XVII IMEKO World Congress
Metrology in the 3rd Millennium
June 22–27, 2003, Dubrovnik, Croatia

NEW SIGNAL PROCESSING METHOD OF TRANSIT TIME MEASUREMENT

Mathias J. Niemann

Metering Systems
RWE Gas AG
Kölner Str. 6, 45661 Recklinghausen, Germany

Abstract – The cross-correlation function is commonly used to measure the transit time between two similar signals. The maximum value of the cross-correlation function corresponds to the transit time. The other way to measure this is to add the two signals. This method is comparable with the well known interference measurement technique. In the spectrum of the added signals after filtering a cosine function can be determined. The frequency of this function is directly proportional to the transit time.

Keywords : Transit time measurement, Spectrum analysis, Addition of two signals

1. Introduction

The calculation of the cross correlation function is normally used to determine the transit time of two similar signals. Two sampled signals $x_n = x(nT_a)$ and $y_n = y(nT_a)$ with the index number n, which can have values between 0 and $N-1$, and the sampling time T_a are given. If both signals are identical and time shifted by the transit time $\tau = iT_a$, the following equation is valid:

$$y(nT_a) = x((n-i)T_a). \tag{1}$$

The cross correlation function of these two signals is defined as:

$$\phi_{xy}(kT_a) = \frac{1}{N}\sum_{n=1}^{N} x(nT_a)y((n-k)T_a) = \frac{1}{N}\sum_{n=1}^{N} x_n y_{n-k}$$

$$= \frac{1}{N}\sum_{n=1}^{N} x(nT_a)x((n-i-k)T_a) = \phi_{xx}((k+i)T_a) \tag{2}$$

This function is identical with the autocorrelation function of the original signal shifted by the transit time $\tau = iT_a$. So the maximum of the cross correlation function corresponds with the transit time $\tau_{\max} = iT_a$, "Fig. 1".

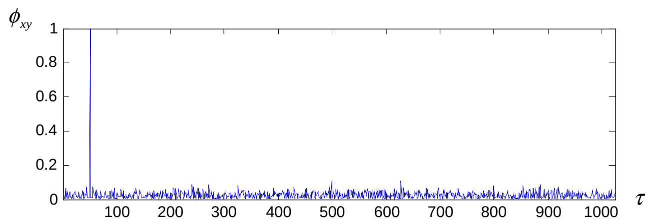

Fig. 1. Cross correlation function

The alternative method for determining the transit time by calculation of the cross correlation function is an analysis of the addition of both signals. The superposition of the signals leads to a cosinusoidal oscillation in the spectrum. There is a directly relationship between its frequency and the transit time.

3. Transit time calculation

The mathematical theory will be shown for two identical time shifted signals: The discrete Fourier transform of a signal $x_n = x(nT_a)$ is defined as

$$X_d(j\omega_k) = X_d\left(j\frac{2\pi}{NT_a}k\right) = \sum_{n=0}^{N-1} x(nT_a)\cdot e^{-j\omega_k nT_a}$$
$$= \sum_{n=0}^{N-1} x(nT_a)\cdot e^{-j2\pi kn/N} \tag{3}$$

with $\omega_k = \dfrac{2\pi}{NT_a}k$ and $k = 0,1,\ldots,N-1$.

The inverse discrete Fourier transform is

$$x(nT_a) = \frac{1}{N}\sum_{k=0}^{N-1} X_d(j\omega_k)\cdot e^{j\omega_k nT_a}$$
$$= \frac{1}{N}\sum_{k=0}^{N-1} X_d(j\omega_k)\cdot e^{j2\pi kn/N} \tag{4}$$

with $\omega_k = \dfrac{2\pi}{NT_a}k$ and $n = 0,1,\ldots,N-1$.

