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A BEM-Approach for Simulation and Optimization of Capacitive Sensor Topologies

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Abstract – In order to optimize front-end topologies of capacitive sensors precise knowledge of the coupling capacitances is important. These capacitances are not easily accessible by means of measurement without significantly altering them, and therefore a numeric method for determining them is advantageous. In this paper we use a Boundary element formulation for this task. Certain aspects of numerical accuracy of the formulation used are addressed. Results from simulations and optimizations are presented and compared with experimental results for validation of the approach.

Keywords: Capacitive Sensor, BEM, Optimization

1. INTRODUCTION

Capacitive measurement techniques are used in a variety of applications in science and under industrial environment. Advances in electronics continuously extend the fields where they are applicable. Main advantages of capacitive technology are simplicity, high accuracy and resolution over the total measurement range, excessive temperature operating capabilities, insensitivity to dirt, moisture and dew [1,2].

This paper is organized as follows: First we will give a brief overview of the working principle of the sensor topologies we investigated and of the BEM formulation we used as well as a problem description. In the following section accuracy-related aspects are discussed. In section 3 results for some numeric examples are presented and compared to experimental data. Section 4 provides the conclusion and an outlook on future work.

A. Working Principle of the Sensor

Fig. 1 shows the working principle of the ratiometric position sensors followed in this paper. One stator plate is used as transmitter with several transmitting segments; the other stator carries one single receiver electrode. The rotor leads ground potential; due to this circumstance it acts as an electric shield lowering the capacitance between transmitters and receiver for those segments covered by the rotor.

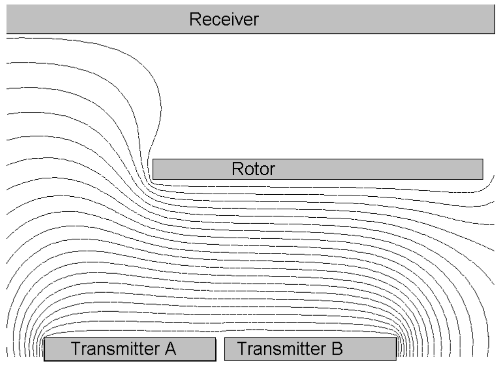

Figure 1: Sensor Principle: The uncovered areas of the transmitter segments determine the coupling capacitances

The position x can be calculated by

$$x = K\,\frac{C_A - C_B}{C_A + C_B} \tag{1}$$

where C_A respectively C_B are the capacitances between transmitters and the receiver. K is a geometry-dependent constant. The reference position x = 0 is attained when the rotor covers same portions of both transmitters and $C_A = C_B$. Because the position is determined by ratios of capacitances, environmental changes equally affecting all capacitances will cancel out. With further subdivision of the segments and dynamic grouping of them, algorithms capable of compensating amplifier offsets and drifts thus providing high accuracy are available [1,10] .

Besides the approach followed in this paper, other sensor topologies and reconstruction methods are commonly used. These comprise systems with dielectric, floating conductive and sinusoidally shaped rotors [5,7]. The stators of all these different technologies are usually quite similar. Therefore the presented approach is also applicable to such sensors.

B. The BEM Formulation

For the optimization of sensor topologies knowledge of the complex interactions between coupling capacitances is important, especially with advanced techniques like virtual grounding (ref. section 3B) and planar electrode structures (ref. section 2A) coming into play. These capacitances, which are often far below 1 pF, are not easily accessible by means of measurement, particularly without altering them. With an accurate numeric estimation of these capacitances optimization is eased and additionally possible at an early stage in development of a sensor. Fig. 2 shows a gray-scale image of coupling capacitances on a logarithmic scale. In row 18 the capacitances between transmitters and the first rotor blade are depicted. Obviously not only the covered segments significantly radiate the rotor but rather unshielded do as well. This becomes essential for so-called "virtual grounding", which will be discussed in section 3.

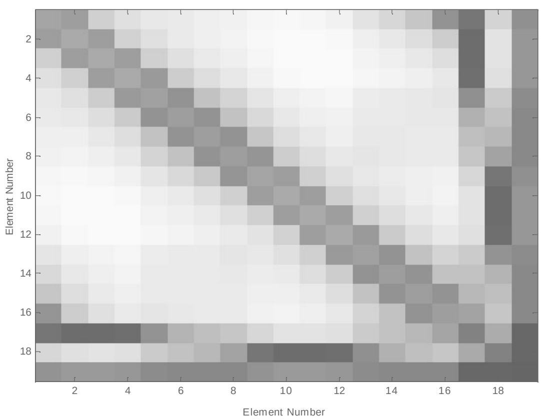

Figure 2: Logarithmic gray-scale image of coupling capacitances. Row/Column 1-16 represent transmitter segments, 17 and 18 two rotor blades, 20 the receiver.

