XVII IMEKO World Congress
Metrology in the 3rd Millennium
June 22–27, 2003, Dubrovnik, Croatia

NEW EVALUATING METHOD FOR TESTING OF SIMPLE GEOMETRY AND WORK PIECES

Blaz Santic

Dr. B. Santic Developments, Augsburg, Germany

Abstract: The paper describes a new approach based on Chebyshev method, which is stabilized by an arbitrary small weighing of least-squares principle in order to permit the reliable and repeatable examination of the best fit geometric elements in a simple and very complex geometry. Field of application of new method is geometry quality control in automotive and machinery building industry, in plastic forming etc. The method can be also used for further investigations of margin and stability conditions of Chebyshev method.

Keywords: best-fit element, co-ordinate metrology

1. INTRODUCTION

Application of Chebyshev method is not satisfactory enough in the case of some CMM measurements. This problem is present on non-precise parts of work pieces and in measurements with scanning robots or coordinate measuring machines with lower accuracy. This is especially true when dealing with plastic work pieces. We have found a solution, with which this problems may be successfully solved. The basic idea was to mix an arbitrary small weighting of the least squares and Chebyshev methods.

2. EXPERIMENTS

We set in the first step the goal of finding a solution, by which the shape of the evaluated element hardly differs from the shape of the pure Chebyshev element. For that purpose we developed a new, joint objective to combine the least squares and the Chebyshev method objectives by means of different weighting parameters (P1, P2).

For the experimentation we have used the measuring data of the work pieces with lower accuracy, plastic work pieces and the data for testing coordinate measuring machine algorithms [1].

We have jointed to the least square method the parameter P1, to Chebyshev method the parameter P2.

At beginning of computation we started with parameter $\mathbf{P_1}$ set on zero and parameter $\mathbf{P_2}$ set on 1. If no result couldn't be found, the parameter P1 was continuously and automatically increased from one trial to another until the first result would be obtained. The increasing value was very small, for instance 1E-5 or less, while a Chebyshev adjusting parameter $\mathbf{P_2}$ was held up at the value 1.

The experiment was successful.

It was found that all results of elements of any geometry converge like the least squares results. The results were repeatable and stable, independently of the measuring point dispersion. This can be explained with the astonishing particularity of the Chebyshev method to obtain a convergence direction by comprehending a secondary factor of an arbitrary small least-squares weighing part.

In this way, the global solution is guaranteed.

It should be noted that the absolute values of the adjusting parameters P1 and 2 are not important, but their relationship. The reciprocally influence of the both objective parts is always continual in full range- from the Chebyshev to the least squares and vice versa (Fig. 1-2 and Table 1).

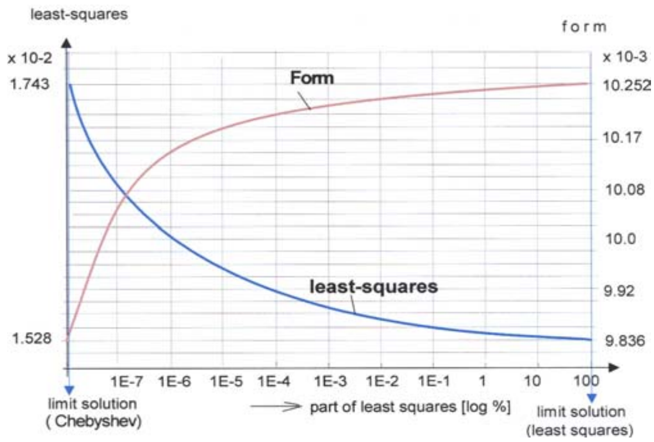

Fig. 1. The course of the form and the least squares depending on partition of the parameter P1, by a cylinder of radius 22.3 mm, height 20 mm, scanned with 7605 points

