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COMPUTER MODELLING OF A PIEZOELECTRIC ANGULAR RATE SENSOR

Saulius Kausinis , Rimantas Barauskas***

*Department of Engineering Mechanics, **Department of System Analysis
Kaunas University of Technology, Kaunas, Lithuania

Abstract - The paper presents the finite element (FE) modelling of operation of a rotational motion sensor that uses a balanced oscillator (tuning fork) to sense the angular rate. The 3D FE model has been employed to examining the sensor's modal properties. The sensitivity functions have been obtained for adjusting the geometric parameters of the quartz element in order to achieve the desired values of natural frequencies. The performance of computations during the dynamic analysis has been enhanced by introducing a dynamically reduced model based on truncation of dynamic contributions of higher modes of the vibrating structure.

Results are presented in terms of sensor performance characteristics for various design parameters and modes of operation. The modelling assumptions adopted are tested experimentally on the vibrating quartz resonator.

Keywords - modelling, piezoelectric sensor.

1. INTRODUCTION

Accurate simulation of piezoelectric transducers requires adequate models that deal not only with fundamental principles underlying measurement systems but also with principles of treatment of deviations and uncertainties based on the analysis of instrument structure and the effects of sensitivity to parameters variations and external influences. Systematic approaches to mathematical modelling of sensors are extensively discussed in the literature [1].

An application of the FE method in modelling the resonance characteristics of the piezoelectric resonators is described [2]. The variation formulation of the problem is based on physical description of the piezoelectric structures behaviour and testing of the model on the longitudinally vibrating quartz resonator.

Our paper reports comparative results of modelling the spatial vibration modes of H-shape piezoelectric structure that is a constituent part of an angular rate sensor. The rate sensor uses the phenomenon of the Coriolis acceleration. In the sensor an oscillatory motion of the vibrating structure is coupled from a primary vibrating mode into a second mode, when the sensor experiences angular rate. A functional description of the gyro rate sensor is available [3, 4].

An application of the FE analysis to modelling a rate sensor produced using shell resonator technology is presented [5].

The use of a simple mechanical coupling model and FE analysis with the aim of reducing the offset output of an H-shape resonator is demonstrated [6]. The results of the analysis have shown, in particular, a correlation between the shape of the vibrator and the ratio between the differing excitation frequency and detection frequency.

We introduce a novel 3D FE computational model and software tools that allow examining the systems' spatial dynamic properties and appreciation of the parametric sensitivity of these. The model is based on physical effects involved and a description in terms of physical geometry and material properties. There is some strong evidence that model developed can be used effectively for analysis and design of the gyro rate sensors.

2. BASIC PRINCIPLE

There are many practical implementations which can be used to produce a rate sensor. The topic describes a computational model of the GyroChip family sensor that uses a balanced oscillator to sense angular rate. The physical principle of operation and performance specifications of the sensor is presented [3, 4]; they are also available on the web site http://www.systron.com.

The sensor has found a wide spectrum of applications in the automotive, aerospace, defence, industrial, commercial, and medical industries. It contains a microminiature double-ended quartz tuning fork and supporting structure, all fabricated chemically from a single wafer of monocrystalline piezoelectric quartz, Fig. 1. A tine undergoing linear motion (along X axis) in a frame of reference, which is rotating about the sensor's longitudinal axis, experiences the Coriolis acceleration (along Y axis) that is directly proportional to the rate of rotation.

The drive tines constitute the active portion of the sensor; they are driven by an oscillator circuit at precise amplitude. Each tine will have Coriolis force acting on it of:

$$F = 2 \cdot m \cdot \Omega \cdot V_r , \quad (1)$$

where m = tine mass, V_r = instantaneous radial velocity, and Ω = input rate.