The second signal is shifted by transit time $\tau = iT_a$:

$$y(nT_a) = x((n-i)T_a). \tag{5}$$

The signal is sampled at the points at $t_n = nT_a - \tau = (n-i)T_a$ related to the first signal, so the inverse discrete Fourier transform results in

$$x((n-i)T_a) = \frac{1}{N}\sum_{k=0}^{N-1} X_d(j\omega_k)\cdot e^{j\omega_k(n-i)T_a}$$
$$= \frac{1}{N}\sum_{k=0}^{N-1}\left[X_d(j\omega_k)\cdot e^{-j2\pi ki/N}\right]\cdot e^{j2\pi kn/N}. \tag{9}$$

The Fourier transform of the time-shifted signal yields

$$x((n-i)T_a) \circ\!\!-\!\!\bullet\; X_d(j\omega_k)\cdot e^{-j2\pi ki/N}. \tag{10}$$

On condition of linearity the Fourier transform of the addition of the time signals $x(nT_a)$ and $x((n-i)T_a)$ is equal to the addition of the Fourier transforms of both signals:

$$x(nT_a) + x((n-i)T_a) \circ\!\!-\!\!\bullet$$
$$X_d(j\omega_k) + X_d(j\omega_k)\cdot e^{-j2\pi ki/N}$$
$$= X_d(j\omega_k)\cdot\left(1 + e^{-j2\pi ki/N}\right). \tag{11}$$

That implies for the absolute value of the spectrum

$$\left|X_d(j\omega_k)\cdot\left(1+e^{-j2\pi ki/N}\right)\right|$$
$$= \left|X_d(j\omega_k)\right|\cdot\sqrt{2+2\cos(2\pi ki/N)}$$
$$= 2\cdot\left|X_d(j\omega_k)\right|\cdot\sqrt{\frac{1}{2}+\frac{1}{2}\cos(2\pi ki/N)} \tag{12}$$
$$= 2\cdot\left|X_d(j\omega_k)\right|\cdot\cos(\pi ki/N)$$

The absolute value of the second signal is identical with the first signal:

$$\left|Y_d(j\omega_k)\right| = \left|X_d(j\omega_k)e^{-j2\pi ki/N}\right| = \left|X_d(j\omega_k)\right| \tag{13}$$

So the absolute value of the added spectrum yields

$$x(nT_a) + y(nT_a)$$
$$\circ\!\!-\!\!\bullet \tag{14}$$
$$\left(\left|Y_d(j\omega_k)\right| + \left|X_d(j\omega_k)\right|\right)\cos(\pi ki/N).$$

The sum of the absolute value of both signals is multiplied by $\cos(2\pi ki/N)$. This term oscillates between zero and one. It is interesting to note that the frequency of the oscillation is directly connected with the transit time $\tau = iT_a$ between both signals. The term $\cos(2\pi ki/N)$ contains the time shift $\tau = iT_a$. Therefore the frequency in the added spectrum is a function of transit time between both signals. The main task of the signal processing is the detection of cosine frequency.

2. Simulations

To illustrate the mathematical context, the measurement of transit time is simulated. Two identical signals are shifted by a defined transit time and consist of normal distributed random numbers with a large bandwidth. Because of the linearity, the adding of the signals in either the time domain or in the frequency domain leads to the same result:

$$x(nT_a) + y(nT_a) \circ\!\!-\!\!\bullet\; X_d(j\omega_k) + Y_d(j\omega_k). \tag{15}$$