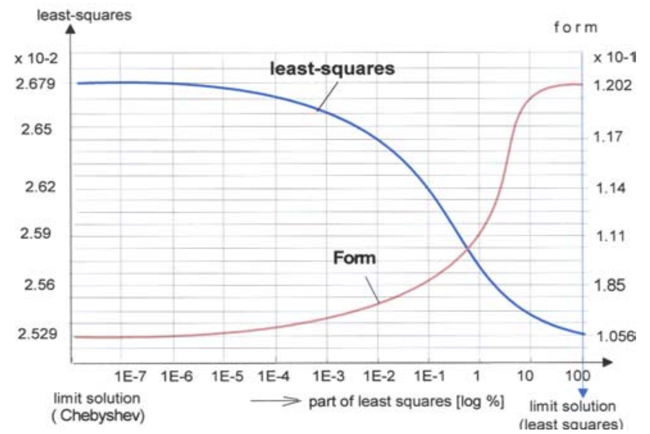

Fig. 2. The course of the form and the least squares depending on partition of the parameter P1, by a cylinder of radius 104.36 mm, height 230 mm, scanned with 25 points

Table 1. Review of results computed for a cylinder "data.b16" [1]

	P1	P2	SQ	Form
Chebyshev	0	1	7,038877E-04	1,971436E-02
	1,00E-06	1	6,978170E-04	1,972961E-02
	1,00E-05	1	5,832237E-04	2,035522E-02
	1,00E-04	1	5,374335E-04	2,081299E-02
	1,00E-03	1	4,960136E-04	2,235413E-02
	1,00E-02	1	4,919769E-04	2,323914E-02
	1,00E-01	1	4,922738E-04	2,336121E-02
	1	1	4,922039E-04	2,337646E-02
Least squares	1	0	4,922039E-04	2,337646E-02

Furthermore, our investigations show that the direction vectors of most Chebyshevian elements can be inclined by a small angle and/or shifted to some small extent without having any practical effect on the best form. This mentioned changing of the best Chebyshev elements is effected by setting a very small weighing of least-square parameter in the Chebyshev algorithm.
Depend on the dissipation and number of measuring points, the change of the form was hardly noticeable (see Fig. 3).

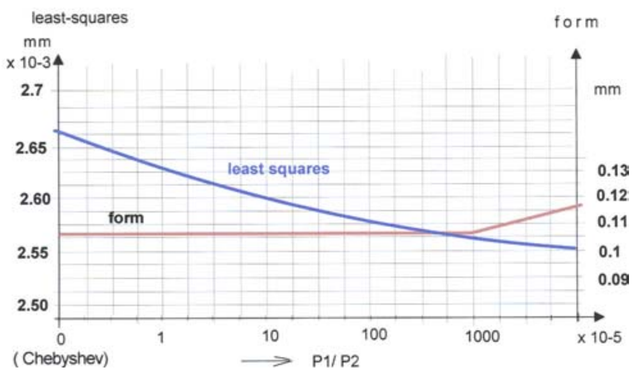

Fig. 3. Influence of weighing parameters P1/P2 on the cylinder form

Lastly we set the new received parameters for the starting parameter in the Chebyshev subroutine. After a few improvement steps in the iteration, the global Chebyshev solution was found.
The not solvable problems according to Chebyshev becoming now solvable.

3. NEW METHOD

The new method makes it possible to find the single and precise limit solution (or any desired intermediate solution) either by the method of least squares or Chebyshev.
The setting of P1 and P2 parameters can be a function of the surface quality of the work piece, or self-generated automatically in order to find the best possible norm-conform solution. It must be emphasized here that no intermediate solution is as good as the pure Chebyshev solution!

The most cases, which were previously defined as not solvable with Chebyshev method, are solvable with very small part (1/100000) of least square in the common objective. The aberrance caused by such one small weighing of the least squares on the exact Chebyshev solution can be estimated only by the measuring data of the geometric elements with a small standard deviation and normal dissipation of measuring points. It can be proved, that the difference to the Chebyshev solution can be practically neglected.
On the other hand, the parameters of the geometric elements received by such a near method guarantee the quick and stable finding of the exact Chebyshev solution.

Regarding the influence of weighing of both parameters on the element's form, it is generally found, that the effect of the least squares weighing is much greater than Chebyshev weighing. This influence becomes larger with the increase of the number of measuring points. For that reason, it is advisable to set the increasing step of least squares parameter at 1/ (No. of points x 100) or less.

