*XVII IMEKO World Congress*
*Metrology in the 3rd Millennium*
*June 22-27, 2003, Dubrovnik, Croatia*

# ANALYSIS OF THE CONCEPTS OF MEASUREMENT, INFORMATION AND KNOWLEDGE

*Ludwik Finkelstein*

Measurement and Instrumentation Centre, City University, London, UK

**Abstract -** The paper presents an outline of the philosophical and formal logical analysis of the fundamental concepts of measurement, information, knowledge and knowledge application, It considers the relation between them.

Keywords: measurement fundamental concepts. information fundamental concepts, knowledge fundamental concepts.

## 1. INTRODUCTION

Those concerned with the application of measurement, and with the instrumentation by which it is implemented, frequently make the claim that measurement is a powerful means of acquisition of knowledge.

Commonly this claim is supported by the well known statement by Lord Kelvin: "I often say that when you can measure what you are speaking about and express it in numbers you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be." [1].

Notwithstanding the frequent assertion of this claim, the community engaged in measurement and instrumentation science and technology has not been much concerned with analysis of the underlying philosophical concepts.

However, the philosophical aspects of measurement have recently been discussed at the Symposium of the IMEKO Technical Committee on Measurement Science held in June 2002, in Cracow, Poland [2]. The evolution of the fundamental concepts of measurement has been discussed in outline by the present author [3]. Mari analysed the epistemology of measurement [4]. Sydenham raised important and far ranging issues of the relation of measurement with the concepts of information, knowledge and wisdom [5]. The above papers are to be published in updated and extended form in a special issue of the IMEKO Journal *Measurement.*

The present paper builds on these enquiries and outlines a formal analysis of the concepts of measurement, information, knowledge and wisdom from the point of view of philosophy and the model theory of logic.

## 2. MEASUREMENT

### *2.1 Informal definitions*

The basis of the discussion in this paper is the definition of measurement in the wide sense.

Measurement in the wide sense is defined as any process of empirical, objective assignment of symbols to attributes of objects and events of the real world, in such a way as to describe them.

By description we mean that the symbols assigned to the attributes imply, and are implied by, relations between the attributes to which they are assigned.

We propose to distinguish between measurement in the strong sense and measurement in the weak sense.

Measurement in the strong sense is defined as one that conforms to the normative paradigm of the physical sciences. In particular the symbol assignment is a mapping into the real number line, on which the operation of addition is defined.

Measurement in the weak sense is defined as one which, while it is an objective and empirically based symbolic description, does not have some of the properties of measurement in the strong sense.

A detailed discussion of the concepts of widely, strongly and weakly defined measurement is given in [3] and the paper to be published that has been developed from it.

### *2.2 Outline of the formal theory of measurement*

For the sake completeness an outline of the formal theory of measurement will be presented using the representational or model theory approach. The presentation follows that given in [6] and the literature quoted therein..

The treatment is in terms of the wide sense definition of measurement. The treatment of strongly defined measurement, and its comparison with weakly defined measurement is given in [3] and the paper to be published that has been developed from it.

A representational theory of measurement has four parts:
(i) an empirical relational system corresponding to a quality.
(ii) a symbolic relational system
(iii) a representation condition
(iv) a uniqueness condition.

These will now be considered.

(i) Quality as an empirical relational system

A quality is a property of an object.

Consider some quality and let $q_i$ represent an individual manifestation of the quality Q, so that we can define a set of all possible manifestations as

$$Q=\{q_1.........\} \quad (1)1$$

Let there be on Q a family R of empirical relations $R_i$

$$R=\{R_{1,}.....,R_n\} \quad (2)$$

Then the quality is represented by an empirical relational system

$$\mathcal{Q}= < Q,R > \quad (3)$$

(ii) Symbolic relational system

A symbol is an object, or event which represents another object or event termed the referent, by bearing a known relation to it.

Let Z represent a class of symbols, that is some set of objects or events to be used for the purposes of representation.

$$Z=\{z_1.........\} \quad (4)$$

Let there be on Z a family *P* of relations

$$P=\{P_{1,}.....,P_n\}. \quad (5)$$

Then

$$\mathcal{Z}= < Z,P> \quad (6)$$

represents a symbolic relational system.

(iii) Representation condition

The representation condition requires that measurement be the establishment of a correspondence between quality manifestations and symbols in such a way that the relations between the referent property manifestations imply and are implied by the relations between their images in the symbol set.

