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FAST AND PRECISE ALGORITHM FOR TRACKING SINUSOIDAL SIGNALS

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Abstract – The paper describes an algorithm for fast and precise tracking of sinusoidal signals. The implementation is designed for a system where the acquisition and processing of the input signal samples are performed in real time. Few instructions are required at every step, so the algorithm can be applied to signals having a relative high frequency.

Keywords: Signal processing, Phase locked loop, Acquisition system.

1. INTRODUCTION

In many instruments for the generation, the measurement and the processing of electrical signals in real time, it can be useful to track the internal reference of the instrument to an external one. Traditionally, in electronic analog systems, this operation is performed by phase locked loop devices (PLL). The same method can also be applied in digital systems for real time signal processing by means of a suitable simulation of the PLL [1]. This method has some advantages: it is simple and the simulation requires only a few instructions, which make it suitable for a fast processing of the signal. However the simulated phase locked loop has also some drawbacks, because there is a trade-off between the settling time and the possible range of the signal that can be locked to a external reference. In case of a digitized signals, where a more elaborate processing is possible by means of a computer or a digital signal processor (DSP), other methods can be adopted for tracking the external reference and the results can be more satisfactory.

In this paper a new type of algorithm for detecting the frequency and the phase difference between two sinusoidal signals is investigated and its characteristic is evaluated both analytically and experimentally.

2. THE ACQUISITION SYSTEM

The basic circuit of a system that can be used for the acquisition of the reference to be tracked and the generation the output signal is shown in Fig. 1. The hardware of this system consists of an analog to digital converter (ADC), whose input is connected to an external sinusoidal reference. The system utilizes an internal DSP, which processes the signal in real time and produces the suitable codes for the output. The system can be embedded in a board within a computer or can be independent and connected to an external computer that prepares, by means of a suitable program, all the codes necessary for the initialization and the generation of the required signals.

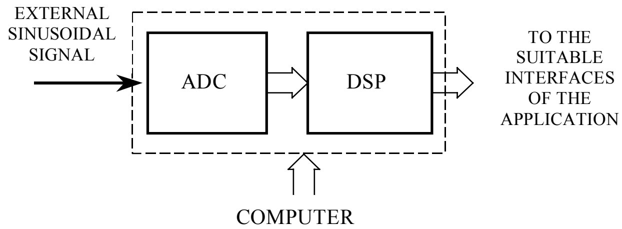

Fig. 1 Basic hardware circuit for the acquisition of the reference in the tracking system.

The internal program for the processing of the signals, driven by the DSP, operating in real time, can utilize successive interrupts, produced at regular interval by the internal timer. During each interrupt the suitable operations are performed to acquire the inputs, to process the signals and to generate the outputs. To operate in real time the algorithm for tracking the external signal must have a very limited number of instructions, because it has to be fast enough to be computed during the time between two samples.

3. THE ALGORITHM

3.1. Theory of operation

In the system represented in Fig. 1 the determination of the phase increment for the synchronization in real time with an external reference requires some preliminary operations for the initialization of the program and then for the acquisition and the processing of the samples.

The method proposed is based on synchronous detection technique and the consecutive digital filtering by means of a low pass and a notch filter centered on the second harmonic. In this way, it is almost cancelled the second harmonic component, which is the main problem of a synchronous technique.

The structure of the algorithm is simple. At fixed intervals T, the amplitude of the external signal to be tracked is acquired, while the internal virtual sinusoidal signal is at every sample represented by the contents of a register, which acts as its instantaneous phase.

The processing of the samples for reaching the synchronization is based on the fact that, when the external reference signal has thesame frequency of the internal virtual signal, the product of the two values is a fix value added to a sinusoidal signal with a frequency which is double of that of the reference. This signal at double frequency is removed by adding the same signal with a delay equal to phase rotation of half period of the second harmonic component.

The detection part of this algorithm can be schematically represented in Fig. 2

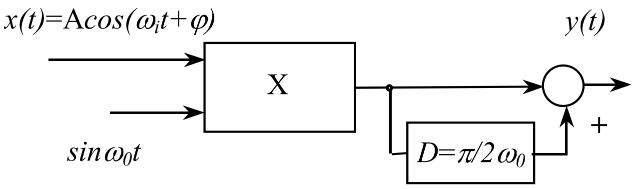

Fig. 2 Schematic diagram of the detection part of the algorithm

The result at the output of the circuit represented in Fig. 2 is given by:

$$y(t) = x(t) \cdot sin\omega_0 t + x(t-D) \cdot sin(\omega_0 t - D) \quad (1)$$

and, if the input is given by a sinusoidal signal:

$$x(t) = A \cdot \cos(\omega_i t + \varphi), \quad (2)$$

then the output $y(t)$ can be used to modify the frequency of the internal generator and to obtain the tracking.
In fact, when two frequencies are exactly the same and:

