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THE CONCEPTS OF ANALOG AND DIGITAL CODING IN TERMS OF MEASUREMENT THEORY

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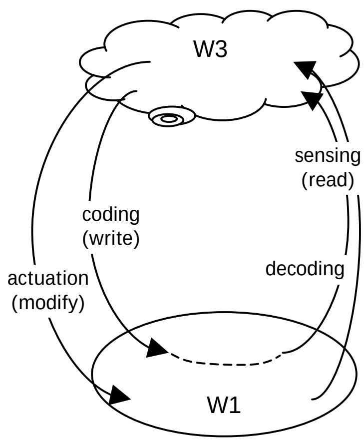
Abstract – The concepts of analog and digital coding are usually employed with a vague, even if not wrong, meaning, according to the more or less explicit assumption that they are synonymous of continuous and discrete coding respectively. This work is aimed at showing how such concepts can be instead properly defined in terms of Measurement Theory, in the context of the fundamental duality between coding and sensing.

Keywords: Measurement Theory, Analog coding, Digital coding.

1. INTRODUCTION

Measurement is recognized to be a peculiar means to bridge the physical world, to which the thing under measurement belongs, and the information world, to which the measurement result belongs. These two worlds (the “world 1”, W1, and the “world 3”, W3, according to the Popper's interpretation, the “world 2” being populated by human minds and the related subjective experiences [1]) have so radically distinct features that any operation establishing a relation between them is epistemologically significant. Two fundamental categories of such operations can be identified in terms of their pragmatics: those aimed at exploiting (a part of the) physical world to store some available information, and those meant to retrieve some information that is assumed to be stored in (a part of the) physical world. With the aim of concisely denoting these operations of *generalized writing* and *generalized reading*, let us call them *coding* and *sensing* respectively (while measurement is, in this sense, a sensing operation, we recognize sensing itself as a more general concept than measurement, because the former does not imply some characteristics of the latter, and in particular the usage of an instrument calibrated to a suitably traceable standard).

Let us shortly point out the context of such operations: in the category of W3→W1 operations, coding, whose goal is to store information by means of a physical support, is complemented by actuation, aimed at modifying a physical system according to information (for example, driving a car requires many actuations and basically no coding); on the other hand, in the category of W1→W3 operations sensing can be performed on either previously coded information, with the goal of retrieving the information stored in a physical support, or to obtain information on a physical system. In this sense decoding appears to be a sub-category of sensing.



Given this duality of coding and sensing [2], it should not be amazing that some features of sensing *reversely appear in coding*. It is particularly significant considering in this view the condition that abstractly characterizes any sensing operation, i.e., the homomorphic preservation of the structure of W1 entities in the mapping to W3 entities, as formalized in Measurement Theory (this seems to be a reasonable interpretation for the well-known Stevens' assertion, according to which «measurement is the assignment of numerals to objects or events according to rule, any rule» [3], even though we believe that a so weak “definition” should be more aptly applied generically to sensing and not only to measurement). While in W1→W3 mappings such a condition is aimed at assuring the meaningfulness of the obtained results [4], in the case of coding it pragmatically guarantees the adequacy of the physical system adopted as support for the information to be stored. Indeed, if for actuation the goal of the operation drives the physical transformation to be performed, in coding such a transformation is not pre-determined and different options are allowed to obtain the result of writing the information on the physical support. These options can be basically categorized according to two fundamental, and for many aspects opposite, strategies: *analog* and *digital* coding.

In the following we will discuss how such strategies can be properly defined by means of Measurement Theory.

2. THE CONCEPTS OF ANALOG AND DIGITAL CODING

Preliminarily, we should discuss why the definitions usually adopted for the concepts of analog and digital, more or less explicitly assuming that they are synonymous of continuous and discrete coding respectively (such that, e.g., "digital" would stand for «operating on data represented as series of binary units or in similar discrete form»), cannot be maintained. Indeed, such a claim derives from the basic results of metrology: from the observation that the resolution and the range of any instrument are finite (we are considering here instruments for both writing on physical supports and reading from them), it follows that the result of any sensing operation cannot be but discrete. Continuous (and therefore analog, according to the usual interpretation) representations would be just a kind of idealization to formalize what was empirically obtained in discrete form. Furthermore, the current awareness of the unavoidability of some epistemic content in the expression of measurement results, witnessed particularly by the acceptance of the concept of *intrinsic uncertainty* within the ISO GUM [5], should allow us to distinguish an *economic attitude* (e.g.: the representation by means of real numbers is useful because of the amount of available mathematical tools based on them) from a *metaphysical principle* (e.g.: despite any metrological limitations, quantities of physical systems in reality vary in continuous way). We do believe that neither "digital as approximation of analog" nor "analog as idealization of digital" are able to explain the conceptual and technological polarities that the concepts of analog and digital actually represent. In other terms, the opposition analog-digital should be maintained as related to pragmatics and technology, not the (ancient!) ontological issue whether «*natura non facit saltus*» (nature does not make jumps, and therefore is inherently continuous) or not.