This force is perpendicular to both the input rate and the instantaneous radial velocity. The two drives tines move in

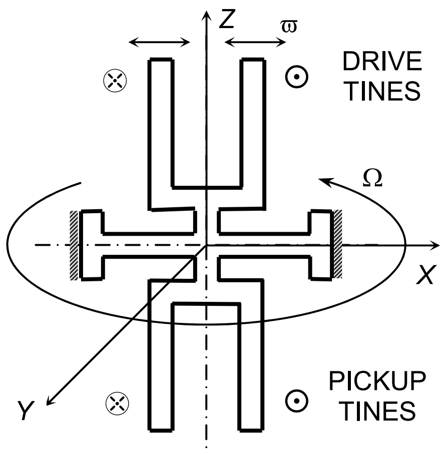


Fig. 1 Vibrating structure of the sensor

opposite directions; the resultant forces are perpendicular to the plane of the fork assembly and in opposite directions as well. This produces a torque that is proportional to the input rotational rate.

The pickup tines, being the sensing portion of the sensor, respond to the oscillating torque by moving into and out of plane producing an output signal proportional to the angular rate.

2. FINITE ELEMENT MODEL

The tuning fork is a vibrating piezoelectric plate of the complex geometric shape the side surfaces of which are laminated by electrodes enabling to create an electric field inside of the material.

In this practical situation the electric field created in the material may be considered as being prescribed, so the piezoelectric phenomena in the plate are governed by the single linear piezoelectricity equation as

$$\{\sigma\} = \left[c^E\right]\{\varepsilon\} - [e]\{E\}; \qquad (2)$$

where $\{\sigma\}$ and $\{\varepsilon\}$ are vectors containing the components of elastic stress and strain, $\{E\}$ = vectors containing the components of the electric field, $\left[c^E\right]$ = stiffness tensor under constant electric field, $[e]$ = piezoelectric stress tensor.

As relative displacements of the tuning fork with respect to the rigid rotating frame are being considered, the full acceleration $\{a_F\} = \{a\} + \{a_N\} + \{a_T\} + \{a_C\}$ is being used in the virtual work equation of the finite element as

$$\int_V \delta\{\varepsilon\}^T \{\sigma\} dV + \int_V \rho\, \delta\{u\}^T \{a_F\} dV = \delta\{U\}^T \{R\}, \qquad (3)$$

where symbol δ denotes the virtual quantity, $\{u\} = [N]\{U\}$ = displacement vector of a particle inside the finite element expressed in terms of the form function matrix $[N]$ and the nodal displacement vector $\{U\}$, ρ = density of the material, $\{R\}$ = vector of nodal interaction forces, $\{a\}$ = relative acceleration with respect to the rotating frame; $\{a_N\}$, $\{a_T\}$ = normal and tangential accelerations due to the rotation of the frame; $\{a_C\}$ = Coriolis acceleration.

The structural dynamic equation of the finite element of the H shape vibrator is obtained as [7, 8]:

$$[M]\{\ddot{U}\} + ([C] + 2\omega[G])\{\dot{U}\} + \left([K] - \omega^2[K_1] + \varepsilon[K_2]\right)\{U\} =$$
$$= \{R\} + \{F\} + \omega^2[K_1]\{X\} - \varepsilon[K_2]\{X\}, \qquad (4)$$

where $[M]$, $[C]$, $[G]$, $[K]$, $[K_1]$, $[K_2]$ are matrices of the finite element, $\{U\}$ is the nodal displacement vector, $\{F\}$ is the vector of nodal excitation forces caused by the piezoelectric effect, $\{X\}$ is the vector of nodal coordinates of the finite element, $\{R\}$ is the vector of nodal interaction forces, ω and ε correspondingly are the angular velocity and angular acceleration of the fork rotation about its longitudinal axis.

