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A REFLECTION MODEL FOR ULTRASONIC TOF MEASUREMENTS FOR INDOOR APPLICATIONS

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Abstract − An acoustic position measurement system is based on a time-of-flight (ToF) measurement of an acoustic tone burst. In indoor situations, the accuracy of the measurement is often affected by echoes generated by nearby objects around the measurement set-up. The problem how to model these echoes is addressed in the paper. Such a model is useful for the development of ToF measurement methods that can deal effectively with the echoes. The proposed model is a zero mean, non-stationary stochastic process statistically defined by its autocovariance function. We are able to specify this function by means of a few parameters. Experiments performed validate the proposed model. We show that the proposed autocovariance function and the one deduced from experimental data can approximately be brought in accordance.

Keywords: ultrasonic position measurement, reflection model, time-of-flight.

1. INTRODUCTION

An acoustic position measurement system consists of a transmitter located at the position that must be measured, and a number of acoustic beacons located at reference positions. The distances between the transmitter and the beacons are determined by measuring the 'Time-of-Flight' (ToF) of a tone burst, sent by the transmitter and received by the beacons. The quality of the position estimation is directly related to the quality of the ToF measurements. The accuracy of these measurements depends on whether the shapes of the received waveforms are predictable or not. In open air, the shape mainly depends on the characteristics of the tone burst, the transmitter and the receiver of the system. In that case the shape of the observed waveform (the so-called direct response) is well predictable and the ToF can be measured accurately. However, in indoor situations, there are many reflective objects near the transmitter and receiver, or near the acoustic path. The echoes from these objects may interfere with the direct response. As a result, the observed waveform will be hardly predictable in a deterministic sense.

Fig. 1 shows observed waveforms in the measurement system. The left waveform in Fig. 1a is acquired in a room without reflective objects in the vicinity of the measurement set-up. Apart from sensor noise and acoustic background noise there are no disturbances in this waveform. Hence, it can be regarded as the direct response of the acoustic system. The right waveform in Fig. 1a is obtained with the transmitter and the receiver positioned about 28 cm above the floor. The distance between transmitter and receiver is 200 cm. With such a set-up one expects to observe the direct response together with a strong second response due to a reflection against the floor and delayed reflections from other objects. It can be calculated that the second response will occur here about 0.26 ms after the direct response. Since the duration of the direct response is on the order of 1 ms, a heavy overlap between the direct response and the second response occurs. The influence of the reflections on the ToF measurement is quite large. Fig. 1b shows the two waveforms after passing a matched filter (correlator). The ToF is measured by detecting the time point at which these signals obtain the maximum value. The reflections in the

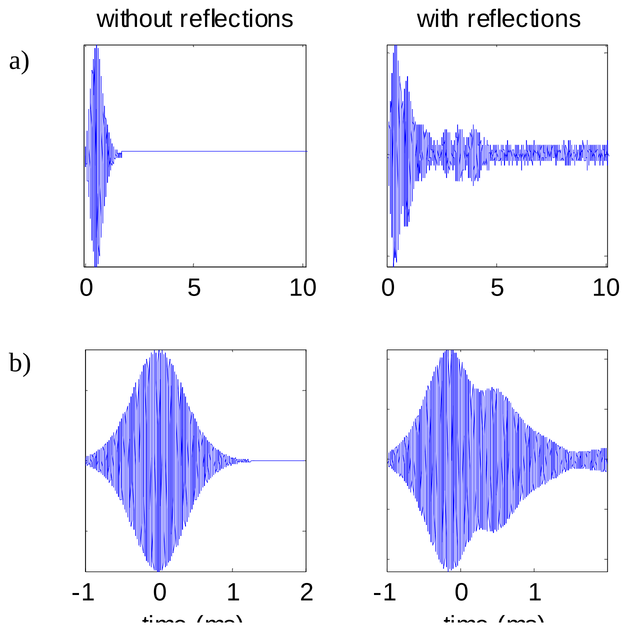

Fig. 1 Waveforms measured with an acoustic distance measurement system; a) Received waveforms, b) Responses of a matched filter.

second waveform introduce an error of 0.15 ms, equivalent to 4.5 cm.