The new evaluation method takes advantage of both:

- The method of least squares which is stable and single- defined, and
- Chebyshev method for norm-conform minimization of the amount intervals.

It is assured that the method converges with both a minimum as well as a very high number of points. Furthermore, convergence happens with minimum as well as very large standard deviations. This is very important in the field of the production measuring technique.

A secondary feature of the new method is a possibility of the testing as well the least-squares (Gaussian) as the Chebyshev algorithm.
For testing of least square software, the free parameters P1 to be set on 1, while the parameter P2 should be increased from zero in very small steps. For testing of Chebyshev software, the free parameters P2 to be set on 1, while the parameter P1 should be changed from zero in very small steps. The results can be arranged in the tables or show in the diagrams, like Fig 1 - 3. If any maximum or minimum is present, there is an error in the software.

4. NEW EVALUATING ALGORITHM FOR A SIMPLE ELEMENT

In the following the new evaluation algorithm for a simple element will be described in four steps.
We recommend to use the simple, fast and flexible automatic computation of the starting parameters according to [2, 3].

STEP 1: Least square iterative evaluation

The well-known objective of least squares of a geometric element measured by a sample of ***np*** measuring points, which is based on the objective function for the squares of the point distances f_i, reads as follows:

$$\mathbf{F} \equiv \sum_{i=1}^{np} fi^2 \Rightarrow \text{Min.} \quad (1)$$

STEP 2: Norm- conform best-fit evaluation

The objective for a best-fit evaluation accoding to Chebyshev method reads as follows:

$$\mathbf{F} \equiv \left| \nabla_0^{np} \right|_i \Rightarrow \text{Min.} \quad (2)$$

There are a lot of existing best-fit evaluation methods according to Chebyshev, such as: Monte-Carlo method, Simplex-method [4, 5] and so called Lp approximation.

All these methods are suitable for combination with the least square method.

The objective of the Lp approximation reads as follows:

$$\mathbf{F} \equiv \left(\sum_{i=1}^{n} fi^p\right)^{1/p} \Rightarrow \text{Min.} \quad (3)$$

Where $\boldsymbol{p}$ is a large number.

STEP 3: Evaluation of the new method for a simple element

The following expressions describe the new evaluation algorithm.

The new common objective for a simple element is:

$$\mathbf{F} \equiv P_1 \sum_{i=1}^{np} fi^2 \Delta + P_2 \left| \nabla_0^{n} \right| + \lambda_1 \ (\underline{\text{mat}}) \Rightarrow \text{Min.} \quad (4)$$

or with the Lp- approximation part:

$$\mathbf{F} \equiv P_1 \sum_{i=1}^{np} fi^2 + P_2 \left(\sum_{i=1}^{np} (fi^p)^{1/p} + \lambda_1 \ (\underline{\text{mat}}) \Rightarrow \text{Min.} \quad (5)$$

where $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ are adjusting parameters; **p** is a large number; λ_1 is Lagrange multiplier; the expression **(<u>mat</u>)** is a mathematical necessary condition.

The equation (4) or (5) to be developed by the partial differentiation with respect to nine variables $\mathbf{a}_i$ in a known system of the linear equations:

$$\underline{\mathbf{N}} = \underline{\mathbf{A}}^{\mathrm{T}} \cdot \underline{\mathbf{A}}\,; \quad \underline{\mathbf{x}} = -\,\underline{\mathbf{N}}^{-1} \cdot \underline{\mathbf{B}}, \quad (6)$$

where $\underline{\mathbf{A}}$ is a determinante (Jacobiane), $\underline{\mathbf{B}}$ is a vector.

Iterative solution of this system using of the Newton method is here not showed.

The evaluated parameters to be used as starting parameters for final step.

STEP 4: Best-fit evaluation using method (2) or (3)

Since the results of the evaluation by using the previously steps hardly differ from the Chebyshev solution, it is obvious, only a few improvement steps in the iteration are needed to reach an exact Chebyshev solution.

