round an insulating stick in the form of a single-layered short coil has been analysed (Fig. 1a). The winding is separated from the investigated medium by an insulating shield.

The influence of the environment upon the processing function has been stabilized by locating the coil of the transducer in a shield with conductive and magnetic properties (Fig. 1b).

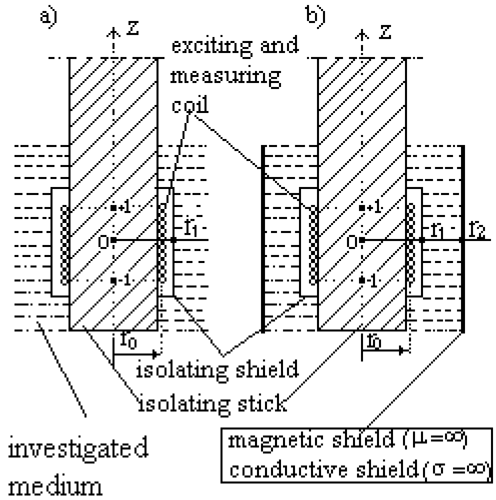

Fig. 1. Computational models of immersion transducers

2. COMPUTATIONAL MODELS

In order to solve the considered problem, construction models of the transducer in the immersion version have been used (Figs. 1a,b).

In the considered models, the parameters of particular media are the following:

– the investigated medium

$$\left.\begin{array}{l} \text{a) } r > r_1, \\ \text{b) } r_1 \le r \le r_2, \end{array}\right\} \quad s > 0, \quad m = m_0,$$

– the exciting coil

$$r = r_0, \quad i = \begin{cases} \frac{Iw}{2l}, & \text{for } |z| < l, \\ 0, & \text{for } |z| > l, \end{cases}$$

– currentless areas

$$\text{a)} \left. \begin{array}{l} 0 \le r < r_0, \\ r_0 < r \le r_1, \end{array} \right\} \; s = 0, \quad m = m_0,$$

$$\text{b)} \left. \begin{array}{l} 0 \le r < r_0, \\ r_0 < r < r_1, \\ r > r_2, \end{array} \right\} \; s = 0, \quad m = m_0,$$

where s is the conductivity and w is the number of turns of coil winding.

The properties of the shield have been assumed as ideal ones; hence, its thickness has not been taken into account in the model.

In the computational models, the initial equations are Maxwell equations [4, 5] whose solutions are searched by means of an angle component of vector potential $A_j\,(r, z)$ in the cylindrical coordinate system. It results from the axial symmetry of the transducer and the assumed model of the inducing coil in the form of surface currents with angle components flowing on the surface of the cylinder characterized by the radius r_0 and height $2l$. Vector potential A_j fulfils – in currentless areas – Laplace equation [5] ($\nabla^2 A_j = 0$) and in the area filled with the investigated media – Helmholtz equation [5]:

$$\nabla^2 A_j = k^2 A_j, \quad k = \sqrt{jwm_0 s}, \quad j^2 = -1.$$

In order to realize the processing function, the vector potential in the currentless area ($r_0 \le r \le r_1$) in which the winding of the coil occurs has been determined. To facilitate numerical calculations, the following solution has been applied:

1) first, the distribution of exciting field associated with the density of the current flowing through the winding of the coil has been calculated,
2) in currentless areas, a superposition of exciting field and eddy-current field fulfilling Laplace equation ($\nabla^2 A_j = 0$) has been assumed,
3) in the area of induced eddy currents, a direct determination of the field – as a result of the solution of Helmholtz equation ($\nabla^2 A_j = k^2 A_j$) – has been used.

For determining the solution of the aforementioned equations, integral Fourier transform [6] with respect to variable z has been applied and – for each subarea – an appropriate first-order Bessel equation has been obtained. The solutions of these equations can be expressed by means of modified Bessel functions of the first and second types with appropriate integration constants. They can be determined on the basis of boundary conditions, which concern the continuity of fields A_j on the surfaces of separation of computational areas ($r = r_0$ and $r = r_1$) as well as the comparison of tangent components of the magnetic field intensity ($m^{-1}\nabla \times A_j$) on these surfaces (Fig. 1a).