ToF measurement methods range from simple but less accurate ones to the more involved methods. For instance, adaptive threshold methods ([1], [2], [3]), curve fitting ([2], [3], [4]) and correlation ([2], [5]) provide rather accurate measurements using a predicted echo signal. However, all these methods rely on the assumption that the observed waveform is predictable. When the echo shows a complex pattern, accurate distance measurements become difficult and a more advanced method needs to be explored.

Such an advanced method is achieved by setting up the problem within the framework of estimation theory. In its simplest form such an approach leads to a cross correlation of the actually observed signal with a template signal and the determination of the moment of maximal correlation. In fact, the template signal is the signal that is forecasted by some model. Some models are based on an empirical expression for the arriving waveform [2], [3], [4]:

$$w(t) = a\,(t-\tau)^m \exp\left(-(t-\tau)/T\right)\cos(2\pi\, ft+\varphi) + n(t) \qquad (1)$$

where τ is the ToF. a and φ are the amplitude and the phase. m is a constant that is usually chosen between 1 and 3. f is the frequency of the carrier, and T is a time constant that determines at what rate the waveform is damped. $n(t)$ represents the noise and the modelling errors. Other models apply a variant of (1) to model the envelope of the carrier as a Gaussian process [7]. These models are empirical and not physically based since features of the measurement set-up are not incorporated.

Some authors ([6], [8], [9]) model the phenomenon of multiple echoes as follows:

$$w(t) = \sum_{i=1}^{N} a_i h(t-\tau_i) + n(t) \qquad (2)$$

where $h(\,)$ is the nominal response of a single echo, and τ_i are the ToF of the various reflections. However, no attempt has been made to use this model for the construction of a ToF estimator that can cope with multiple echoes.

In this paper, we introduce a new model that can handle waveforms that include such a multiple set of echoes. This model comprises two random processes: a Poisson impulse process that describes the occurrence of the arrival of echoes per unit of time, and a Gaussian process that characterises the set of attenuation factors associated with the echoes. The model is defined by four parameters describing the two processes. Indeed, our model assumes that the echo pattern is a non-stationary random process characterised by its autocovariance function. In fact, because the model compromises between accuracy and simplicity, such a model can provide the base of a new ToF estimator. On one hand, it describes the phenomena adequately and precisely (in a statistical sense). On the other hand, the model is mathematically tractable allowing for the development of a new ToF estimator. This paper solely addresses the problem of statistical modelling. The design of the new estimator is addressed in an accompanying paper [10].

Section 2 introduces the stochastic reflection model. Section 3 deals with the experimentation and the validation of the model. Section 4 finalises the paper with a conclusion.

2. THE STOCHASTIC REFLECTION MODEL

The purpose of this section is to develop a stochastic model that describes the arrival of a tone burst and its various echoes at a receiver of an acoustic measurement system. The measurement set-up is as follows. An electric tone burst $u(t)$ induced by a signal generator is fed to a transmitter and causes an acoustic burst. The objects and walls in the room, where the acoustic measurement system stands, reflect the burst in many directions giving rise to a multitude of reflections $r(t)$. Together with the direct response, these reflections generate a signal $z(t)$ at the output of the receiver. We model the reflections $r(t)$ as a stochastic process that is composed of a (possibly) infinite, countable number of echoes occurring after the transmission of the tone burst.

2.1. Modelling the arrival of echoes as a Poisson Process

Due to the interaction of the acoustic signal with the walls as described above, we can assume the presence of an avalanche effect of echoed tone bursts that accumulate in the received signal. When no objects block the direct path between transmitter and receiver, the first received signal will always be the direct one. It has the shortest path from transmitter to receiver. The part of the wave that missed the receiver travels through the room, and will eventually hit an object, e.g. a wall, the floor, etc. The wave partly reflects back into air and continues its way through the room. The echoes entering the receiver shortly after the direct response have probably reflected only once because of their relative short path lengths. The expectation is that the number of these ‘short’ reflection paths is relatively small. Accordingly, the number of the echoes associated with these primary reflections is also relatively small. However, shortly after the arrival of the first horde of these primary reflections, a second horde will come caused by secondary reflections, i.e. echoes that interacted twice. These echoes need also more time to reach the receiver. The assumption is made that as time proceeds the number of echoes entering the receiver will grow and that this process goes on infinitely. Every reflection path has its own attenuation that can be positive or negative (due to phase reversal). We assume that the attenuations are zero-mean random variables whose variances decay to zero as time proceeds.

