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MEASUREMENT QUALITY CHARACTERISTICS IN METROLOGY: SYSTEMIC APPROACH

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Abstract - Problem of measurement quality analysis is stated as a systemic one. Measurement procedure is investigated as a complex system of various elements, including real objects and mathematical models. Several kinds of systemic decomposition are performed.

Quality of measurement is investigated in main aspects; both quality of the real objects and quality of the mathematical models are studied on the unified methodological basis.

Traditional accuracy analysis of measurement is complemented by the investigation of the main groups of quality characteristics for measurement procedure. In particular, reliability and complexity characteristics are studied.

Complete mathematical model of measurement procedure is formulated and studied. It relies on the models of the physical objects, as well as on the models of transformations, including the algorithms and software for data processing. Quality characteristics of the algorithms and software are estimated according to their role in measurement. The model quality characteristics are also investigated, including adequacy characteristics.

Keywords: Measurement, quality, model

1. INTRODUCTION

Problem of measurement quality analysis can be stated in accordance with several levels of generalization for the measurement as a basic notion.

Foremost these levels include the following ones: subject, formal, and operating levels. Accordingly, there are four types of the surrounding, such as subject, model, algorithmic, and computing (software) environment.

The statement of the measurement quality problem requires for investigation of the complete measurement procedure. This procedure is regarded as a complex system of various elements, being studied in a number of aspects. It is generally in agreement with the contemporary trends in measurement theory [1-2].

Generally, the elements of measurement can be divided into two main groups. The first one consists of the real objects, such as physical bodies and measuring instruments, and also of the bodies' properties, operations and processes. Mathematical models for the first group elements form the second group.

The quality of the measurement as a whole is determined by the quality of both real and model elements given above, and also by their interrelations.

Traditionally, analysis of measurement quality is restricted in two aspects. Firstly, the quality characteristics are only studied for real object. In particular, specific metrological procedures are developed for evaluation of measuring instruments quality. However, it does not settle the measurement quality problem completely. Quality of the model elements is as well important for the measurements.

The second restriction is due to the limited set of the quality characteristics under investigation. Usually, only the accuracy characteristics of measurement are evaluated. Not detracting the significance of this kind of characteristics, it is also important to study other groups of characteristics, which are as well significant. For instance, reliability and complexity characteristics must be investigated.

In this report an attempt is made to expand the measurement quality analysis in the directions mentioned above.

2. GENERAL METHODOLOGY

The quality evaluation problems stated above are topical for measurement theory and applications. On the one hand, the quality analysis is of prime importance for measurements. On the other hand, general methodology for the error evaluation has been created in metrology, so it can be expanded to create a basis for the quality analysis and evaluation in much wider area.

The peculiarity of the measurement quality study is due to the multi-aspect nature of the measurement procedure and result. So a number of aspects or properties must be taken into consideration. It may be carried out on the basis of the systemic decomposition of the measurement procedure by the set of aspects. In particular, the measurement quality study produces an increasing set of systemic models, which are mutually coordinated. The procedure of the model construction may be described as adaptive process [3, 4].

So, measurement quality evaluation is a systemic problem, and it is to be investigated within systemic approach [5].

The traditional representation of the measurement result is the pair of values: $\{\hat{A}, \delta A\}$; so the physical quantity estimate $\hat{A}$ is accompanied by the accuracy (error) characteristic δA. Such a representation stresses an importance of the accuracy characteristic. But it can also give a wrong impression of the completeness of the result representation; so it conceals the problem of the model adequacy and measurement quality. The reason is that the estimates are obtained just within the assumed model of measurement procedure; so the measurand A, measurement result Â, and accuracy characteristic δA are all valid only within this model. Thus the quality of measurement model is of key importance.

The traditional measurement accuracy analysis can be taken as a starting point for fulfilling the complete quality analysis of measurement. In this report an expansion is carrying out in two main directions. First, the quality of the mathematical models is studied as well as the quality of the real objects. Second, the main groups of quality characteristics, which are defined in general quality theory [6], are applied for the measurement procedure. These two directions are closely connected.

Thus, the following aspects of the measurements quality evaluation are presented below on a base of the general systemic approach:

- generalized error analysis;
- principles of model quality analysis.