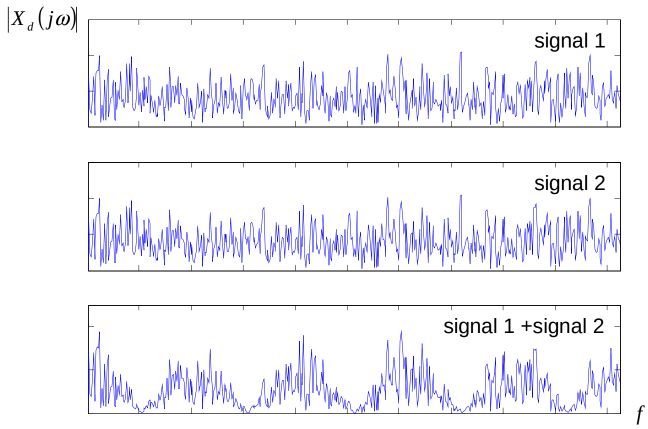

Fig. 2. Spectra of the two signals of and of the addition signal

In the spectrum of the sum signal a cosine oscillation is superposed. “Fig. 2” shows the absolute values of the spectrum for both single signals and for the sum signal. The cosine oscillation extends over the whole spectrum.

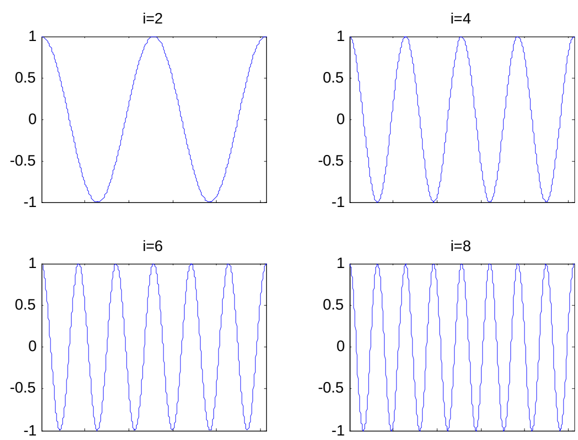

Fig. 3. Influence of the transit time on the frequency of the cosine oscillation

The frequency is a function of transit time between both signals and can be separated from the absolute value of the sum spectrum $\left(\left|Y_d\left(j\omega_k\right)\right|+\left|X_d\left(j\omega_k\right)\right|\right)\cdot\cos\left(\pi ki/N\right)$.

The frequency of the cosine oscillation, witch depends on transit time, is shown in "Fig. 3" for four different time shifts. Wit increasing transit time the frequency is rising. The cycle duration corresponds with the first zero crossing of the first derivative. Knowing the period time the frequency can be calculated.

In practice the two signals are often not identical. Nevertheless the cosine term in the spectrum can be separated. "Fig. 4" shows the cosine function after adding two none identical but similar signals.

A gauge for the similarity of both signals is the height of the standardized cross correlation function. In this case both signals are similar with a value of 87 %.

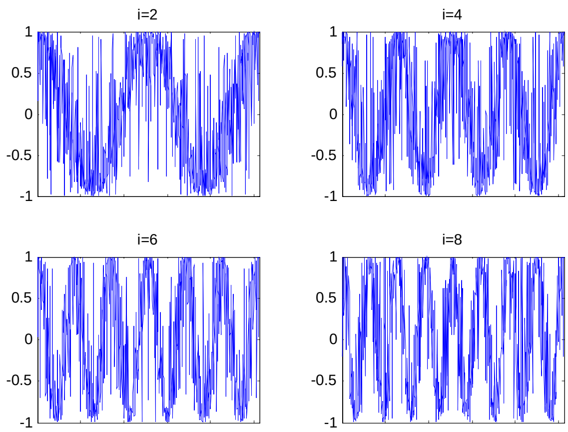

Fig. 4. Cosine oscillation disturbed by noise

The oscillation for all transit times is disturbed by noise. The main task of signal processing is the detection of the cosine term.

3. Practical applications

Besides the signal-to-noise ratio the bandwidth of the time signals has a major influence on the determination of the oscillation frequency. Practical examples for similar signals with a low bandwidth are the ultrasonic measurement results of streaming gas flow. Natural structures occurring in streaming fluid modulate a sinusoidal ultrasonic carrier signal in both amplitude and in phase. The signals of two barriers downstream in a distance d are not identically modulated because of the dissipation of the structures. The modulation is only limited by a small frequency band so the bandwidth of the demodulated signals is very low. In "Fig. 5" the absolute value of the spectrum of both signals and of the added signals are shown.