Due to the complex shapes of the boundaries, analytic solutions for the potential problem to be solved in order to calculate the capacitances are not easy to find and frequently do not exist. Therefore a numeric method has to be used. While in FEM the entire volume has to be discretized, in the Boundary Element Method (BEM) only the boundaries require a discretization. When parts have to be moved, like the rotor in the case of capacitive sensors, then this represents a major advantage of the BEM, because the discretization can remain the same. The BEM has been successfully applied to coupling capacitance extraction for many different applications especially in the field of integrated circuits design (e.g. [8])

The boundary integral equation used in this paper is given by

$$c_j U_k + \sum_i \int_{\Gamma_i} \frac{\partial G_{ki}}{\partial n_i} U_i \, d\Gamma = \sum_i Eps \int_{\Gamma_i} \frac{\partial U_i}{\partial n_i} G_{ki} \, d\Gamma \qquad (2)$$

with

$$\text{Eps} = \begin{cases} \frac{\varepsilon_m}{\varepsilon_j} & \text{for m > j} \\ 1 & \text{for m < j} \end{cases} \qquad (3)$$

for domain Ω_j and surface Γ_k, where j, m denote domain numbers, k, j are indices for the surfaces Γ. G stands for the Green's function for respective source and field points. U denotes the potential and c_j the singularity value for boundary Γ_k of domain W_j [3,9].

Second order isoparametric rectangular elements with 8 nodes per element are used for the discretization of the boundaries. With these elements, curvilinear boundaries can be approximated.

2. ACCURACY ANALYSIS

In order to obtain relevant optimization results from BEM analysis, it is important to verify the accuracy of the calculation. In this section it is shown how some well-known numeric problems affect the accuracy and how this can be estimated from the calculation results.

A measure for the accuracy of the calculation can be obtained from the coupling capacitance matrix. The capacitance between two elements can be calculated by integration over the charge of either element. Consequently, the coupling capacitance can be calculated twice; the results differ due to numerical inaccuracies. This is shown in Fig. 3. For a certain rotor position the receiver was rotated while the rotor was not moved. Of course, this rotation of it does not change the coupling capacitances, but may change the discretization error. The error is not equal for the two different calculations of the coupling capacitances between two elements and the difference can provide a simple yet useful measure for the accuracy. We accepted calculations with a maximum relative error of less than 0.5% which is in the range of the measurement error of our test equipment and sufficiently low, as our experiments (ref. section 3) have shown.

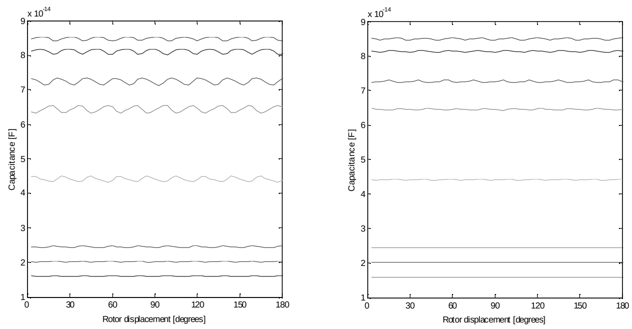

Figure 3: Rotating Receiver: The capacitances should remain constant but show a dependency on the receiver displacement. Left: Capacitance seen from the transmitter, Right: Capacitance seen from receiver

A. Edge Effects

Although algorithms with adaptive grid generation are available [6] we use macro-element-modeling because we need to remain on the same grid in order to avoid discontinuities in the target function of the optimization procedures. From a library of common objects (e.g. electrodes and rotor) new models are readily constructed by the use of these building blocks. The thin but wide electrodes and the distances between objects may pose problems for the stability of the simulation. Thus these and edge effects must be taken into account for the pre-discretization of boundary elements. We studied those effects on planar sensor topologies.