3. MEASUREMENT ERROR DECOMPOSITION

In metrology, traditional approach to measurement analysis is based on the error decomposition and investigation of error components [1, 7, 8]. When passing to the general quality analysis, it is necessary to carry out the following tasks:

a) reveal the main factors, causing the errors, and single out the corresponding error components;

b) investigate the error models as functions or stochastic processes in time domain;

c) define additional quality characteristics of measurement, in particular, reflecting the time-domain properties of errors.

The error decompositions can be derived in several aspects and realized on various levels. Firstly, it is necessary to investigate the inadequacy error, which is due to the limitations of the adopted measurement model. So the total error of the measurement result can be represented as follows:

$$\delta Q = \delta_c Q + \delta_a Q , \qquad (1)$$

where $\delta_c Q = \delta \{Q \mid \Omega_0\}$ - the component, obtained under assumption of the measurement model Ω_0;
$\delta_a Q$ - the inadequacy error, due to the deviation of the real measurand from the value, which is determined by the assumed model Ω_0 [4]

These two error components are determined and evaluated in different ways. First, the “inside” error $\delta_c Q$ is well defined within the limits of the assumed model Ω_0. The evaluation of the measurand Q and the error $\delta_c Q$ could be rather complicated, but these are well defined and clearly stated problems providing model Ω_0.

The error $\delta_c Q$ may be further decomposed as follows:

$$\delta_c Q = \delta_m Q + \delta_t Q \qquad (2)$$

where $\delta_m Q = \delta_m \{Q \mid \Omega_0\}$ – methodical error, due to properties of method of measurement;
$\delta_t Q = \delta_t \{Q \mid \Omega_0\}$ - transformed error, due to properties of the data processing algorithm and the model of data under processing.

These components are formed within the assumed model Ω_0, and they are evaluated within the procedure of the data processing algorithm certification [9].

But that does not settle the matter of error evaluation completely. The non-adequacy component $\delta_a Q$ is not so clearly defined and evaluated. It could be evaluated only by extending the initial model. So the problem of non-adequacy error includes the formulation of the proper extended model and evaluation of discrepancies between measurement results, obtained within initial model Ω_0 and the extended one [4].

The error decomposition mentioned above could be further specified in some aspects. The most complete and detailed decomposition of the total measurement error can be derived and analyzed when the measurement procedure is considered in time domain; so the measurand A is represented as the transformation of the input signal:

$$A = \Phi_0 \{B_0[X(t)]\}, \qquad (3)$$

where B_0 - ideal operator of measuring instrument,
Φ_0 - functional transformation of the signal,
$X(t) = (X_1(t), \ldots, X_n(t))$ - ideal input signal.

But in realization of measurement procedure we only obtain approximate (non-ideal) operator $\widetilde{B}$, and real transformation $\widetilde{\Phi}$; moreover, the real input signal $x(t)$ is corrupted with noise. So the measurement result $\tilde{A}$ is represented as follows

$$\tilde{A} = \widetilde{\Phi} \{ \widetilde{B} [x(t)] + \mu(t)\}. \qquad (4)$$

Therefore, the measurement error is decomposed into the following components:

$$\delta(t) = \widetilde{\Phi} \{ \widetilde{B} [x(t)] + \mu(t)\} - \Phi_0 \{B_0[X(t)]\} =$$
$$\Phi_0[\delta \widetilde{B} (x)] + \Phi_0[B_0(\delta x)] + \delta\Phi[\widetilde{B} (X)] + \widetilde{\Phi} [\mu(t)], \qquad (5)$$

where $\delta \widetilde{B} (x)$ and $\delta\Phi$ are the distortions of the operators, $\delta x(t)$ and $\mu(t)$ - distortions of the input and output signals of the measuring instrument.

The decomposition (5) forms a natural basis for describing the time-domain properties of errors and defining the corresponding quality characteristics of measurement.

First, it is a basis for unification all kinds of models, which are suitable for the description of the partial elements of measurement and for specifying the main stages of measurement. Second, some new quality characteristics can be defined, in particular, reflecting time-domain properties of errors and measurement elements.

Certainly, various kinds of models for the object under investigation could be constructed; that results in the corresponding types of the models for ideal and real input signals. Further, there are a number of the models for the measuring instruments, which could be expressed as linear or non-linear operators in relevant functional spaces. In particular, metrological characteristics of the measuring instruments form an essential part of the instrument model.