The FE model of a non-rotating structure has been developed in ANSYS by using SHELL43 elements and used for obtaining the modes of the tuning fork. In order to take into account the gyroscopic effects caused by rotation of the fork the model has been extended by adding the gyroscopic matrix $[G]$, correction terms $[K_1]$, $[K_2]$ of the stiffness matrix, and angular frequency dependent forcing terms $\omega^2[K_1]\{X\} - \varepsilon[K_2]\{X\}$ as presented in equation (4). In order to analyse vibrations of the tuning fork in terms of slowly varying amplitudes the time averaging method has been applied. Results are presented in terms of sensor performance characteristics for various design parameters and modes of operation.

By the use of finite element analysis the operation specifics of the sensor and the quantitative evaluation of the relationship of the output signal against the angular velocity of the outer frame have been modelled. The modelling assumptions adopted were tested experimentally on the vibrating quartz resonator.

The tuning fork as a continuous structure has theoretically infinite number of vibration modes. A finite number of modes of a non-rotating fork are obtained by solving the characteristic equation

$$\left([K] - \omega^2[M]\right)\{U\} = 0\,. \qquad (5)$$

Natural frequencies of ten lower modes are presented in table 1.

TABLE 1. Lower range of natural frequencies of the fork

Mode number	Natural frequency (Hz)
1	3.529
2	4.048
3	8.110
4	8.113
5	11.215
6	12.052
7	25.085
8	25.398
9	25.400
10	27.183

The fundamental vibration modes of interest are 3rd, 4th, 9th and 10th ones; they play the key role in the resonant dynamic behaviour of the rate sensor and provide the in-plane and out-of-plane vibration coupling of the fork rotating about the Oz axis. The shapes of these modes are presented in Fig.2 and Fig. 3.