In case of the model of Fig. 1b, on the surface $r = r_2$, additionally occurs: a) zeroing of the normal component of magnetic induction as for ideally conductive surface ($s_1 = \infty$):

$$A_j = 0, \quad \text{for } r = r_2,$$

and b) zeroing of the tangent component of magnetic induction as for the surface of ideal magnetic properties ($m = \infty$):

$$B_z = \frac{1}{r}\frac{\partial(rA_j)}{\partial r} = 0, \quad \text{for } r = r_2.$$

The solution taking into account the aforementioned boundary conditions has provided dependencies determining the vector potential on the surface modelling the coil, which – in these models – is both the exciting and measuring coil:

$$A_j\,(r_0, z) = \frac{m_0 Iwr_0}{pl}\int_0^\infty f_{1-3}(l) I_1(lr_0)\frac{\sin ll \cos lz}{l}\,dl, \quad (1)$$

where:

– for the model of Fig. 1a:

$$f_1(l) = K_1(lr_0) + \frac{lK_0(lr_1)K_1(qr_1) - qK_0(qr_1)K_1(lr_1)}{lI_0(lr_1)K_1(qr_1) + qK_0(qr_1)I_1(lr_1)} I_1(lr_0),$$

– for the model of Fig. 1b (for $s_1 = \infty$):

$$f_2(l) = K_1(lr_0) + \frac{lK_0(lr_1)A_1(l) + qK_1(lr_1)A_2(l)}{lI_0(lr_1)A_1(l) - qI_1(lr_1)A_2(l)} I_1(lr_0),$$

– for the model of Fig. 1b (for $m = \infty$):

$$f_3(l) = K_1(lr_0) + \frac{lK_0(lr_1)B_1(l) - qK_1(lr_1)B_2(l)}{lI_0(lr_1)B_1(l) + qI_1(lr_1)B_2(l)} I_1(lr_0),$$

and

$$\begin{aligned} A_1(l) &= K_1(qr_2)I_1(qr_1) - K_1(qr_1)I_1(qr_2), \\ A_2(l) &= K_1(qr_2)I_0(qr_1) + K_0(qr_1)I_1(qr_2), \\ B_1(l) &= K_0(qr_2)I_1(qr_1) + K_1(qr_1)I_0(qr_2), \\ B_2(l) &= K_0(qr_1)I_0(qr_2) - K_0(qr_2)I_0(qr_1), \\ q^2 &= l^2 + jwm_0 s, \end{aligned} \quad (2)$$

l – a variable resulting from Fourier transform,

I_1, I_0, K_1, K_0 – modified Bessel functions.

In turn, the tangent component of the electric field has been determined:

$$E_j\,(r_0,z) = -jwA_j\,(r_0,z)$$

and then the voltage at the coil terminals has been calculated:

$$U = \frac{2pr_0w}{2l}\int_{-l}^{l} E_j\,(r_0,z)\,dz$$

as well as the impedance – on the basis of the formula $Z = \frac{U}{I}$ – has been obtained.

3. PROCESSING FORMULAS

The processing formula has been defined as follows:

$$D^0 Z = \frac{Z - Z_0}{|Z_0|} = D^0 R + jD^0 X \;, \qquad (3)$$

where Z, Z_0 – the coil impedance with and without the sample, respectively.

In order to assure the universality of the processing formulas, new variable $y = \frac{l}{\sqrt{wm_0 s}}$ has been introduced, and the remaining parameters have been expressed in a relative form: $d = r_0\sqrt{wm_0 s}$ and

a) for the model of Fig. 1a: $a = \frac{r_1}{r_0},\ x = \frac{l}{r_0}$,

b) for the model of Fig. 1b: $a_1 = \frac{r_0}{r_1},\ a_2 = \frac{r_1}{r_2},\ x = \frac{l}{r_0}$.

After introducing these changes, the processing formulas assume the form:

– for Fig. 1a:

$$\Delta^0 Z = \frac{j\int_0^\infty F_1(y)[I_1(yd)\frac{\sin ydx}{y}]^2\,dy}{\int_0^\infty I_1(yd)K_1(yd)[\frac{\sin ydx}{y}]^2\,dy}, \qquad (4)$$

where

$$F_1(y) = \frac{yK_0(x_1)K_1(x_2) - \sqrt{y^2+j}\;K_0(x_2)K_1(x_1)}{yI_0(x_1)K_1(x_2) + \sqrt{y^2+j}\;K_0(x_2)I_1(x_1)},$$

$$x_1 = day\,, \quad x_2 = da\sqrt{y^2+j}\,,$$

– for Fig. 1b with $s_1 = \infty$:

$$\Delta^0 Z = \frac{j\int_0^\infty F_2(y)[I_1(yd)\frac{\sin ydx}{y}]^2\,dy}{\int_0^\infty I_1(yd)K_1(yd)[\frac{\sin ydx}{y}]^2\,dy}, \qquad (5)$$

where

$$F_2(y) = \frac{yK_0(yd/a_1)A_1(y) + \sqrt{y^2+j}\;K_1(yd/a_1)A_2(y)}{yI_0(yd/a_1)A_1(y) - \sqrt{y^2+j}\;I_1(yd/a_1)A_2(y)},$$