Formally, measurement is defined as an objective empirical operation M

$$M: Q\rightarrow Z \quad (7)$$

so that

$$z_n=M(q_n) \quad (8)$$

such that $\mathcal{Q}$= < Q,*R* > is mapped homomorphically into (onto) $\mathcal{Z}$= < Z,P>

The above homomorphism is the representation condition.

Firstly it implies that if $q_n$ is related to $q_m$ by an empirical relation $R_{k_k}$, that is $R_{k_k}(q_n,q_m)$, $P_k$ is symbolic relation corresponding to $R_{k,,}$, $z_n = M(q_n)$ is the image of $q_n$ in Z under M then $R_{k_k}(q_n,q_m)$ implies and is implied by $P_{k_k}(z_n,z_m)$

Measurement is a homomorphism, rather than an isomorphism, because M is not one-to-one, it maps separate but indistinguishable property manifestations to the same number.

$$S=<Q,Z,M> \quad (9)$$

constitutes a scale of measurement for *Q*.

$z_n = M(q_n)$, the image of $q_n$ in Z under M is called the measure of qj on scale *S*

(iv) Uniqueness condition

The requirement that the fundamental measurement procedure of a scale should map the empirical relational system *Q* homomorphically into the symbolic relational system .*N* does not determine the mapping uniquely.

There is an element of arbitrary choice in the setting up of scales of measurement.

The requirement of homomorphism thus defines a class of scales that may be called equivalent. The class of transformations, which transform one member of a class of equivalent scales into another, is called the class of admissible transformations. The conditions which admissible transformations must satisfy, are known as the uniqueness conditions.

### *2.3 General representation by symbols*

Measurement as defined above can be seen as a special case of general representation of entities by symbols. Since a discussion of information and knowledge involves a consideration of such representation it is proposed to outline the principles underlying it.

Let $q_n$ be a referent entity. Consider further that $q_n$ is a member of a family or set of similar entities Q,

$$Q=\{q_{1_1}.........\}, \quad (10)$$

Q is termed the referent set. Let there be on Q a family R of relations

$$R=\{R_{1,}.....,R_n\}. \quad (11)$$

We may term $\mathcal{Q}$= < Q,*R* > , the referent relational system.

Let now $z_n$ be a symbol entity. Consider further that $z_n$ is a member of a family or set of similar entities Z,

$$Z=\{z_1,.........\} \quad (12)$$

Z is termed the symbol set. Let there be on Z a family *P* of relations

$$P=\{P_{1,}.....,P_n\} \quad (13)$$

We may term $\mathcal{Z}$= < Z,*P*> the symbol relational system.

Let there be a mapping

$$M: Q\rightarrow Z \quad (14)$$

so that $z_i=M(q_i)$. Further let there be a mapping

$$F: R\rightarrow P \quad (15)$$

so that $P_n=F(R_n)$.

. We may define

$$C=<\mathcal{Q},\mathcal{Z},M,F,> \quad (16)$$

where *C* is the representation code, and its inverse, the interpretation code. $z_n$ is termed a symbol of or for $q_n$.

## 3. INFORMATION

The most important concept of information can be defined and explained from the above formulation of general symbolic representation. [6]

Information about the referent consists of a symbol for the referent together with the representation relation. The foregoing informal definition can be expressed formally and more widely as follows:

If zi is a symbol of or for qi. then information about qi given zi can be denoted by *J*(qi|zi) where

$$J(q_i|z_i) = <z_i, C> \quad (17)$$

In Measurement Science discussion on information is generally based on the so called Information Theory [7].

In the Information Theory founded by Shannon [8,9,10.11] we consider an information transmission channel which transform symbols $x_i$, elements of a symbol set X, acting as inputs, into symbols $y_i$, elements of a symbol set Y, constituting outputs. The transformation is in general many-to-many. The quantity of information about an input $x_k$ provided by the occurrence of an $y_i$ output is given by :

$$I(x_k; y_I)= \log[P(x_k/y_I)/P(x_k)] \quad (18)$$

The base of the logarithm defines the unit of the scale. The definition of quantity of information given by information theory and the definition of information based on symbolic representation theory are consistent, and indeed similar. The information theoretic definition presupposes the concepts of representation by symbols. In both information is knowledge about an entity provided by an image of the entity under a mapping Information theory deals with a restricted class of problems. It does not provide insight into the wider problems of the information carried by symbols, in particular semantic and pragmatic problems. They will be considered later in this paper in terms of language and knowledge.