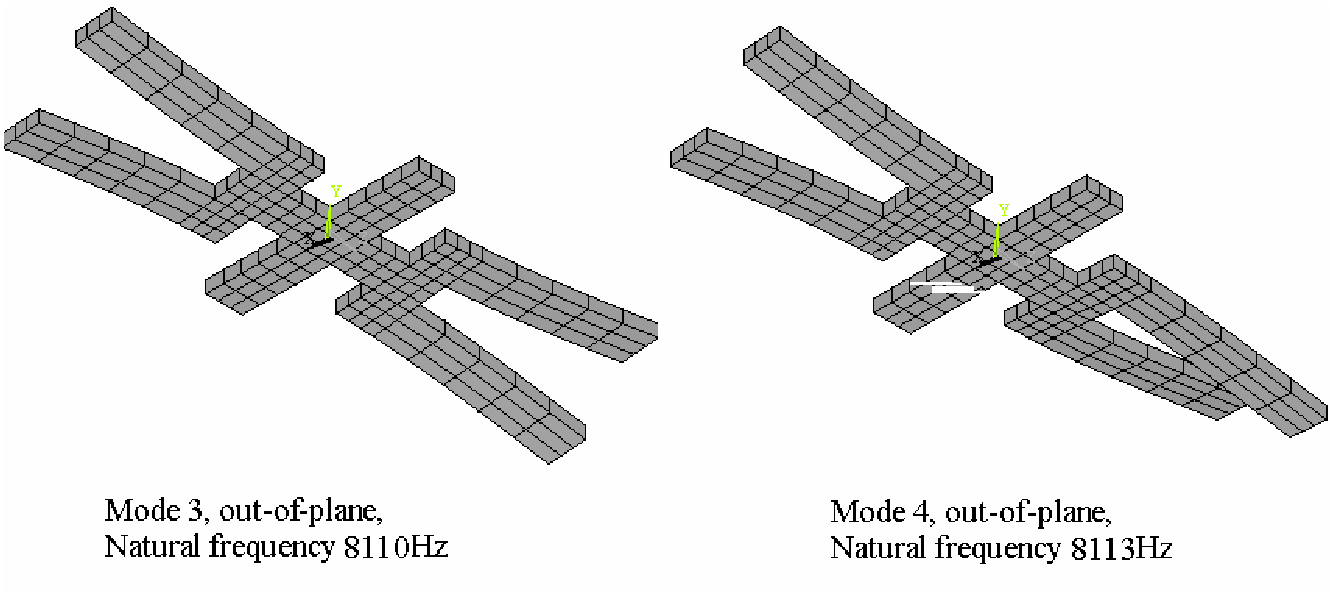


Fig. 2. 3th and 4th (out-of-plane) modes of the tuning fork

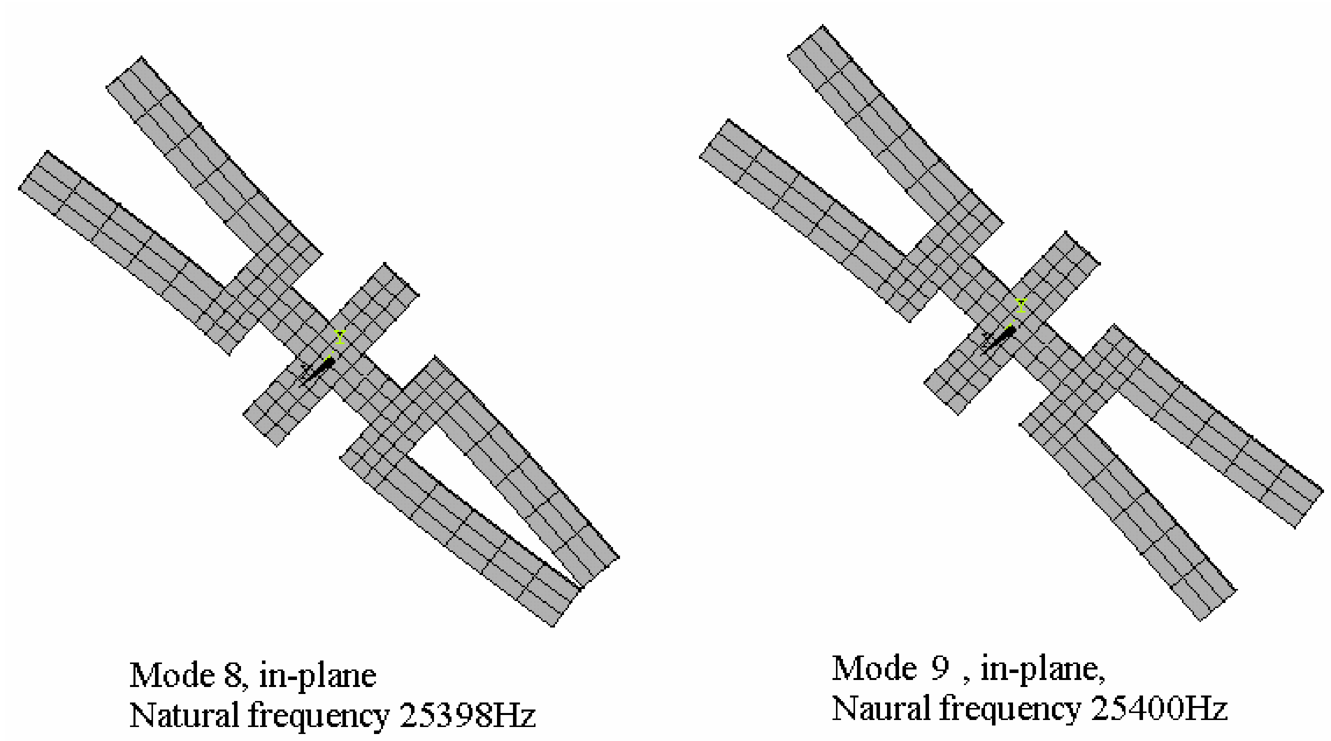


Fig. 3. 8th and 9th (in-plane) modes of the tuning fork

3. PARAMETER SENSITIVITY

As the tuning fork is a mechanical vibrating system with high value of mechanical Q-factor, its performance and dynamic features depend substantially upon natural frequencies and shapes of vibration. The influence of the design parameters can be most effectively carried out by employing gradient techniques.