This way of reasoning provides a considerable extension of the quality characteristic set, as compared with traditional accuracy characteristics. But more essential expansion of this set may be based on the quality analyses of the mathematical models in measurements.

4. MODELS QUALITY ANALYSIS IN MEASUREMENT

4.1. Main kinds of quality analysis procedures

The mathematical models in measurements are of various nature; so three main groups are as follows:

1) models of physical objects or devices, including the model of the physical object under investigation, physical quantity and measurand, scale and unit of physical quantity, models of measuring instruments, so on;

2) models of information nature, including models of signals, metrological properties of measuring instruments, observation and measurement results, measurement errors (or uncertainties);

3) transformations and operations models, including initial equations of measurement, data processing algorithms and software.

The quality of the real and model objects must be studied on unified methodological basis. So various mathematical tools are to be investigated in quality aspect as the significant part of the measurement. In particular, quality analysis is significant for the algorithms and software that are the most important kinds of the data transformations models.

The practical employment of the models is undoubtedly accompanied with the problem of model adequacy or validity. So the important part of the model quality analysis is just the adequacy estimation. In particular, the inadequacy characteristics of the objects or devices models are to be included into the set of the quality characteristics of the measurements.

In this paper several kinds of the quality analysis procedures are investigated. First of all, they are as follows:

certification of the data processing algorithms [9],
certification of the corresponding software [10];
evaluation of the model characteristics [4].

These procedures are presented as similar ones within the framework of the systemic approach mentioned above. The similarity of the procedures is useful both for the realization of the procedures and for the integration of the quality analysis results.

There are two main groups of the model quality characteristics to be studied. The first group characteristics are intended for describing the degree of the model conformity with the object. Those are inadequacy characteristics for various kinds of models, such as statistical, functional, etc.

The second group of model characteristics describe the possible “limiting” accuracy, accessible within the limits of the model adopted. There is a similarity with the “physical limits” of measurements, due to the physical nature of the real objects and their interactions with measuring instruments. So this is an additional aspect of limiting potentialities of measurements, that is, model aspect.

4.2. Certification of the data processing algorithms

Certification of a data processing algorithm is considered as a procedure of algorithm characteristics estimation for selected typical models of initial data. The formal scheme of the certification is described in [9]. As a result, the values of the algorithm characteristics $\Pi_1,\ldots,\ \Pi_n$ are computed or estimated for typical data models: $u_1,\ldots,\ u_m$:

$$\pi\ (i, j) = \Pi_i\ (a,\ u_j\ ). \qquad (6)$$

The algorithms quality characteristics $\Pi_1,\ldots,\ \Pi_n$ are the norms or functionals, which are useful for describing the algorithms properties within a homogeneous group of algorithms $A =\{\ a\ \}$. The characteristics are used both for comparison of algorithms in this group and for estimation of the measurement errors (assuming data model).

These characteristics include three main groups:

a) Characteristics of precision, intended for result error component estimation (including traditional metrological ones, such as standard deviation or variance of random error; limit or confidence limit for error);

b) Characteristics of stability or reliability, defining the region of normal operation (including the tolerable fraction of initial data distortion, breakdown point and the bounds of data parameter region for normal operation of the algorithm);

c) Characteristics of complexity, specifying computing and temporal expenses for the algorithm realization.

The set of characteristics is not limited, but the characteristics, which may be useful for algorithms comparing and error estimating, may be added (according the aim of certification).

4.3. Certification of the data processing software

If there is need to certify the data processing software**,** or data processing program (DPP), as a separate unit, it is carried out according the same general scheme, as for the algorithms. It may be topical question, perhaps, for the general-purpose programs.

General metrological principles for solving DPP metrological certification problem could be formulated as follows [10]:

1) DPP as the element of a metrological system is to be certified as well as all other elements.

2) DPP as a certification object is analogous to measuring instrument or MP.

3) DPP is to be certified separately only if it is impossible to certify the metrological object, containing DPP, as a whole.

4) DPP is to be certified in a form of release product.

5) DPP shall be certified in the form ensuring integrity (inalterability) of its algorithm.

6) DPP shall be certified in the measuring-computational environment, which DPP is to operate in.

Practically, quality of the media-resident software is described by the corresponding algorithm characteristics with adding up the system performance and environment characteristics [9, 10]. There is undoubtedly a strong interrelation between these groups of characteristics. So the initial algorithm characteristics could be transformed as applying to programs, or they have an influence on the specific program characteristics.