For the sake of completeness attention is drawn here to some important concepts discussed in [6]. Measurement is an information process. Measuring instruments are information machines. An information machine is defined as a machine which transforms input information-carrying symbols into output symbols, in accordance with a prescribed functional relation.

## 4. LANGUAGE

Language is the essential tool for formulating, expressing and manipulating knowledge. For consideration of knowledge must basic aspects of language must be outlined here.

Most generally knowledge is expressed in natural language. Natural languages are highly complex systems. Event an outline of the linguistics of natural would be excessive here. For introductions to the subject and the relevant problems reference may be made to [12, 13, 14].

The key problems of use of natural language for handling knowledge are: ill-defined meaning, including vagueness and ambiguity, as well as the affective aspects of language.

Knowledge based on measurement has well defined meaning. For this reason we can confine our consideration to a limited class of language, which has such meaning. This will now be discussed in formal terms.[ 15, 16, 17]

A language *L* is a subset of all finite strings, that is concatenations, of elements of an alphabet or set of symbols, A.

In the context of the symbolisation *C* the alphabet consists of the elements of Z and *P*, supplemented by function symbols.

A string of the language is termed a sentence. A typical sentence in the language considered above is:

$$Z_n = P_i(z_1,....,z_m) \quad (19)$$

A language *L*, thus defined, may consist of a large number of strings. A grammar, or syntax, *G*, is a set of rules, which operating on the alphabet A, can generate all the strings of *L.* An alphabet and grammar constitute a compact description of a language.

A code, such as *C* above, describing the correspondence of the linguistic symbols to the real world, or more generally to extra-linguistic entities and their relations, constitutes the semantics of the language.

A sentence in a language with descriptive semantics, carries information about the portion of the world of which it is an image.

A sentence that is declarative and asserts something true, or false, is termed a proposition.

## 5. THEORY

To provide a basis for the analysis of knowledge it is proposed to discuss briefly the concept of a theory

The term theory is used with a wide meaning to denote a system of concepts and propositions attempting to bind together knowledge of the world of experience [18].

It is proposed here to use it in the more restricted formal terms of model theory [15].

A theory *T* is then defined a set of sentences of the language *L*.

It may be possible to present *T* as an axiomatic system, in which all sentences of the theory are clearly derived from a set of axioms, that is considered to be true. A fully formalised axiomatic system consists of (i) primitive symbols, (Z, in the notation adopted above); (ii) rules for the formation of well-formed expressions (*G*); (iii) axioms; (iv) rules of inference; (v) a code *C* establishing the definition of the symbols and the interpretation of symbols and sentences [15, 19, 20].

It is to be noted that the domain (usually extra-linguistic) for which the theory holds true is termed in logic a model of the theory. This is opposite to the usage in science which terms the theory a model of reality.

## 6. KNOWLEDGE

### *6.1 Definition of knowledge in philosophy*

Knowledge is a fundamental philosophical concept, the subject of the branch of philosophy termed epistemology [21, 22, 23]. Knowledge from the point of view of information processing is reviewed in [24]. The classical and plausible view is that it consists of a set of propositions which are true, which a subject believes to be true and which the subject is justified in believing to be true. The main problem in this view of knowledge is the nature of the justification of the belief.

The sufficiency of this definition has been disproved by Gettier who showed by counter-examples that mat have justified true belief in *p*, without knowing *p* [25].

[21] outlines some theories which provide a basis of justification. Two significant in the present contest are based on causality and reliability. According to causal theories, knowledge consists of true belief that bears a true relation to the fact in question. According to reliability theories

knowledge requires that it be acquired by a reliable process or method.

*6.2 Knowledge based on measurement.*

The result of a measurement constitutes knowledge.

A measurement result is a statement in the form

$$z_k = < z_l, S > \quad (20)$$

where $z_k$ is the symbolic designation of the measurement, $z_l$ is the symbol assigned by to the measurand by the measurement process and $S$ is the scale on which the measurement is carried out.

Knowledge obtained by measurement satisfies the conditions of valid knowledge, meeting the requirements of the theories discussed above.

Firstly, measurement is obtained by empirical observation.

Secondly, the process is objective. That is the result of measurement is independent of the observer.

The process of measurement ensures the causal relation between the measurement result and underlying fact.

The process of measurement meets the condition of reliability.

*6.3 Declarative and procedural knowledge*

In the context of the present paper we may. distinguish between declarative and procedural knowledge [26, 27].