Sensitivity is defined in terms of a sensitivity function, which denotes the sensitivity of the system design decisions to variations in the system geometric parameters presented in Fig. 4. The finite element matrices of the tuning fork can be presented as functions $[K(b_i)],[M(b_i)]$ of the geometric parameters b_i of the vibrating structure [9]. The relations between small variations of design parameters and corresponding variations of natural frequencies of vibration are obtained by using the free vibration equation (5). The first variation of (5) gives the following relations:

$$\partial\{\zeta_i\} = [C_i]\partial\{b\} \quad , \tag{6}$$

where $[C_i] = \{y_i\}^T \left(\frac{\partial[K]}{\partial\{b\}} - \zeta_i \frac{\partial[M]}{\partial\{b\}} \right) \{y_i\}$ is the matrix of sensitivity coefficients, $\zeta_i = \omega_i^2$ = square of the i-th angular natural frequency, $\{y_i\}$ = the vector describing the *i*-th shape of vibration.

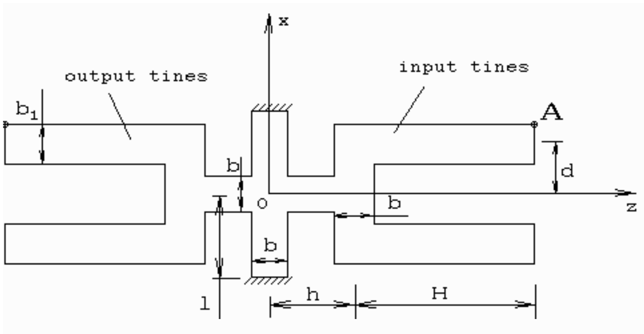


Fig. 4. Geometric parameters of the vibrating structure

The sensitivity functions obtained may further be used to bring about modifications needed in the structure's dynamic properties and determine which modifications would be the most effective for the desired change.

The analysis of obtained sensitivity coefficients indicates that the stiffness of the supporting bar described by its length and width has the main influence upon the difference of the 3rd and 4th natural frequencies. The plot of the values of the two frequencies against the value of parameter l (half length of the supporting bar) is presented in Fig. 5. At parameter value $l \approx 1.54mm$ natural frequencies of 3rd and 4th modes are equal, and this requires a special attention in dynamic analysis as here the magnitudes of the two natural frequencies may counterchange as a result of small variation of the parameter l .

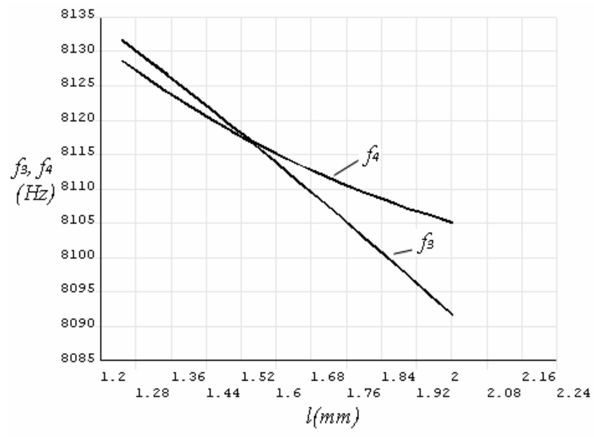


Fig. 5. Natural frequencies of 3rd and 4th modes vs. length l

4. REDUCTION OF DYNAMIC EQUATIONS

In case of moderate velocities of rotation of the frame the modal coupling and resonance phenomena of the fork in the rotating frame can be much better understood by expressing the equations in modal coordinates of the non-damped and non-rotating structure, the vibration modes of which have a clear interpretation.

Moreover, the aim of having the modal properties of a non-rotating structure as target functions in geometrical design of the fork is seen as more natural and convenient.