– for Fig. 1b with $m = \infty$:

$$\Delta^0 Z = \frac{j\int_0^\infty F_3(y)[I_1(yd)\frac{\sin ydx}{y}]^2\,dy}{\int_0^\infty I_1(yd)K_1(yd)[\frac{\sin ydx}{y}]^2\,dy}, \qquad (6)$$

where

$$F_3(y) = \frac{yK_0(yd/a_1)B_1(y) - \sqrt{y^2+j}\;K_1(yd/a_1)B_2(y)}{yI_0(yd/a_1)B_1(y) + \sqrt{y^2+j}\;I_1(yd/a_1)B_2(y)}.$$

Expressions $A_1(y), A_2(y), B_1(y)$ and $B_2(y)$ can be obtained from formulas (2) after replacing arguments qr_2 and qr_1 by $\frac{d}{a_1 a_2}\sqrt{y^2+j}$ and $\frac{d}{a_1}\sqrt{y^2+j}$, respectively.

4. RESULTS OF COMPUTATIONS

In order to reveal metrological properties resulting from formulas (4), (5), and (6), they have been subject to numerical analysis with the use of programs written in MATLAB environment.

Fig. 2 presents the numerical results obtained on the basis of formula (4). They express the changes of particular components of the impedance increment ($\Delta^0 Z$) as functions of argument d in a wide range of changes of its value.

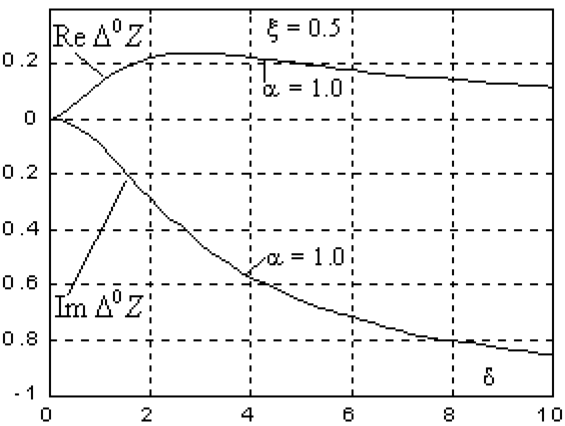

Fig. 2. Transfer functions of the transducer in a wide range of changes of argument d

The plots of Fig. 2 show that the resistance component loses the character of monotonic function for $d > 2$ whereas the reactance component for $d > 4$ becomes more and more flat and, thus, less sensitive to the changes in argument d. The plots of Fig. 2 provide informations regarding the selection of an optimal range of argument d, in which both components have the largest slope in regard to d axis. For component $\mathrm{Re}(\ddot{A}^0 Z)$, an optimal range of d is the interval (0.4, 2), and for component $\mathrm{Im}(\ddot{A}^0 Z)$ – the interval (0.4, 4). In connection with these facts, further calculations have been carried out for the values of d from the interval (0, 2); they are presented in Fig. 3.

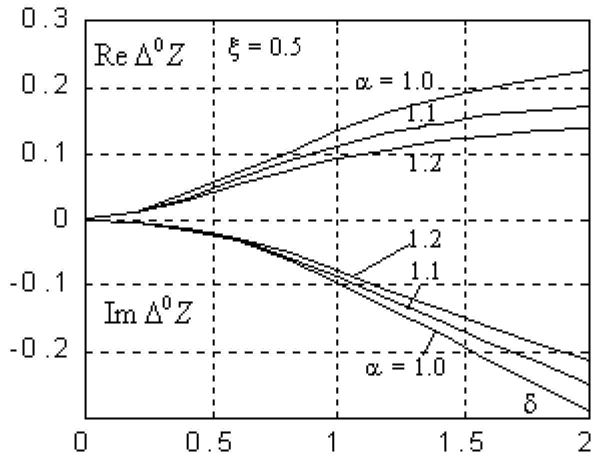

Fig. 3. Transfer functions of the transducer in the area of practical applications

The plots of Fig. 3 additionally show the influence of the insulating shield thickness upon the transfer function (parameter a).