Furthermore, we assume that only the magnitude and the delay of a reflected burst is affected, but not its shape. The size of the transducers, together with their alignment on the acoustic path, has a minor influence on the shape of the observed waveform. The type of reflecting object (e.g. wall, corner or edge) neither affects the shape [7]. The attenuation is constant within the bandwidth of the waveform. The medium itself influences the ToF but the attenuation is

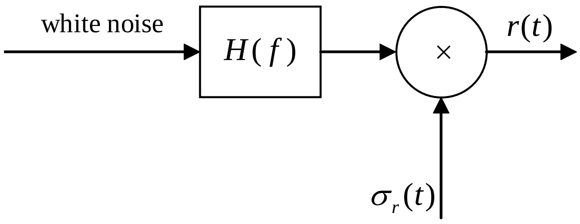

Fig. 2 The stochastic reflection model

smooth over the frequency [9]. Consequently, it does not affect the shape neither.

Therefore under these assumptions, we associate the arrival of echoes with varying attenuation with a *non-uniform*, *generalised* Poisson impulse process [11]. The term "non-uniform" refers to the situation of having a time variant density (= mean number of echoes per unit time). The term "generalised" refers to having events (=arrivals of an echo) with differently weighted intensities (attenuation factors).

Suppose that at time $t=0$ a tone burst is inputted at the transmitter. At the output of the receiver, the direct response to the burst is $a\,h(t-\tau_{TOF})$. Here, τ_{TOF} is the time of flight of the response and a is the amplitude of the signal. $h(t)$ is the nominal response of the acoustic system to the particular input signal. By definition, the nominal response has the property that:

$$\|h(t)\| \overset{def}{=} \sqrt{\int_{-\infty}^{\infty} h^2(t)dt} = 1 \tag{3}$$

Next, we consider the sequence τ_k with $k=1,2,3,\cdots$ of time points of arrival of the first echo, the second echo, and so on. We regard this sequence as a Poisson impulse process with non-uniform density $\lambda(t)$. Since no echo can occur before the direct response, $\lambda(t)$ must be zero before the arrival of the direct response. Because of the avalanche effect, it must be a monotonically increasing function after the arrival of this response. We also assume that the acoustic system is linear and time invariant. Therefore, the output signal of the receiver is:

$$z(t) = a\left(h(t-\tau_{TOF}) + r(t-\tau_{TOF})\right) + n(t) \tag{4}$$

where $n(t)$ is the sensor noise and $r(t)$ represents the reflections:

$$r(t) = \sum_{k=1}^{\infty} d_k(\tau_k)h(t-\tau_k) \tag{5}$$

The random sequence $d_k(\tau_k)$ is the set of attenuation factors associated with the echoes. Since, as pointed out before, the echoes are weaker as time proceeds, we model their variances, $\sigma_d^2(\tau)$ as non-stationary, i.e. a monotonically decreasing function of time.

2.2. The autocovariance function of the Poisson process

Because $d_k(\tau_k)$ is assumed to be zero mean, the stochastic process $r(t)$ is also zero mean. We thus derive the autocovariance function of $r(t)$ as follows (see also[11], [12], [13]):

$$\begin{aligned} R_{rr}(t,u) &= \mathrm{E}\left[r(t)r(u)\right] \\ &= \mathrm{E}\left[\sum_{k=1}^{\infty}\sum_{m=1}^{\infty} d_k(\tau_k)d_m(\tau_m)h(t-\tau_k)h(u-\tau_m)\right] = \\ &= \mathrm{E}\left[\sum_{k=1}^{\infty} d_k^2(\tau_k)h(t-\tau_k)h(u-\tau_k)\right] \\ &= \int_{\tau=-\infty}^{\infty} \sigma_d^2(\tau)\lambda(\tau)h(t-\tau)h(u-\tau)d\tau \\ &\approx \sigma_d^2(t)\lambda(t)\int_{\tau=-\infty}^{\infty} h(t-\tau)h(u-\tau)ds \end{aligned} \tag{6}$$