5. APPLICATION OF NEW ALGORITHM FOR TESTING OF WORK PIECES WITH DEFINED TOLERANCE ZONES (COMPLETE GAUGING)

The importance of such an evaluation becomes visible by complete gauging of the work pieces [6, 7]. There it is absolutely necessary that all geometric elements must be norm-conform evaluated. Every one prescribed tolerated connection condition, which has been given in input, practically is acting as a restriction which inclines and/or shifts the axis of the best surface in the required direction. Since the different elements have different surface qualities and/or different marginal and connecting conditions, the Chebyshev solution can not always be guaranteed. There it is further absolutely necessary that all geometric element must be stable on each improvement step during the iterative process of solving very large non- linear equations.

As previously mentioned we find, that the direction vector of many standard geometric elements can be inclined by a very small angle and/or shifted by small amount and nevertheless its best Chebyshev form was practically not changed. This finding, proved by a complete gauging according to [7], enables a quality inspection of complex work pieces within tolerance zones as defined by [8, 9] with absolutely the least fault zones.

6. NEW ALGORITHM FOR A WORKPIECE

In the following the new evaluation algorithm for a work piece will be described in two steps.

STEP I: Evaluation of the new method for testing a work piece

The new common fundamental objective for testing a work-piece under enveloping conditions is to minimize the polynomials consisting of several separate common objectives (one for each surface of the work piece with corresponding measuring points), wherein each part of the separate objectives (least squares and Chebyshev) is multiplied with the adjusting parameters **P1** and **P2**, including the weighing factors according to importance of the surfaces and all necessary mathematical conditions as well as all prescribed connecting conditions between the surfaces with its tolerances in accordance with the drawing:

$$\mathbf{F} \equiv \boldsymbol{P1} * \sum_{0}^{m} \left\{ \mathbf{g}_1 * (\Delta_1^2) + \mathbf{g}_2 * (\Delta_2^2) + .. + \mathbf{g}_m * (\Delta_m^2) \right\} +$$
$$\boldsymbol{P2} \sum_{0}^{m} \left\{ \mathbf{g}_1 |\nabla|_1 + \mathbf{g}_2 |\nabla|_2 + ... + \mathbf{g}_m |\nabla|_m \right\} + \lambda_v \ (\underline{\mathbf{mat}}_v) +$$
$$+ \lambda_k \left(\mathbf{g}_k \, \underline{\mathbf{Con}}_k + \varphi_D + \mathbf{tol}_k . \sin(\alpha_k) \right) \Rightarrow \text{Min.}, \quad (7)$$

where ***P1*** and ***P2*** are adjusting parameters (multipliers),

(Δ_i^2) are $\sum_0^{np} fi^2 \Rightarrow$ Min., a least squares partial objective for each one surface,

$|\nabla|_i = \left| \nabla_0^{np} \right|_i \Rightarrow$ Min., a best-fit objective for each of one **m** surface; **np** is number of the associated measured points for each element; **k** is number of the prescribed and tolerated connections; **v** is number of the necessary mathematical connections; $\mathbf{g_m}$ are weighing factors of the individual surfaces according to their importance; $\mathbf{g_k}$ are weighing factors of the individual conditions according to its importance; λ_v , λ_k Lagrange multipliers; the expressions $(\underline{\mathbf{mat}}_v)$ are mathematical necessary conditions; the expressions $(\underline{\mathbf{Con}}_k + \varphi_D + \mathbf{tol}_k . \sin(\alpha_k))$ are prescribed connecting conditions of interrelated surfaces in accordance with the drawing; $\underline{\mathbf{Con}}_k$ are angle conditions (such as parallelism, orthogonality, inclination) or distance conditions between the interrelated surfaces in accordance with the drawing; φ_D is the average difference range of size or angle. If a prescribed tolerated range is not symmetrical, then it will be set at an average tolerance range and the eccentricity (positive or negative difference between the 0-value and the average value) will be added to the respectively prescribed size or angle condition;

$\mathbf{tol}_k$ are tolerance´s ranges (boundaries) of the connections of interrelated surfaces in accordance with the drawing;

α_k are auxiliary parameters ($0 < \alpha_k < 2\pi$), which also to be improved by iteration; $\sin(\alpha_k)$ sine or cosine function, with which to be ensured, that the prescribed tolerated range can not be exceeded.