Let us denote by $\omega_1, \omega_2, \ldots \omega_n$ the natural frequencies of the fork and by the matrix [Y] containing in its columns the natural forms of vibration of a non-rotating fork. By neglecting the effects caused by angular acceleration and centripetal forces, the steady vibration of the fork in the non-rotating reference frame is governed by the equation in terms of modal displacements $\{z\}$ as

$$\begin{aligned} &\{\ddot{z}\} + [diag(\mu)]\{\dot{z}\} + [diag(\omega^2)]\{z\} = \\ &= -2\Omega[Y]^T[G][Y]\{z\} + [Y]^T\{F\} \end{aligned} \quad , \tag{7}$$

where the modal damping coefficients $\mu_1, \mu_2, \ldots \mu_n$ are obtained as $\mu_i = \alpha + \beta\omega_i^2$, and the substitution $\{U\} = [Y]\{z\}$ and relations $[Y]^T[M][Y] = [I]$, $[Y]^T[K][Y] = [diag(\omega^2)]$ have been employed. Though in modal coordinates, the equations are still coupled as the matrix $[Y]^T[G][Y]$ is non-diagonal.

Further simplification of the equations can be carried out by neglecting the dynamic contributions of higher modes of the fork [10]. We partition the modes and modal displacements into two sets so that the displacement vector can be presented as $\{U\} = [Y_1]\{z_1\} + [Y_2]\{z_2\}$, and truncate the terms corresponding to inertial and damping forces of the second modal set. Finally the following equation in terms of modal displacements of only first set is obtained as

$$\{\ddot{z}_1\}+\left(\left[diag(\mu_1)\right]+2\Omega[Y_1]^T[G][Y_1]\right)\{\dot{z}_1\}+\left[diag(\omega_1^2)\right]\{z_1\}=$$
$$-2\Omega[Y_1]^T[G][S_k]\{\dot{F}\}+[Y_1]^T\{F\}, \quad (8)$$

where $[S_k]=[K]^{-1}-[Y_1]\left[diag(1/\omega_1^2)\right][Y_1]^T$ is the higher modes compliance matrix.

The model consisting of piezoelectric shell elements has been programmed in ANSYS and FOTRAN. The reason of application several different programming environments was that the ANSYS program does not allow obtaining the harmonic vibration response of the rotating structure. Therefore the structural stiffness and mass matrices have been printed to files and taken into a FORTRAN program that was written to perform calculations by using the above-presented dynamically reduced model.

The accuracy of the solution depends upon the number of modes taken into consideration. We always may obtain the exact solution when solving equations with all modes participating with their dynamic contributions. If the solution is close to the exact one with taking into account the dynamic contributions of only few modes, the participating modes are decisive for the operation law of the angular rate meter.

Fig.6 presents the amplitude frequency characteristics (AFCH) of the characteristic points (A) and (B) on the input (driving) and output (pickup) tines of the tuning fork by taking first 4 modes, first 9 modes and the full dynamic model. The detailed analysis of the influence of modes by adding them one by one to the dynamic model leads to the conclusion that the 9x9 reduced dynamic model obtained by taking into account the dynamic contributions of only first 9 modes is accurate enough and can be used instead of the full 1326x1326 original model. It should be noticed that 8th and 9th dynamic modal contributions though excited far below the resonance are important for proper representation of the dynamic features of the system.

The comparison of the results obtained by using the reduced and full models demonstrates their good coincidence. The reduced model runs 50-100 times faster and saves computing time considerably when obtaining the amplitude- and phase-frequency characteristics that require multiple calculations of the forced harmonic response.