The approximation is only valid if $h(t)$ has a short duration relative to $\sigma_d^2(t)\lambda(t)$. For the moment, the assumption is that the approximation is valid. The variance of $r(t)$ is $R_{rr}(t,t)$. Due to (3), the approximation in (6) shows that $R_{rr}(t,t) \approx \sigma_d^2(t)\lambda(t)$. Therefore, the standard deviation is about:

$$\sigma_r(t) \overset{def}{=} \sigma_d(t)\sqrt{\lambda(t)} \tag{7}$$

In section 3.4, the validity of the approximation will be discussed further.

Fig. 2 shows another view of the (approximate) model. Here, we have white Gaussian noise that passes a band-pass filter whose impulse response is $h(t)$. The output of the filter is stationary noise with a power spectrum $|H(f)|^2$ where $H(f) = FT\{h(t)\}$. Next, the coloured noise is modulated by $\sigma_r(t) = \sigma_d(t)\sqrt{\lambda(t)}$. The resulting waveform has an autocovariance function given by:

$$R_{rr}(t,u) = \sigma_r(t)\sigma_r(u)\int_{\tau=-\infty}^{\infty} h(t-\tau)h(u-\tau)ds \tag{8}$$

This function takes the same form as the approximation in (6). The advantage of (8) over (6) is that it preserves the necessary symmetry property $R_{rr}(t,u) = R_{rr}(u,t)$, even if the requirement for the approximation is not fully met.

2.3. A parametric model

The next step is to transform the model for the autocovariance function into an empirical, parametric model. We formulate our model such that it only meets the requirement qualitatively. It is not tractable to base the development of such a model on the physics of the problem because most room environments will be too complex. The function $h(t)$ does not need much further development since it depends on the selected tone burst together with an appropriate model of the transfer functions of the transmitter and the receiver. Therefore, this section focuses on (7). The goal now is to find a suitable model described by a few parameters that can be evaluated empirically by means of an estimation procedure. In other words, we are looking for a function $\hat{\sigma}_r(t,\mathbf{p})$ that can empirically be fitted to $\sigma_r(t)$. The parametric model that we consider particularly is defined as

$$\hat{\sigma}_r(t,\mathbf{p}) = \begin{cases} 0 & t \le 0 \\ A \cdot \left(\frac{t}{T}\right)^b \cdot \exp\left(-\frac{b}{c}\left(\left(\frac{t}{T}\right)^c - 1\right)\right) & t > 0 \end{cases} \quad (9)$$

with $\mathbf{p} = [A \quad b \quad c \quad T]$ a parameter vector. The rationality behind (9) is as follows. We have already established that no echo can occur before $t = 0$. Hence $\sigma_r(t) = 0$ if $t \le 0$. The factor t^b describes the avalanche effect of echoes. The parameter $p_2 = b$ controls the rate of growth of this effect. The factor $\exp(-(t/T)^c)$ describes the decay of echoes controlled by the parameters $p_3 = c$ and a time constant $p_4 = T$. The function in (9) reaches its maximum at $t = T$. The maximum value is $\hat{\sigma}_r(T,\mathbf{p}) = A$. $p_1 = A$ is a measure for the overall intensity of the reflections

3. EXPERIMENTS

Experiments have been carried out to validate the proposed reflection model. We recorded 150 waveforms at different locations and under various conditions. These records have been analysed collectively to compare their statistical properties with those predicted by the proposed model.