4.4. Evaluation of the model characteristics

The analysis of the quality characteristics for the objects (devices) models is of prime importance for measurements. There are two groups of characteristics, as stated above. The inadequacy characteristics are analyzed for various kinds of models [3, 4], such as statistical, functional, etc.

The second group of the model characteristics, describing the "limiting" accuracy within the limits of the model, may be illustrated for an important special case. There is the output signal y presented as linear transformation B of the input signal x:

$$y = B\,x + \xi \; . \qquad (7)$$

The pair of operators {B, R} represents the data model, where R is the correlation operator of the noise ξ (assuming that $M\xi=0$).

Let the measurand Q be defined as a linear operator of input signal: $Q = L\,x$. Then the measurement result is also a linear transformation of the observed signal:

$$z = F\,y = F\,B\,x + F\,\xi \; . \qquad (8)$$

Thus limiting accuracy accessible under the model B is evaluated as

$$\pi_0\,(B) = \inf D\,(F, B) \qquad (9)$$

where "inf" is taken over all the unbiased operators F, which satisfy condition: $M\,\{z\} = F\,B\,x = Q = L\,x$.

The model quality characteristics introduced in this way emphasize the duality of models and algorithms within the data processing procedure. It allows comparing various models on the aspect of the limiting accuracy or effectiveness.

5. DUALITY OF MODELS AND ALGORITHMS

The general systemic approach can provide relevant formal tools for solving various problems and also give a new vision of the procedures interrelations.

In particular, data processing algorithm is just a particular case of the model. So the general approach to the model investigation is quite valid to the algorithms. At the same time, algorithm is just a "typical" kind of model, and a vast set of models admit algorithmic representation, which is as follows:

$$\Omega = \{\, X, Y, A, B, T, S, V, U \,\}, \qquad (10)$$

where X and Y are the sets of input and output signals;
A - set of constant parameters;
B - state space;
T - time parameter space;
$S: (X, A, T) \to B$ – operator of the state determination;
$V: (X, A, T, B) \to Y$ and $U: (X, A, T) \to Y$ – operators of the input – output transformations.

This is a clearly algorithmic representation. On the other hand, it is quite similar to the unified representation of the algorithms by Markov. In particular, data processing algorithm is determined by 7 structural elements:

$$\Omega = \{\, D_0, R_0, Y, f_0, (f_i), \rho, f^* \,\}, \qquad (11)$$

which are as follows:
D_0 - set of initial data (signals) under processing;
R_0 - set of the data (signals) processing results;
Y - set of the intermediate processing results;
f_0 - origination rule;
f_i - rules of immediate treatment;
ρ - termination rule (terminator);
f^* - result extraction rule.

Thus the data processing algorithm can be represented as a superposition of the mentioned mappings:

$$F = f^* * \Pi f_i * f_0 \; : \qquad D_0 \to R_0 \, . \qquad (12)$$

The mentioned elements define the algorithm completely.

These representations describe a basic systemic duality of the models and algorithms. This relation is interesting both for theory, and for applications, especially for the algorithms and software certification procedures.

In particular, the characteristics of algorithms are represented in symmetrical form (6). For the given algorithm the values of characteristics $\Pi_1,\ldots,\ \Pi_n$ are evaluated at the chosen data models $u_1,\ldots,\ u_m$. So it is possible to compare the properties of the given algorithm for various models.

Likewise, it would be possible to select a model and then evaluate the characteristics for a set of the algorithms. So it would be possible to compare the properties of several algorithms on the fixed model. By the way, it is quite a topical question for some measurement problems.

6. CONCLUSIONS

In this report the main kinds of the quality analysis procedures are presented for the major elements of measurements.

Starting from the traditional analysis of the measurement accuracy, two main aspects of the measurement quality analysis are outlined. First, the quality of the mathematical models is studied as well as the quality of the real objects. Second, the main groups of quality characteristics are applied for the measurement procedure. These two directions are closely connected.

The development of these aspects is essentially based on general systemic approach. The quality of measurement procedure is represented as a systemic unification of the partial characteristics of quality on the common methodological basis.

This approach stresses the systemic character of the quality problem and creates the uniform basis for solving this problem both in theoretical and applied aspects.

In particular, certification of the data processing algorithms (software), and evaluation of the model characteristics are analyzed as similar procedures within the general systemic approach.

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