5. ANALYSIS OF THE DYNAMIC BEHAVIOUR

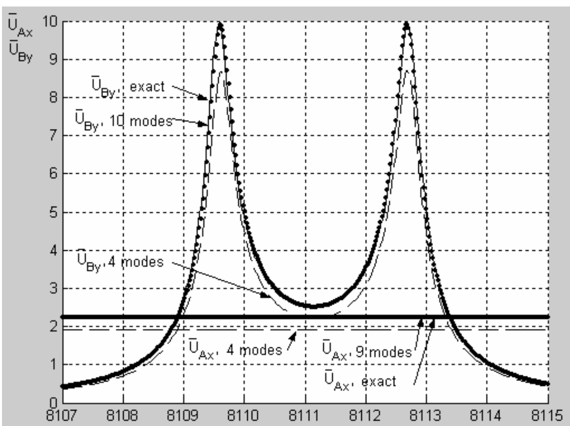


Fig. 6. Frequency response of the vibrating structure.
$\bar{U}$ = the dimensionless vibration amplitude

In order to perform its function as the sensitive element of the rate meter, the tuning fork is excited by means of the applied electric voltage over one half of the fork (input tines). The frequency of excitation of the fork is close to the natural frequencies of resonant out-of-plane modes 3 and 4 (Fig. 2), though the modes are not excited because of the in-plane action of electromechanical excitation forces. For in-plane vibration, the excitation frequency is far below the resonance, so they may be regarded as non-resonant.

While the frame is rotating, in-plane vibration of input tines excites both neighbouring modes 3 and 4. The vibration law of output tines depends upon the mutual position of values of natural frequencies f_3 and f_4 on the frequency axis.

If $f_3=f_4$, the out-of-plane vibration of output tines will not be excited because of eliminating each other contributions of modes 3 and 4. However, the resonant out-of-plane vibration will be excited in input tines. This mode of operation is based on a very narrow allowable frequency range in order to keep the output vibrations essentially on the peak of AFCH curve. Similar dynamic properties can be obtained in case when the fork is excited over all the surface of the fork. In a rotating frame, the in-plane vibration would excite only mode 3 of out-of-plane vibration;

The optimum separation of natural frequencies f_3 and f_4 by selecting proper geometrical parameters allows obtaining out-of-plane vibration of output tines, the AFCH of which has a plateau or a local minimum on its top. The tolerance of the excitation frequency is allowable in wider range, Fig. 7a, where $\bar{U}_A, \bar{U}_B$ and ϕ_A, ϕ_B denote the amplitudes and phase angles of input and output tines' vibration.

If f_3 is very close but not equal to f_4 , the out-of-plane vibration of output tines will be excited by rotation of the frame. The phase angle of vibration of output tines will depend upon the mutual positions of natural frequencies of symmetrical (f_3) and anti-symmetrical (f_4) out-of-plane modes. As the frequency values of them interchange (i.e., f_3 becomes greater as f_4), the phase angle of output tines vibration changes through value π , Fig. 7b.

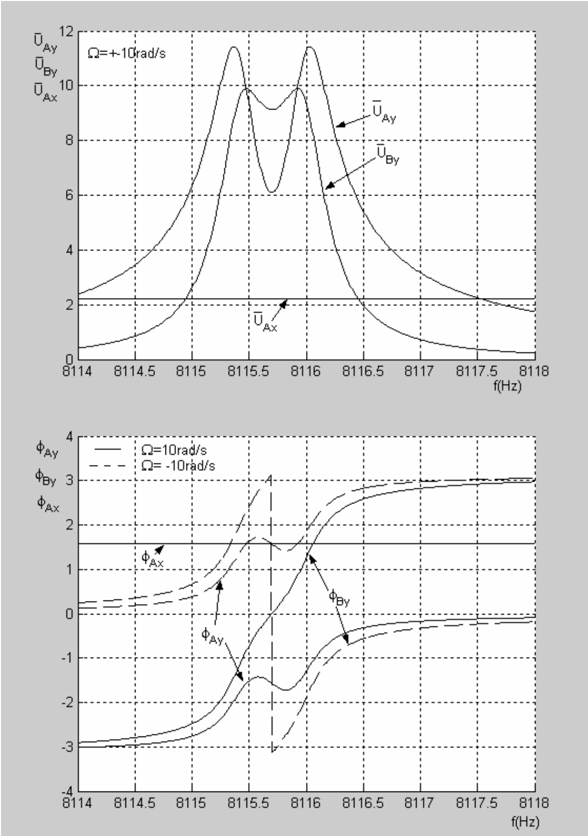
This leads to the same effect as the change of the sign of angular of rotation of the frame and can cause a misinterpretation of the direction of rotation.