3.1. Experimental set-up

Using an acoustic measurement system, data records have been acquired under a number of conditions in two rooms, a laboratory room and a classroom. The acoustic system used two air ultrasonic ceramic transducers, a transmitter (400ST100) and a receiver (400SR100), mounted on pedals in a face-to-face direction. A waveform generator (HP33120A) applied a 40 *kHz* sinusoidal tone burst consisting of twenty cycles to the transmitter. A digital oscilloscope (TDS3014) acquired the received waveform using a sampling period Δ of 2 μs. The bandwidth (-6 *dB*) of the transmitter and the receiver is 2.5 and 3.0 *kHz*, respectively. The centre frequency is 40.0 ± 1.0 *kHz*. The conditions of the rooms were not special. The measurement set-up was located in the vicinity of the usual furnishings, i.e. near walls, tables, chairs, cupboards, etc. The number of people in the vicinity of the set-up varied from record to record. We also altered a number of objects located near the transducers. Other factors that were varied from record to record are height above the desktop (or floor), the distance between transducers and the location of the system within the room. Table 1 gives an overview of the selected factors. In addition to the 150 records, also a special record was acquired in an anechoic room. This waveform, shown in Fig. 1, is used as a reference waveform from which the nominal response $h(t)$ can be derived.

3.2. Preprocessing of the records

Before the real analysis, we pre-processed all records individually. The ToFs of the records can be found because

we know the distance between the transmitter and the receiver. In addition, from the geometry (height and distance) of a record we can deduce a minimal delay of time of the first echo with respect to the direct response.

Table 1 Factors of the experiments

Factors	**Variations**		
Room	Laboratory	Class	
Height (*m*)	H_1 = 0.2	H_2 = 0.3	H_3 = 0.4
Distance (*m*)	D_1 = 1.0	D_2 = 2.0	D_3 = 3.0
Location	L_1	L_2	L_3

Although this delay can be quite small (e.g. 100 μs), it is just sufficient to estimate the ToF because we already have a clue where to detect it. Using this minimal delay we can also estimate a: matching $a\,h(t - \tau_{TOF})$ to the observed waveform within the given interval. Once the direct response is identified, the reflections (with noise included) are found. We also estimated the variances of the noise from the (removed) header of the waveform; they are denoted by $\hat{\sigma}^2_{n,m}$. The result of the process is a set of $M = 150$ records:

$$y_m(k\Delta) = r_m(k\Delta) + \frac{n_m(k\Delta)}{a_m} \quad (10)$$

where $m = 1,\cdots,M$ enumerates the various records, k is a discrete time index and Δ is the sampling period.

3.3. Statistical analysis of the records

On adoption of the approximate model stated in (6) the factor $\sigma_r(k\Delta)$ is the standard deviation of the ensemble of the reflections. Its square, $\sigma_r^2(k\Delta)$, can be estimated from the population variance $S^2(k\Delta)$ of the set of records:

$$S^2(k\Delta) = \frac{1}{M}\sum_{m=1}^{M} y_m^2(k\Delta) \quad (11)$$

Our proposed model predicts a variance of:

$$\hat{\sigma}_r^2(k\Delta,\mathbf{p}) + \sigma^2_{n,ens} \quad (12)$$

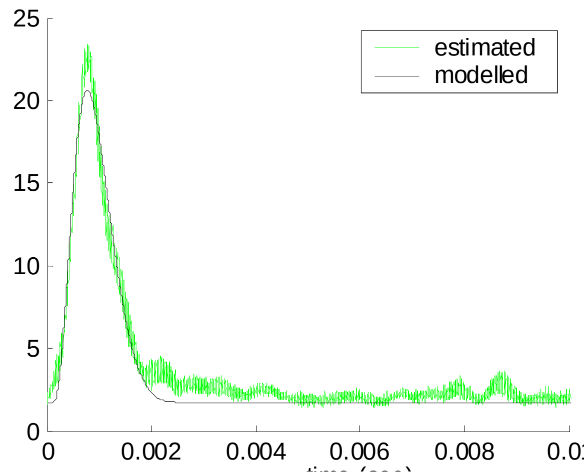

Fig. 3 The modelled and the directly estimated standard deviation of the ensemble.