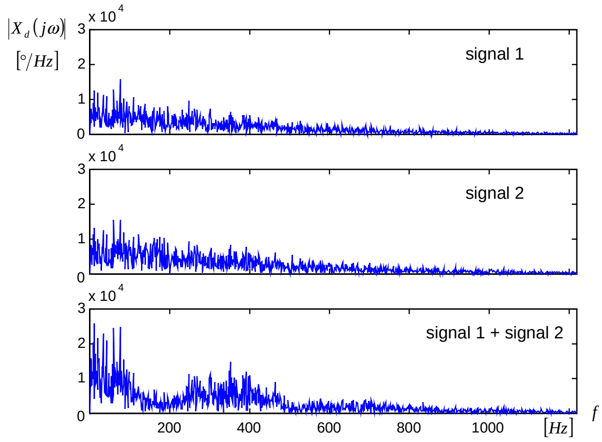

Fig. 5. Spectrum of signals with a low bandwidth

The separated cosine oscillation for four different transit times caused by different flow velocities is shown in "Fig. 6".

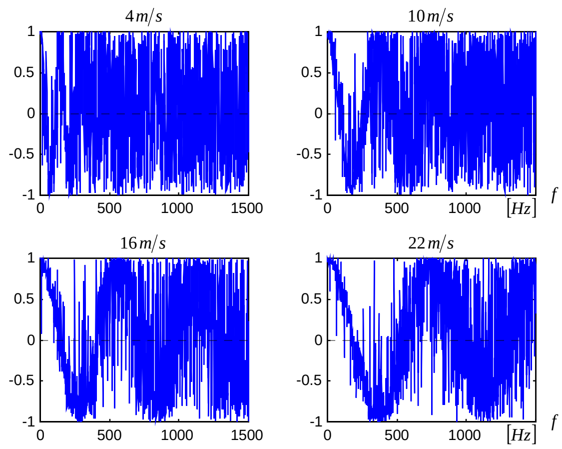

Fig. 6. Cosine oscillation disturbed by noise caused by two not identical signals

The signals are strongly disturbed by noise so the oscillation disappears after two consecutive periods. The easiest way to determine the filtering of the time signal is by calculating the auto-correlation function.. The auto-correlation function leads to a cosine oscillation with much less noise than the original time signal, Fig. 7".

Especially in the case of low flow velocities success is realized because there is still recognition of the cosine oscillation after the occurrence of several periods. But the frequency of the oscillation can not be determined with these signals

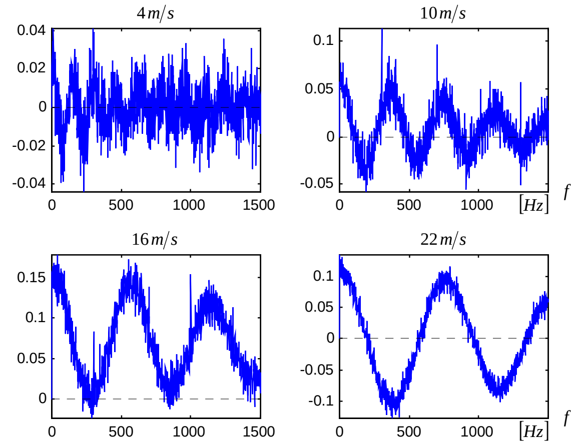

Fig. 7. Cosine oscillation after filtering with autocorrelation function

For further signal processing the spectrum of the filtrated cosine oscillation is calculated, shown in “Fig. 8”.