Coming back to the problem of the strategies for coding, it is easy to recognize that the information to be written on a physical support is often *more than classificatory only*, and allows the identification of not only an equivalence relation (such that the given set of symbols S is partitioned into a set of classes, $S_1,\ldots,S_n$, satisfying the conditions, $S_i \cap S_j = \varnothing$ and $\cup_i S_i = S$) but also, e.g., an ordering and / or a metric: the symbols we deal with usually convey both "nominal information", *à la* Shannon, and structural, meta-information [6]. In other terms, such symbols are defined in a scale whose type is algebraically richer than the nominal one.

The fundamental issue then arises of *how to maintain the meta-information* when storing the information on a physical support, so that from the observation of such a support both the coded information and the related meta-information can be inferred. To solve this problem, two general strategies can be envisioned:

* *the meta-information is stored on the physical support*: in this case, a support is adopted whose configurations (i.e., its empirically distinguishable states) belong to a set on which a set of relations is defined, each of them corresponding to an informational relation to be maintained. Hence, the coding is performed as a morphism from the symbolic (i.e., informational, i.e., embedded in W3) relational system to a suitable empirical (i.e., embedded in W1) relational system, the condition of morphism ensuring the preservation of the available meta-information;

* *the meta-information is maintained in the coding rule*: in this case, the configuration set of the adopted support must only be able to maintain the distinction among the symbols to be stored, i.e., the coding is performed as an invertible mapping from the symbolic relational system to the empirical configuration set, the only condition imposed in this case being therefore on the cardinality of such a set. On the other hand, the lack of any structure of the configuration set imposes the mapping that implements the coding to be explicitly, i.e., extensionally, known.

We suggest to denote these strategies as *analog* and *digital* coding respectively.

3. AN EXAMPLE

Let us discuss a simple example to compare these strategies while highlighting their peculiarities. A support with a configuration set $C=\{c_j\}$ is adopted, aimed at storing a natural number s chosen in the set $S=\{0,\ldots,3\}$ (let us mention again that the definition of C is based on the empirical possibility to recognize its elements as distinct). As the result of the decoding operation, the symbol identification together with its ordinal positioning must be obtained.

Formally, the symbol s_i is coded to a configuration $c_j=\mathrm{cod}(s_i)$, and such a configuration is decoded to a symbol $s_k=\mathrm{dec}(c_j)=\mathrm{dec}(\mathrm{cod}(s_i))$. Therefore the goal is to maintain in the physical storage not only the information related to the distinction between symbols, if $s_{i'} \neq s_{i''}$ then $\mathrm{dec}(\mathrm{cod}(s_{i'})) \neq \mathrm{dec}(\mathrm{cod}(s_{i''}))$, but also the meta-information on the symbol ordering, if $s_{i'} \leq s_{i''}$ then $\mathrm{dec}(\mathrm{cod}(s_{i'})) \leq \mathrm{dec}(\mathrm{cod}(s_{i''}))$. Operatively, the (a priori unknown) initial symbols $s_{i'}$ and $s_{i''}$ and their ordering must be reconstructed by decoding the observable configurations $c_{j'}$ and $c_{j''}$.

The first strategy could be implemented by coding the symbols in terms of a physical quantity characteristic of the support (let us call it α-*rule*): the symbol $s=2$ could be coded by a 2 V electrical potential or a 2 g mass. Once the support configuration has been determined, the retrieval of the stored information and meta-information simply requires the *intensive* knowledge of the coding rule (as it could be expressed by, e.g., "the symbol $s=x$ is coded by generating a potential of x V"), in this case applied in its inverse form ("having observed a potential of x V, the decoded symbol is $s=x$"). The existence of the empirical relational system (in this case such that the relation $c_{j'} \leq c_{j''}$ is empirically observable) guarantees that also the meta-information is maintained on the support, and that can be transferred to symbols by means of a morphic decoding (if $c_{j'} \leq c_{j''}$ is observed then the condition $\mathrm{dec}(c_{j'}) \leq \mathrm{dec}(c_{j''})$ must be verified).

