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MEASUREMENT OF FREQUENCY AND TIME - ERRORS DUE TO SYNCHRONIZATION EFFECTS AND ERROR CORRECTION

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Abstract – After an introduction to the mathematical theory of rheolinear systems the application of the results leads to the fact that due to the synchronization there arises a frequency deviation even outside of the synchronization range. The errors caused by this effect were investigated and consequences for high-precision frequency or time measurement and methods of error-correcting are obtained .

Keywords: rheolinear system, synchronization, high-precision frequency or time measurement.

1. INTRODUCTION

Time measurement is done using a high-precision frequency. Frequency measurement normally compares two frequencies: the unknown frequency Ω_l and the reference frequency Ω_0 using converter (mixer) principle. Because of the inevitable coupling of the two frequency sources the problem is described by a differential equation of *Hills* type (rheolinear system) leading to two effects: a synchronization – often desirable – and an inevitable frequency shifting $\Delta\Omega^*$ outside of the synchronization range. In the literature the last mentioned effects and therefore these errors are neglected that means the difference frequency is assumed to be:

$$\Delta\Omega = \Omega_l - \Omega_0 \tag{1}$$

The paper deals especially with these problems and leads to new results important for praxis of high-precision measurements. Methods to reduce these errors by means of a correcting algorithm were gained too.

2. THEORY OF RHEOLINEAR SYSTEMS

Coupling between the two frequency sources is inevitable because there always is a capacitance between both inputs of the mixer. Due to this fact the coupled voltage leads to two consequences: the amplification of the frequency generator Ω_l and due to the varactor effect of the transistor the frequency deciding capacitance is varied. Therefore the differential equation yields

$$y``(t) + a_1(t)\, y`(t) + a_2(t)\, y(t) = x(t \tag{2}$$

where $a_1(t)$ considers the variation of the damping because of the varying amplification and $a_2(t)$ the variation of the capacitance because of the varactor effect. Using the Transformation

$$z(t) = y(t)\, e^{-\int a(\tau)/2\, d\tau} \tag{3}$$

we get an inhomogeneous *Hill*-equation. For we are only interested in the Eigenvalues the solution of the homogenous *Hill* equation is sufficient [1]

$$z(t)`` + \Phi(t)\, z(t) = 0 \tag{4}$$

To a theorem of *Floquet* [2] solutions are existing

$$z(t) = e^{\mu t} f(t) + e^{-\mu t} g(t) \tag{5}$$

The mathematical solution is very difficult even for the special case of a periodic $\Phi(t)$, the so-called *Mathieu* differential equation [3]. Here the Diagram of *Ince* and *Strutt* demonstrates instable regions with real parts of the exponent μ in equ. (5).

We will use another approach to get informations of the behaviour of the system and especially of the characteristic exponent μ based on physical considerations and approximations as in principle described in [4].

3. PHASE AND ENERGY INVESTIGATIONS

Instead of equation (4) we use the *Mathieu*-equation

$$z(t)`` + \Omega_0^2(1+\sigma \sin\omega\, t)\, z(t) = 0 \tag{6}$$

describing an oscillator with time-varying capacitance. Following investigations given by *Wenke* [5] and *Erdelyi* [6] we get a current in phase with the voltage and one with 90° phase difference leading to a negative damping and to an additional capacitance. This additional capacitance generates a frequency shift, the possible maximum yields the so-called locking or synchronization range $\Delta\Omega_o$. Because this parameter may be easily measured we will use him to describe the problem instead of σ in equation (6). As given in details in [7] the exponent μ is

$$\mu = \sqrt{\Delta\Omega_0{}^2/4 - \overline{\Delta\Omega}^2} \qquad (7)$$

where $\overline{\Delta\Omega}$ means the deviation to the resonance frequency Ω_o.

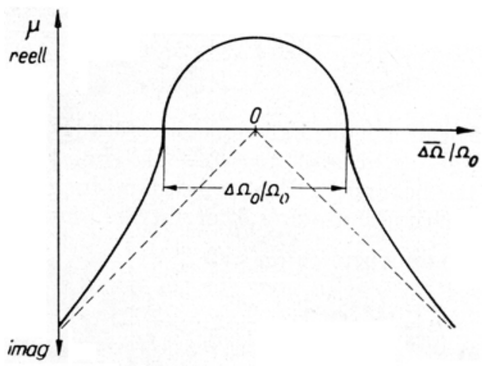

Fig. 1. Course of the characteristic exponent μ

Figure 1 shows the course of μ . Inside the synchronisation range $\Delta\Omega_o$ the exponent μ is real, that means a negative damping and not a frequency difference (synchronization). Outside this range μ is imaginary, that means a frequency deviation (error) in comparison with the ideal difference frequency $\Delta\Omega = \Omega_l - \Omega_0$.

4. ERRORS IN FREQUENCY MEASUREMENT

To gain the error between the ideal difference frequency $\Delta\Omega$ adequate to equation (1) and the real difference frequency we use equation (7) with $\overline{\Delta\Omega} = \Delta\Omega$

$$\mu = \sqrt{\Delta\Omega_o^2/4 - \Delta\Omega^2} = j\sqrt{\Delta\Omega^2 - \Delta\Omega_o^2/4}$$

So the real difference frequency follows

$$\sqrt{\Delta\Omega^2 - \Delta\Omega_o^2/4} = \Delta\Omega\,[1 - 0{,}5(\Delta\Omega_o^2/4\,\Delta\Omega^2)] \qquad (8)$$

and the elative error is

$$-\,0{,}5(\Delta\Omega_o^2/4\,\Delta\Omega^2) \qquad (9)$$

That means: If for instance the difference frequency is by the factor 100 greater than the synchronization range $\Delta\Omega_0$ the relative error still runs to $0{,}125 \cdot 10^{-4}$! In praxis the permissible error in high precision frequency or time measurement may be less than 10^{-10} to 10^{-12} [8]. From this fact it follows that the synchronization range has to be smaller than $0{,}3{*}10^{-4}$ to $0{,}3{*}10^{-5}$ of the difference frequency.

In praxis the synchronization range should be measured before frequency measurement. If $\Delta\Omega_o$ is known equation (8) can be used for error-correction.

If it is not possible to know the synchronization range $\Delta\Omega_o$ one can use the measurement of the distortions to gain $\Delta\Omega_o$, because there exists a relationship between the distortion factor and the synchronization range. The reasons for the distortion are relaxations in the neighbourhood of the synchronization border [9]. Here further investigations are necessary.

5. CONCLUSIONS

At first a short introduction to the theory of systems with time varying parameters – so-called rheolinear systems – is given, leading to a characteristic exponent μ.. Using phase and energy investigations the course of this parameter in relation to the difference frequency can be gained. From these results follows a frequency error even outside the synchronization range, a result with great importance for high-precision frequency or time measurement. The relation of this error with the synchronization range is gained and so it is possible to correct this error if the synchronization range is known. Finally an outlook is given if it is not possible to measure the synchronization range for there exists a connection to the distortion factor.

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