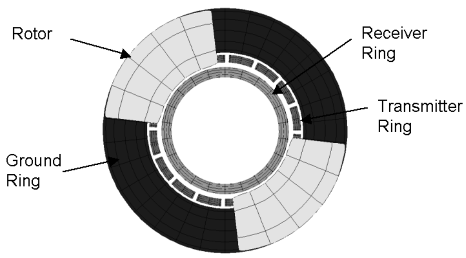

Figure 4: BEM-representation of a planar capacitive sensor. Each rectangular element represents 1 macroelement, which itself is further subdivided into several boundary elements

We refer to a "planar sensor" when transmitter and receiver segments reside on the same plane. Such sensors are more sensitive to variations of mechanical parameters than two plane designs. Therefore a detailed analysis on the impact of parameter changes is important for reliability investigations. The BEM-model of an example planar topology is shown in Fig. 4. Fig 5 shows the calculated capacitances over the displacement angle for different discretizations. Where sharp edges have been used in modeling, the results are doubtable: The capacitance would increase when the rotor covers the transmitter. The effect gets smaller with more elements (straight forward subdivision) on the boundary, but the expected characteristic is still inverted.

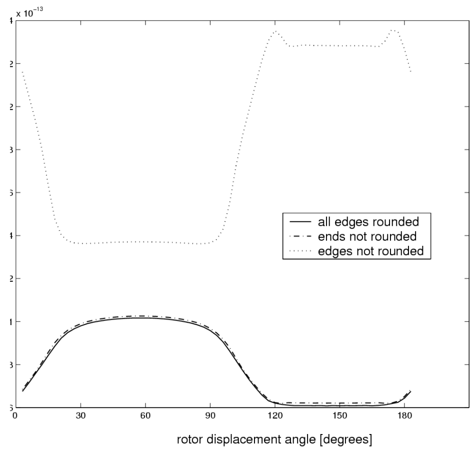

Figure 5: Effects of geometric singularities: By eliminating sharp edges reasonable results can be achieved without increasing costs.

Geometric singularities in the model are deemed the cause of these problems. The sharp edges of the segments theoretically lead to "infinite" charge densities along these edges - something that cannot be accurately approximated by polynomial shape functions used in our numeric approach.

By rounding the edges this shortcoming of the simulation can be bypassed. For isoparametric elements with quadratic shape functions it is possible to find coordinates for middle nodes ensuring C^1 continuity at element boundaries.

An edge of the boundary is given by

$$P(\eta) = \sum_i P_i N_i(\eta) \tag{4}$$

where P_i are the nodes along the edge and N_i the corresponding shape functions; $\eta \in [0,1]$.

For smooth transitions between elements, the tangents at the end nodes of the mapping function must match those of the neighboring elements:

$$\left.\frac{\partial P(\eta)}{\partial \eta}\right|_0 = k_1 \vec{v}_1 \qquad \left.\frac{\partial P(\eta)}{\partial \eta}\right|_1 = k_2 \vec{v}_2 \tag{5}$$

where $\vec{v}_1$ and $\vec{v}_2$ are the tangents at endpoints P_1 and P2 for the neighboring elements; k_1 and k_2 are arbitrary parameters. While solutions for above equations are easily found in 2D, there seems to be no generally valid solution for the 3D case. However, since boundary elements have to match along their borders, the tangents in (5) have to be on the same plane and a unique solution is obtained for point P_3.

With this modification acceptable results have been obtained without any increase in computational costs. Results with rounded objects show expected behavior, as can be seen in Fig. 5. For the rounded models in this figure actually less elements were used than for the sharp-edged type.

B. Other Discretization Errors

Discretization errors in the calculation result in errors even after application of advanced signal processing algorithms used to determine the position of rotors. Such effects are analyzed in the following example.

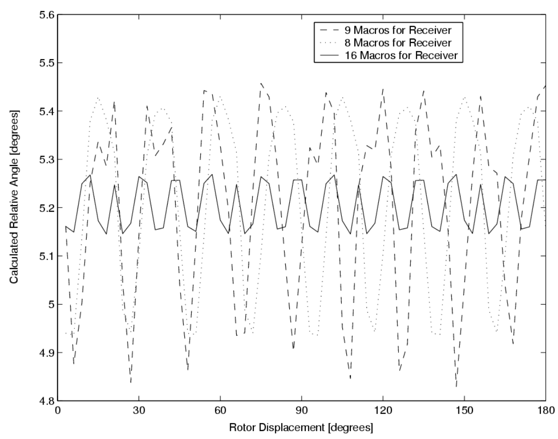

Figure 6: Effects of receiver discretization: The relative angle of 5° has not been changed during simulation.