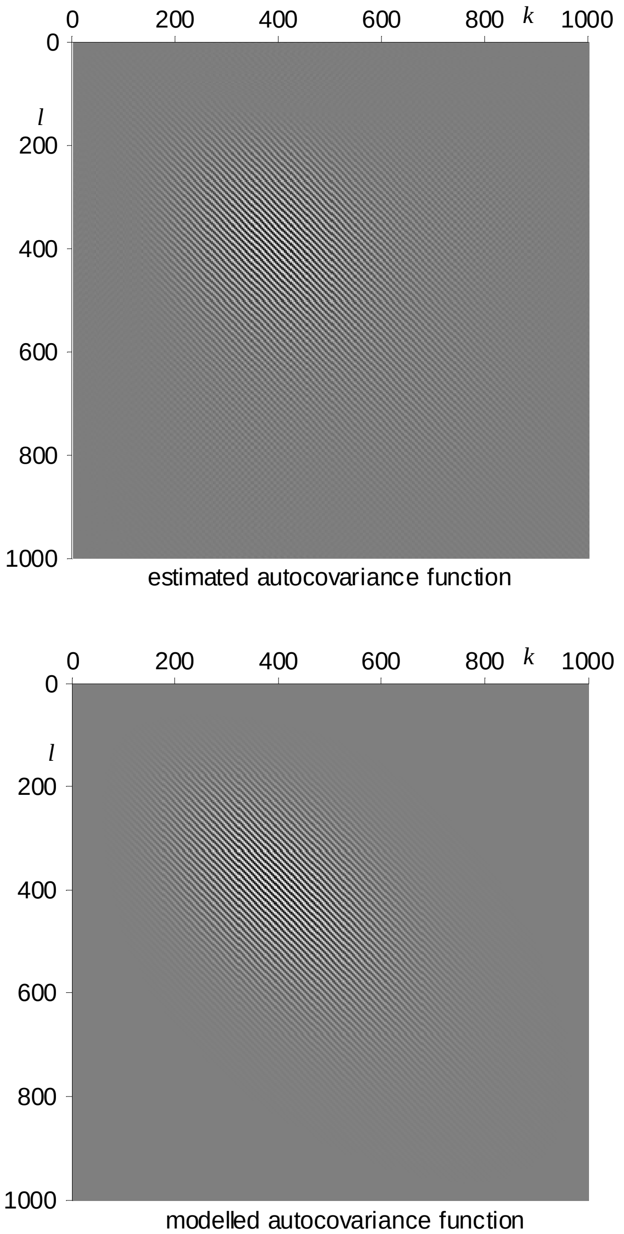

Fig. 4 The modelled and the directly estimated autocovariance function.

Here, $\hat{\sigma}^2_{n,ens}$ is the variance of the noise estimated by:

$$\hat{\sigma}^2_{n,ens} = \frac{1}{M}\sum_{m=1}^{M}\left(\frac{\hat{\sigma}_{n,m}}{a_m}\right)^2 \qquad (13)$$

A least square error fitting procedure was used to find the parameter $\mathbf{p}$ such that the modelled standard deviation obtained from (12) matches the observed population deviation $S(k\Delta)$. The modelled standard deviation obtained in this way is shown in Fig. 3 together with the observed deviation. The best fit was obtained with $\mathbf{p} = [20.5 \quad 4.3 \quad 1.0 \quad 0.78ms]$.

The next step is to compare the modelled autocovariance function $R_{rr}(k\Delta, \ell\Delta)$, (8), with the one estimated from the records. An estimate is obtained by the ensemble average, i.e.:

$$\hat{R}_{yy}(k\Delta, \ell\Delta) = \frac{1}{M}\sum_{m=1}^{M} y_m(k\Delta) y_m(\ell\Delta) \qquad (14)$$