The second strategy does not take advantage of any structure of the support, the only requirement on it being that its configuration set contains at least 4 elements, say ■, ◆, ▲, ● (note that we are using these markers, instead of the more usual $c_1,\ldots,c_4$, to emphasize the lack of structure in C: let us call it δ-*rule*). The observation of two of such configurations just conveys the information whether the coded symbols were equal or not, but it is by no means sufficient to make any inference on the related meta-information. Furthermore, the lack of structure in C prevents the intensive definition of the coding rule, that must be described by a complete and explicit listing, 0→■, 1→◆, 2→▲, 3→●, i.e., the same *extensive* knowledge being required to perform the decoding.

4. HYBRID STRATEGIES

The previous examples of the α-rule and the δ-rule show the extreme options of analog and digital coding respectively. The analysis of two hybrid cases, aimed at solving the problem of coding a natural number $s \in S=\{0,\ldots,3\}$ again, is helpful to make the features of these strategies clearer.

As a modified version of the α-rule (let us call it α'-rule), the symbols are coded by collecting a suitable number of repeated elements that with their cardinality constitute the support, so that the symbol s=5 could be coded by a collection of 5 pebbles. As in the case of the α-rule, the *intensive* knowledge of the coding rule ("the symbol s=x is coded by collecting x pebbles") is sufficient to perform the decoding because of the morphic relation between the symbolic relational system and the empirical one. The basic distinction between the α-rule and the α'-rule emerges when the coded set S is changed, for example as S'={0, 0,5, 1, 1,5, …, 3}. While the α-rule can be maintained as is (provided that the devices adopted to write on and read from the support are able to reach a resolution of at least 0,5 V), in the case of the α'-rule the pebbles are dealt with as atomic entities (in other terms: the coding rule is independent of the dimensions of the pebbles), so that the rule itself must be modified, for example assuming that the symbol s=x is coded by collecting 2x pebbles.

Hence, the α'-rule can be thought of as a case of the α-rule, in which the resolution of the support or the writing / reading devices has been reached. Therefore such a rule stresses the availability of a *discrete analog* strategy.

The usual, and clearly mistaken, correspondence between digital and binary suggests a modified version of the δ-rule (let us call it δ'-rule), in which the support is allowed to assume a number of configurations less than #S and down to 2, say ■ and ◆ in our example, but it is required to be replicated. Each symbol is then coded by a sequence of such "elementary supports", e.g., 0→{■,■}, 1→{■,◆}, 2→{◆,■}, 3→{◆,◆} (note that this "natural" coding rule also implies the ordering of the collections, such that {■,◆} and {◆,■} must be empirically recognizable as different). The coding / decoding can be then performed in two phases: a W3→W1 rule maps "elementary symbols" to "elementary supports" (e.g., 0→■, 1→◆), and a W3→W3 rule maps collections of such "elementary symbols" to symbols in S (e.g., 00→0, 01→1, 10→2, 11→3). The hybrid nature of this *replicated digital* strategy emerges when considering that while the first rule is purely digital (therefore a δ-rule, whose instances must be explicitly stated), the second one can be defined in terms of its properties, i.e., intensively.

5. A COMPARISON / CONCLUSION

The features of the four strategies discussed above can be compared and synthesized in terms of 5 parameters as follows:

Strategy	required to define and deal with the rule	B. Intervention required when the coded set is modified	C. Applicability in dependence of the cardinality of the coded set	dependence of the structure of the coded set	E. Requirements on the support
α-rule (standard analog)	intensive	basically nothing	limited by the resolution / range of the devices / support; even for a priori unbound cardinality	applicable only if a structure is present	applicable only with supports provided with a suitable structure
α'-rule (discrete analog)	intensive	redefinition of the rule	limited by the availability of the repeated elements constituting the support	applicable only if a structure is present	applicable only with supports provided with a suitable structure
δ-rule (standard digital)	extensive	redefinition of the rule	limited by the cardinality of the chosen configuration set	applicable in any case	applicable in any case
δ'-rule (replicated digital)	extensive for the "elementary symbols"; intensive for their collections	change of the number of "elementary supports" in the collections	limited by the length of the collections of "elementary supports"	applicable in any case	applicable with replicable supports

We believe that even a summary analysis of the contents of this table is sufficient to significantly explain the conceptual, technological and social reasons of the nature of *opposition* of the relation between analog and digital.

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