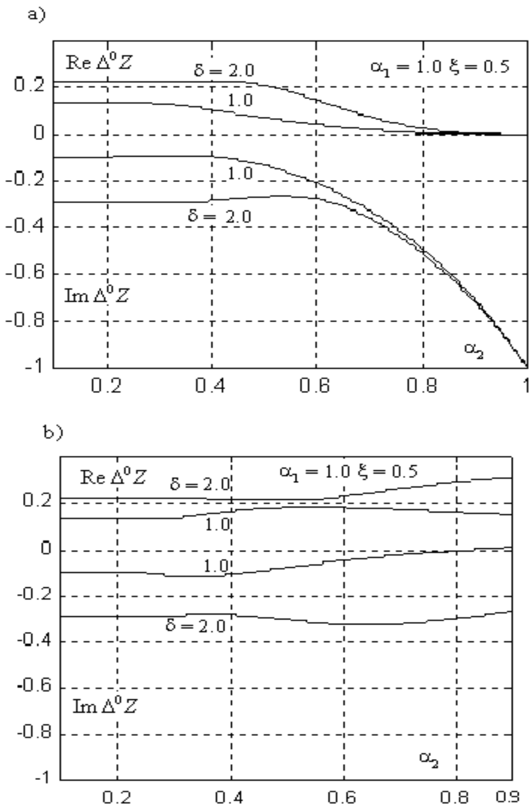

Fig. 4. Influence of the distance of the shield (vessel walls) upon transfer functions for: a) the shield with conductive properties, b) the shield with magnetic properties

The plots of Fig. 4a and Fig. 4b present the influence of the distance of the shield with conductive ($s_1 = \infty$) and magnetic ($m = \infty$) properties upon the transfer function of the transducer. As a shield, one can also consider the walls of the vessel, which contains the investigated medium. On the basis of these plots, it is possible to determine the distance from the vessel walls so as the influence of these walls was lesser than the assumed one.

5. MEASUREMENTS

In the experiment, as the investigated medium, the H_2SO_4 electrolyte solution of 20% concentration has been used. Its conductivity s_0 – measured by means of conductometer equipped with an electrode sensor – is equal to 65.3 S/m. This value has been assumed as a reference value with respect to the results obtained with the use of the inductive transducer prepared according to criteria which follow from the plots presented in Section 4 of this paper. This transducer was made in the form of a single-layered coil. Its copper-wire winding has been directly placed on the surface of an insulating stick (plastic) of 100.00 mm diameter. The winding – from the external side – has been secured by means of insulating shield (plastic) of 2 mm thickness. Therefore, the value of coefficient $a = \frac{r_1}{r_0}$ is close to 1. Coefficient x – determining the length-to-diameter ratio of the coil – is equal to 0.5. The measurements of the changes in electrical parameters $\mathrm{Re}(\ddot{A}^0 Z)$ and $\mathrm{Im}(\ddot{A}^0 Z)$ of the transducer – caused by the immersion of the transducer in the investigated medium – have been carried out by means of RLC transformer bridge under the frequency $f = 1\,\mathrm{MHz}$. The indirectly measured values $\mathrm{Re}(\ddot{A}^0 Z)$ and $\mathrm{Im}(\ddot{A}^0 Z)$ as well as numerically calculated characteristics $\mathrm{Re}(\ddot{A}^0 Z) = f_1(d)$ and $\mathrm{Im}(\ddot{A}^0 Z) = f_2(d)$ enable us to determine the value of argument d and then – from dependence $s = \frac{d^2}{r_0^2 w m_0}$ – the value of the conductivity of the investigated medium. The result $s = 67.2\,\mathrm{S/m}$ has been obtained. Comparing s and the reference value s_0, it can be found that the difference does not exceed 3%.

6. CONCLUSIONS

The formulas obtained and the computer programs developed enable us to perform numerical calculations and – on the basis of their results – an optimal selection of geometrical dimensions of transducers and frequency of power supply so as to work in an advantageous point of characteristics (Fig. 2). If the conductivity of the investigated medium is small, then the advantageous points of characteristics can be reached by increasing the radius of the coil and the frequency of power supply. The results obtained clearly demonstrate that the measurements with the use of the proposed methodology can be carried out in the contacless way and with sufficient accuracy.

7. REFERENCES

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