Declarative knowledge is knowledge that, and has been discussed above.

Procedural knowledge is knowledge how. It is essentially expressed as transformational rules for declarative knowledge.

Knowledge based on measurement is essentially declarative knowledge. However , the implementation of the measurement process is based on procedural knowledge.

*6.4 Knowledge in information technology*

Measurement is generally implemented by information machines [6]. Following sensing and signal conditioning the measurement information acquired is expressed in symbols and processed, generally, by standard information technology equipment.

It is therefore appropriate to say something about knowledge in such equipment.

Firstly the term knowledge, as used in such terms as knowledge processing and knowledge engineering embraces in general not only true knowledge, but also beliefs, held to be probably true but unproven.

Secondly both declarative and procedural knowledge, is utilised in the processing.

The strength of measurement information is that it expresses knowledge in a symbolic language compatible with effective processing.

*6.5 Knowledge, positivism, logical positivism, and operationalism*

The significance of measurement as a basis of knowledge has been a theme of the philosophical movements of positivism, logical positivism, and operationalim [28, 29, 30, 31].

This paper cannot present even an introductory exposition of the main features of these movements, and even in summary must oversimplify arguments. Readers should refer to the references given above and to the literature cited therein.

Essentially positivism arises from empiricism and bases knowledge on scientifically acquired and interpreted experience. Logical positivism, essentially, bases knowledge on objective, empirical observation, expressed and processed in terms of formal logical language. Operationalism essentially is concerned with defining concepts in terms of objective empirical observational operations. In ether words s concept to be meaningful must be based on a measurement operation.

The statement by Kelvin about measurement [1] is essentially operationalist.

Positivism, logical positivism, and operationalism have been extensively criticised in the last half-century. They have made a contribution to philosophy but have run their philosophical course. They have deficiencies and they do njot exhaust all aspects of human knowledge. Nevertheless their concepts greatly influence the practical attitude of scientists and engineers to knowledge.

*6.6 Organisation of knowledge*

Knowledge in the form of an atomic proposition, such as the result of a single measurement, is of limited value.

Knowledge increases in significance as it grows to a set of propositions, covering a domain of the universe under consideration, in a complete, connected and consistent manner.

Knowledge further increases in significance as it provides the basis of further reasoning.

Effective knowledge must be capable of being effectively stored, retrieved and processed.

Knowledge is most usefully organised in theories, particularly well axiomatised theories, as defined above.

Knowledge based on measurement being expressed in the language if mathematics, or other well-defined symbolism, is particularly compatible to the formation of effective theories, or mathematical models. It is also particular suitable for further processing.

## 7. APPLICATION OF KNOWLEDGE

Sydenham in [3] raised the topic of the relation of measurement to wisdom.

Wisdom is not a precisely defined or analysed topic in modern philosophical thought [32]. It is proposed to define it here as the thoughtful application of knowledge, particularly in relation to conduct.

It is suggested here that the application of knowledge takes in general the form of a problem solving or design process [33].

In such a process the basic operations are an analysis of requirements, leading to a value model or objective function, possibly multi-dimensional, Following the generation of candidate solution concepts, which are commonly knowledge based symbolic models, the candidates are analysed in terms of the value- model. On the basis of an analysis of the thus derived values of the solutions, one or more of the candidates are chosen by a decision process to be accepted for further development, or implementation.

The value model is based on a measurement-like process of assigning numbers to attributes [34]. The value model is however subjective reflecting the view of the decision-maker. It is not measurement.

## 8. CONCLUSIONS

The paper discusses some fundamental concepts of measurement, information, language, knowledge and wisdom. It explores relationships between them. It is intended to stimulate awareness and to act as a basis for further analysis and debate.

## 9. ACKNOWLEDGEMENTS

The author wishes to thank Prof. K.T.V. Grattan and Prof. B.M.A. Rahman, the Directors of the Measurement and Instrumentation Centre, for their encouragement and the facilities to carry out the work. He is also grateful Dr S. H. Khan of the Centre for companionship and help.

Finally he is indebted to Prof. Peter Sydenham and Dr, Luca Mari for the inspiration to undertake the work.

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**Author:** Prof. L. Finkelstein OBE FREng, Measurement and Instrumentation Centre, City University, Northampton Square, London EC1V 0HB, UK. Tel: +44 (0)20 7040 8139, Fax: (0)20 7040 8568, e-mail : l.finkelstein@city.ac.uk