For this example we used a two-plane sensor for relative angle measurement proposed in [10]. Two opposite rotors are connected to the same shaft while the other two are e.g. mounted on a torsion bar thus the relative angle between the two is proportional to the applied torque.

The angle between the two rotors was 75°, with 80° being the reference position[1] hence the relative displacement angle was set to 5°. With a rather coarse subdivision of the receiver using 8 macro elements, considerable deviations are the result as can be seen in Fig. 6. Much better accuracy is achieved using 16 elements and gets better with more elements used.

The calculated angle of about 5.2 degrees is higher than the real value of 5° between the two rotors. This is a shortcoming of the used ratiometric algorithm: When the rotors are close, which they are due to the small size of the sensor (outer diameter of 50 mm), the theoretical unshielded maximum capacitance is not achieved through the small gap between the blades. Therefore for the algorithm the rotors appear to be closer together than they actually are. With BEM look up tables to compensate for this shortcoming of the algorithm can be calculated.

Notably, the errors vary continuously with continuous geometry changes. This is a major advantage for optimization, as the quality function derived from capacitances will not be jagged. Jagged quality functions have been reported [8] in literature but so far have not been observed in our calculations.

3. APPLICATIONS AND RESULTS

A. Adaptation of Effective Rotor Width

As mentioned in the previous section, the discretization errors change continuously as long as singularities in the geometry are avoided. In the following example the method will be used to optimize the rotor of the relative angle sensor from described in section 2B.

Looking at the field distribution shown in Fig. 1 one may conclude that the rotor image "projected" onto the receiver is wider than the actual rotor. Measurements approve this assumption. Algorithms based on (1) use a correlation to find the approximate rotor position [1,10]. This correlation works best, when shielded and non-shielded areas are of the same (effective) width. Therefore it is desired to adapt the rotor in order to obtain an "image" on the receiver with a "geometric duty cycle" of 0.5.

The iterative process to find the optimal rotor is performed in three steps:

- Calculate capacitance with respect to angular displacement
- Determine effective rotor width
- Calculate new rotor width

[1] This results in a measurement range of ±10°

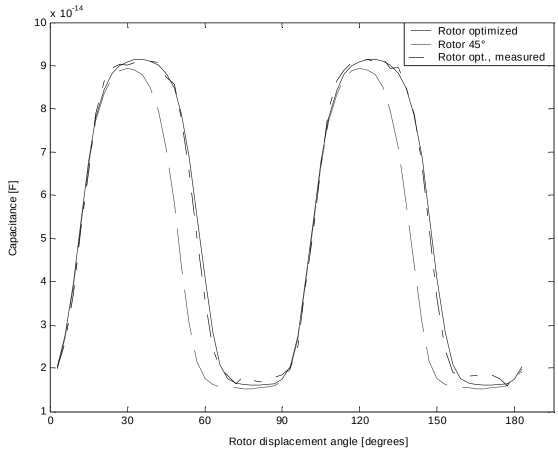

Figure 7: Rotor width optimization: For a rotor with geometrical width of 45°, the unshielded is much smaller than the shielded area. For the optimized rotor, the effective width is close to the desired value of 45°.

Above procedure converges after very few steps of iteration. Fig. 7 depicts the calculated capacitance over the displacement angle for one segment for a rotor of 45° width and both measured[2] and calculated capacitance for an optimized rotor, which show good accordance.

B. Virtual Grounding

The sensor principle described in section one requires a rotor on ground potential. While this is usually maintained over the bearing of an electrically conductive shaft, this may be undesired for some applications, e.g. due to EMC concerns. In this case, virtual grounding offers an alternative [4]. Here the rotor is held on ground potential via high capacitances between it and electrical ground. Since the impedance of this grounding capacitance will be higher than for the conductive case, the rotor will slightly swing with the transmitted signal. Thus the shielding effect is weaker; additionally the indirect coupling over the rotor also increases the received signal for unshielded segments.

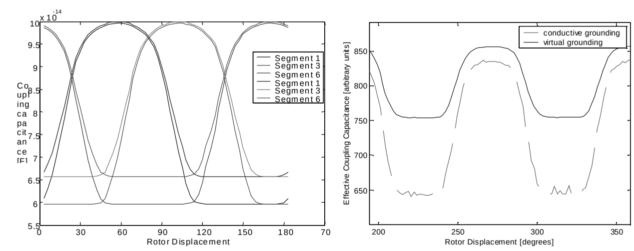

Figure 8: Coupling capacitances for selected segments with conductive (cg) and virtual (vg) grounding. Left: calculated, Right: measured

In Fig 8 conductive and virtual grounding are compared for both calculated and measured coupling capacitances. For the

[2] Arbitrary units of measurement have been scaled to match the calculated units

calculation, additional areas on ground potential outside of the transmitter ring have been introduced in order to maintain a high grounding capacitance (Fig. 4). Due to space limitations this could not be used for the prototype sensor. The measurement results are also depicted in Fig 8. However, the predicted effect can also be observed in the measurement.