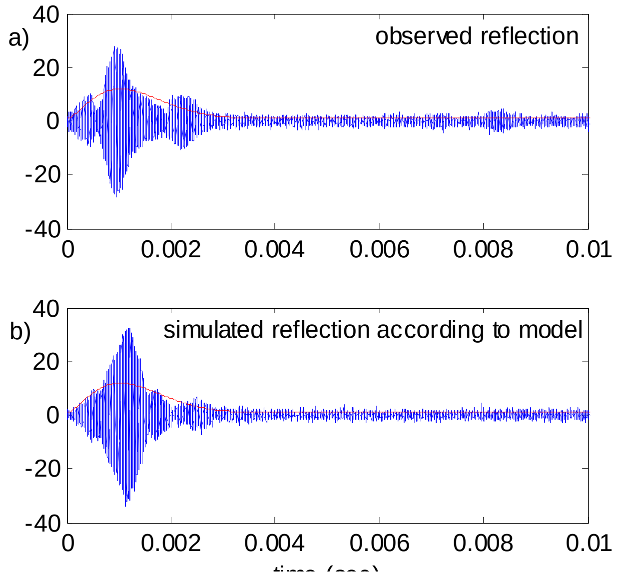

Fig. 5 A record of a reflected waveform obtained directly from an observed waveform, and a simulated reflection obtained by the model depicted in Fig 2.

Fig. 4 shows estimated autocovariance function obtained directly from (14) and the modelled one obtained from the model with the parameters that corresponds with the fit in Fig. 3. A comparison between the two obtained functions does not reveal contradictions except when the autocovariance becomes small. An explanation on this difference will be given soon. Fig. 5 shows two waveforms. Fig5a is one of the 150 observed reflections. Fig5b shows an artificial waveform. It is generated such that it has the same autocovariance function as the one derived from the observations. The time-variant standard deviations of the two waveforms are also depicted.

3.4. *Evaluation of the results*

The first concern is the question whether the modelled autocovariance function agrees with the one estimated from the population of 150 records. Fig. 3 shows that the modelled standard deviation can be fitted well to the estimated one. The uncertainty of the estimated autocovariance can be expressed as a (fixed) multiple, say 3 or 4, of the standard deviation of the estimate:

$$\sigma_{\hat{R}}(t,u) = \sqrt{\frac{1}{N}\left(R_{rr}(t,t)R_{rr}(u,u) + R^2_{rr}(t,u)\right)} \qquad (15)$$

We calculated this uncertainty and compared it with the difference between the estimated and the modelled autocovariance, both shown in Fig 4. It appeared that for the middle range values of t and u the difference is within the uncertainty interval. However, when the envelope of the autocovariance function becomes small, the difference is too large compared with the uncertainty. At those values of t and u, the model fails. Fortunately, this happens only for small values of the autocovariance, and although the relative modelling error is too large, the absolute error is still small. In order to explain this deviancy we have to reconsider our assumption in section 2.2 to obtain the approximation in (6). Fig. 1 and Fig. 3 reveal that the duration of the direct response $h(t)$ and the one of the standard deviation $\sigma_r(t)$ is on the same order, i.e. about $1\,ms$. The assumption is not fully satisfied, and a modelling error results. This is the price that has to be paid for using the much easier approximation instead of the much more involved exact expression.

The model proposed in this paper is an 'overall' model. It describes the statistics of the ensemble. As said before, the stochastic process that we propose is non-stationary. But besides that, the process is also non-ergodic. The parameters of the process cannot be estimated from a single observed waveform. Experimental factors such as 'distance', 'height', 'room', and 'location in a room' have their impacts on the reflected waveform.

4. CONCLUSION

For indoor applications, the reflections of an acoustic tone burst measured by an ultrasonic transducer can be modelled as a non-stationary stochastic process. The physics underlying the process consists of the arrival of a number of echoes each with its own delay and own intensity. An accurate model of this process is difficult because of the many unknown factors involved. However, an approximate model fully statistically defined by its autocovariance function becomes feasible by regarding the echoes as a generalised Poisson process. This model is described by a few parameters.

With 150 records measured under various room conditions, we validate the proposed model by empirically assessing the parameters and by comparing the modelled autocovariance functions with the estimated one. The experiments indicate that the model agrees well with the experimental data except for a small deviancy (for which an explanation has been given).

The further development of a ToF estimator, based on the stochastic reflection model, is described in an accompanying paper [10].

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