The objective (7) to be developed in a well-known non linear equations system. Iterative solution of this system using of the Newton method is not shown here.

The evaluated parameters of all geometric elements to be used as starting parameters for Step II.

STEP II: Best-fit evaluation using the Method for Testing of Work Pieces Using Complete Gauging [7]

The objective is to minimize the polynomial of the maximal deviations for **m** surface's objectives by Chebyshev, including all necessary mathematical conditions as well as all prescribed connecting conditions (relationships) between the surfaces with its tolerances in accordance with the drawing:

$$\mathbf{F} \equiv \sum_{0}^{m} \left\{ \mathbf{g_1}\left|\nabla\right|_1 + \mathbf{g_2}\left|\nabla\right|_2 + ... + \mathbf{g_m}\left|\nabla\right|_m \right\} + \lambda_v\,(\underline{\mathbf{mat}}_v) +$$
$$+ \lambda_k\,(\mathbf{g_k}.\,\underline{\mathbf{Con}}_k + \varphi_D + \mathbf{tol}_k . \sin(\alpha_k)) \Rightarrow \text{Min.,} \quad (8)$$

where $\left|\nabla\right|_i = \left|\nabla\right|_0^{np}{}_i$ = an objective (minimize the maximal deviations for each of one **m** surface)

For the explanation of the expressions see Step I.

Since the parameters of the geometric elements evaluated according to previously described method for testing a work piece hardly differ from the parameters according to Chebyshev solution for each geometric element, it is obvious, only a few improvement steps are needed to reach the exact Chebyshev solutions.

7. SUMMARY

The presented new method is based on Chebyshev principle, which is stabilized by an arbitrary small weighing factor of least-squares principle in order to permit the reliable and repeatable examination of the best fit geometric elements in a simple or a very complex geometry, by non-precise parts of work pieces or by CMM measurements of lower accuracy.

The convergence of the new evaluation method corresponds to the least square principle, while the Chebyshev precision is reached in the full-minimization of the amount intervals.

It can be further proven, that the direction vector of many standard geometric elements can be inclined by a very small angle and/or slightly shifted without changing the best Chebyshev form.

By using all these advantages, the method is successfully implemented by complete gauging of the work-pieces within tolerance zones as defined by ISO 2692 [9]. There is an absolute requirement that the Chebyshev method for all different elements must be stable on each improvement step during the iterative process of solving very large non-linear equations, besides all tolerated conditions and different surface's qualities.

Using presented method for geometry quality control in automotive and machinery building industry, in plastic forming etc., an evaluation is ensured of the best least fault zone of a single element as well as the least shape of each element by testing of work pieces under enveloping conditions [7].

The new method characterises a convergence independent of the measuring point dispersion.

REFFERENCES

[1] "Testing Coordinate Measuring Machine Algorithms, Phase II ", *Commission of the European Communities, bcr information, Applied Metrology, Report EUR 13417, Luxemburg 1991*

[2] G.T. Anthony, H.M. Anthony, M.G. Cox, A.B. Forbes "The parametrization of geometric form", *EUR 13517, 1991*

[3] B. Santic " New algorithm for examination of turbine blades", *doctoral dissertation, Zagreb 1990,*

[4] S. Rao. "Optimisation Theory and Applications", *Wiley Eastern Ltd, India, 1984.*

[5] W. Lotze, "General solution for Chebyshev approximation of form elements in co-ordinate measurement", *Measurement vol. 12 , pp 337-344, 1994*

[6] B. Santic " An integral method for on-line testing of work pieces comprising several forming elements", *Patent DE 196 00 002 C2", 2002*

[7] B. Santic "Computation method for testing of work pieces using complete gauging", *XVII IMEKO, 2003.*

[8] ISO 1101 "Technical drawings-Geometrical tolerancing-Tolerancing of form, location and run-out"

[9] ISO 2692 "Technical drawings-Geometrical tolerancing-Maximum-Material-Principle "