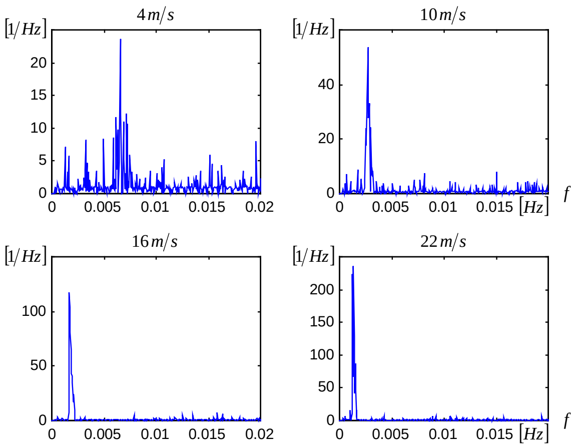

Fig. 8. Spectrum of the filtered cosine oscillation

In this case the noise level appears to be different for all transit times because of the use of varied scales as shown in the figure above. For all velocities the dominating frequency can be clearly detected as a maximum value in the spectrum.

With the value of the dominating frequency in the spectrum the transit time can be calculated. Another possibility to determine the frequency of the cosine oscillation is the measurement of the period. A digital filter allows to pass a frequency range just around the dominating frequency in the spectrum and minimizes noise in the oscillation signal. The result is shown in “Fig. 9”. The cycle duration corresponds with the first zero crossing of the first derivative. With information about the period the frequency can be calculated.

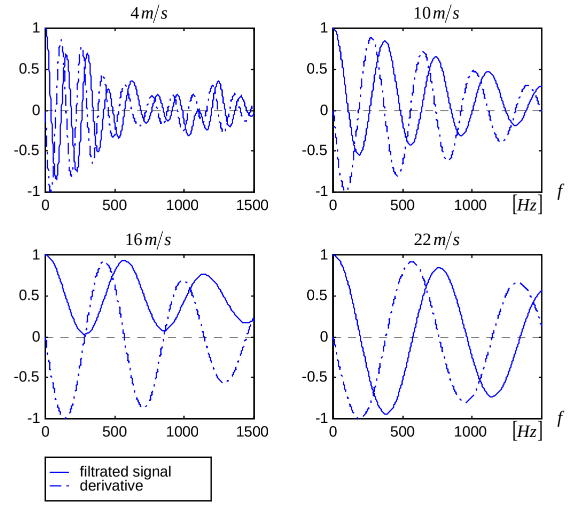

Fig. 9. Filtrated signal with deviation

So there are two different possibilities to detect the frequency of oscillation in the spectrum of the added signals.

4. Conclusion

The results obtained have shown, that the addition of two signals shifted by a transit time and the analysis of the spectrum is an alternative to cross-correlation measurement. The spectrum is overlapped with a cosine oscillation. The frequency value is directly related to the determined transit time. Using a suitable filter technique the noise of the oscillation can be reduced to nearly zero. By this method the frequency and the transit time can be accurately calculated.

In practice the measurement signals are analogue and have to be converted. Typical of the new method is that the analogue digital conversion is limited to only one signal, because the signals of both barriers can be added analogous.

5. Acknowledgement

The research developed during my employment at Institute of Measurement and Control of the University of Essen, Prof. Dr.-Ing. habil. Volker Hans.

REFERENCES

[1] M. Niemann, “Signalverarbeitung in der Ultraschall-Durchflußmessung“, *PhD thesis*, University of Essen, Shaker Verlag, Aachen, 2002.

[2] M. Niemann, “Phase reconstruction of modulated ultrasonic signals and new measurement technique of flow velocity”, *IIIE IMCT 2002*, Proceedings, S. 93-97, Anchorage, 2002.

[3] M. Niemann, “New signal processing methods of ultrasound phase reconstruction for measurement of disturbed flow”, *SCI 2002*, Proceedings, Orlando, 2002.

Author: Dr.-Ing. Mathias J. Niemann, Metering Systems, RWE Gas AG, Kölner Str. 6, 45661 Recklinghausen, Germany, Phone: +49-2361-38-2061, Fax: +49-2361-38-2434, E-Mail: mathias.niemann@rwegas.com.