C. Axial Rotor Displacement

One of the problems associated with planar sensor topologies is the impact of axial rotor displacement on the coupling capacitances as they decrease with increasing distance of the rotor. While this influence cancels out for a sole increase in the distance, problems arise with rotor tilt, as the distances between transmitter and opposite edges of the rotor are no more equal. As depicted in Fig 9 the BEM provide accurate predictions of this effect and therefore allows front-end optimization together with algorithm tuning on simulated data.

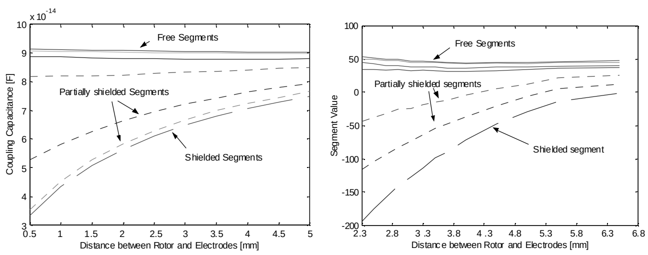

Figure 9: Calculated (left) and measured (right) influence of axial rotor displacement

4. CONCLUSION AND OUTLOOK

The applicability of BEM on analyzing and optimizing capacitive sensors has been studied. Possible sources of errors have been identified and their effects demonstrated by examples. Edge-rounding was found to essentially improve results.

By comparing simulation and measurement results it was shown that the used BEM approach is sufficiently accurate to be applied to sensor optimization with desired accuracies below 0.1°.

Future work will focus on both optimizing front-end topologies and algorithms to further improve robustness of planar and non-planar sensors based on simulated data with proven reliability. Additionally, we will study the applicability of faster solving methods which is important as the parameter space for the optimization is huge.

REFERENCES

[1] G. Brasseur, "How to Build Reliable Capacitive Sensors for an Industrial Environment", *IEEE Conference on Instrumentation on Measurement (IMTC 02)*, pp 141-144, Mai 2002.

[2] A. H. Falkner, "The use of capacitance in the measurement of angular and linear displacement." *IEEE Trans. Instrum. Meas.*, Vol. 43, No. 6, 1994.

[3] C. J. Huber, "Erstellung einer Boundary-Elemente-Software für allgemeine 3D-Potentialprobleme" Masters Thesis, Graz University of Technology, 1994.

[4] G. Brasseur, P. Fulmek, W. Smetana "Virtual Rotor Grounding of Capacitive Angular Position Sensors" *IEEE Conference on Instrumentation and Measurement (IMTC 01*), May 2001.

[5] M. Gasulla, X. Li, G.C.M. Meijer, L. van der Ham, J.W. Spronck, "A Contactless Capacitive Angular-Position Sensor" *Sensors 2002. Proceedings of IEEE* , Vol. 2 2002, pp 880-884

[6] W. Shi, J. Liu, N. Kakani, T. Yu, "A Fast Hierarchical Algorithm for Three-Dimensional Capacitance Extraction", *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, Vol. 21 No. 3 pp 330-336, March 2002

[7] T. Thurner, S. Cermak, G. Brasseur, P.L. Fulmek, "Signal Processing for Capacitive Angular Position Sensors by the Discrete Fourier Transform", *IEEE Conference on Instrumentation and Measurement (IMTC 01*), pp 1135-1138, May 2001

[8] M. Beattie, L. Pileggi, "Bounds for BEM Capacitance Extraction", *Design Automation Conference, 1997. pp 133 – 136, June 1997*

[9] G. Beer, "Programming the Boundary Element Method", *John Wiley & Sons*, 2001

[10] S.Cermak, G. Brasseur, P. Fulmek, F. Wandling, W. Ziarsky, "Capacitive Sensor for Relative Angle Measurement", *IEEE Conference on Instrumentation and Measurement (IMTC 2000*), Vol. 2, pp. 830-833, 2000

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