By selecting a rational difference between natural frequencies of the out-of-plane vibration of the fork the peaks of AFCH may be made wider and less sharp in order to decrease the possibility of misinterpretation of the measured angular velocity direction on the base of the phase angle of vibration of the output tines.

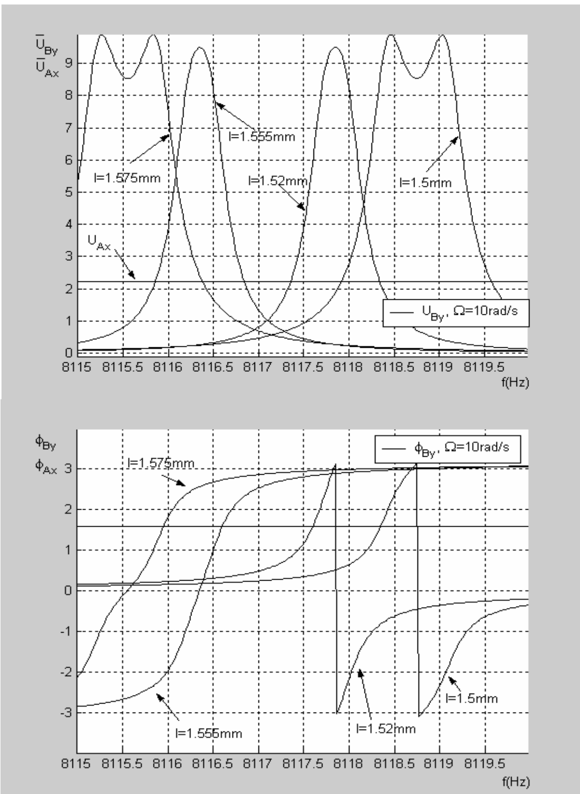
6. CONCLUSION

The dynamic behaviour of the balanced H-shape oscillator of the piezoelectric rate sensor has been analysed by means of a finite element model including the gyroscopic effects upon vibrating structure in a rotating frame.

Structural vibration problems present a major hazard and design limitation for interpretation of the system properties and their impact on system performance.



a) f_3=8115,3 Hz; f_4=8116 Hz.



b) frequencies f_3.and f_4. for each curve correspond to the values of length l indicated in Fig. 5

Fig. 7. Effects of frequencies f_3 and f_4 separation on the system frequency response

A novel computation model and software tools have been developed that allow both accurate simulation of the dynamic behaviour of the sensor and appreciation of the parametric sensitivity.

It has been demonstrated that investigation of the dynamic behaviour of an angular rate sensor by employing the 3D FE model facilitates considerably the understanding of the operation specifics of the sensor and allows the quantitative evaluation of the input-output relationship.

The modelling approach worked out can be also applied to the education of advanced level specialists in the field of measurement and instrumentation in the design-oriented framework.

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Professor Saulius Kausinis, Department of Engineering Mechanics, Kaunas University of Technology, K. Donelaicio Str. 73, LT-3006,
Tel./Fax.: +370 37 323855
E-mail: Saulius.Kausinis@mf.ktu.lt

Professor Rimantas Barauskas, Department of System Analysis, Kaunas University of Technology, K. Donelaicio Str. 73, LT-3006, Kaunas, Lithuania
Tel.: +370 37 300357
Fax: +370-37-323876
E-mail: *Rimantas.Barauskas@mf.ktu.lt*