$$\omega_i t + \varphi = \omega_0 t + \theta(t) \quad (3)$$

Then

$$y(t) = -A \cdot sin(\vartheta(t)) \quad (4)$$

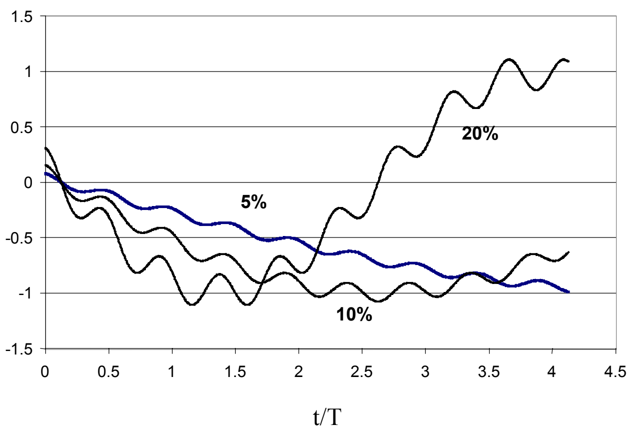

Fig. 3 Output of the detection algorithm as a function of time in periods of the internal reference for different input frequency (the initial phase is zero). The labels indicate the relative difference between the external reference and the internal reference.

In Fig.3 the output of the detection algorithm is given as a function of the difference between the internal frequency and the input frequency as a function of the time expressed in period of the reference signal.

3.2.. Practical implementation

A practical implementation of this algorithm was firstly experimented for producing a signal made by a combination of calibrating harmonic components and then tracking them to a sinusoidal signal given as a reference [1]. However, the algorithm can be also applied in other instruments for metrological purpose such as, for example, lock-in amplifiers and special calibrators or spectrum analyzers.

The simpler implementation of the algorithm can be built with a structure represented schematically in Fig. 4. SIN and MULT are two circular buffers. The registers PH, ST and RT are register and their upper part point to the current address of the two circular buffers. In particular ST is the register that points to the buffer MULT when a datum has to be stored and RT when a datum has to be retrieved.

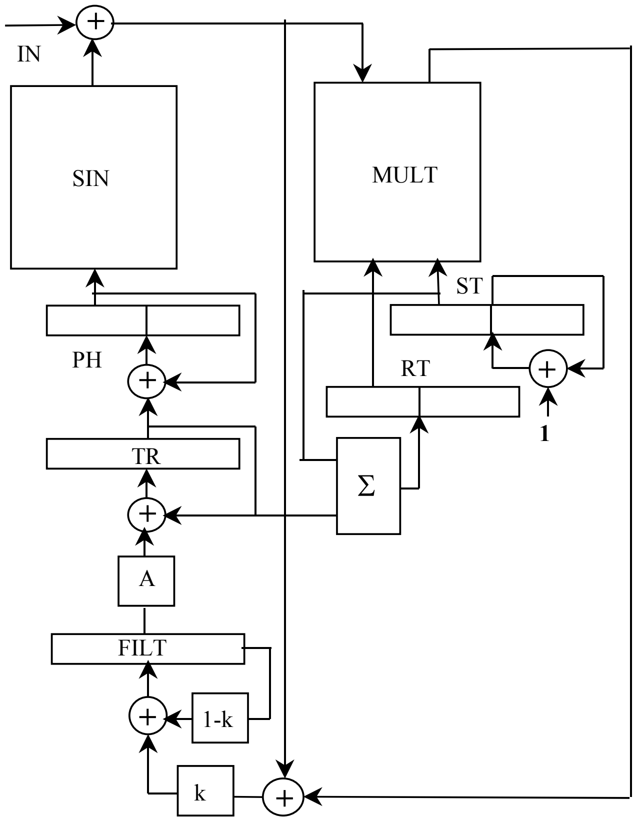

Fig. 4 Schematic representation of the algorithm.

The algorithm consists of some steps performed at every sample. Each of them can be realized generally by a single instruction in particular DSPs or computers. These steps are:

- The input is acquired by means of the ADC and the input multiplied by the sine of the internal virtual reference is stored in the circular buffer MULT.
- The input multiplied by the sine of the internal virtual reference is added to the retrieved in the circular buffer MULT and used to evaluate the output of the filter FIL.

- The relative value of the instantaneous period is evaluated.
- The pointer for the storage and the retrieved data on the MULT buffer and the PH register are updated.

The operations can be listed as instructions for a possible program on a DSP:

$IN*SIN(PH_U) \Rightarrow MULT(ST_U)$
$(1-k)*FILT+k*(IN*SIN(PH_U)+MULT(RT_U)) \Rightarrow FILT$
$TR+G*FILT \Rightarrow TR$
$ST+1 \Rightarrow ST$
$ST-N/(4*TR) \Rightarrow RET$
$PH+TR \Rightarrow PH$

In the instructions above ST_U and RT_U are respectively the upper part of the register ST and RT, k and G two values that can selected for adjusting respectively the band of the digital filter and the gain of the feedback loop.

This basic algorithm, which uses the circular buffer like in a DDS generation system [1], [2], can be refined with some additional instructions evaluating for example the interpolation between two samples in the SIN and MULT buffer, taking into account the lower part of respectively the PH and the RT register [1].

4. ADVANTAGES OF THE ALGORITHM AND RESULTS

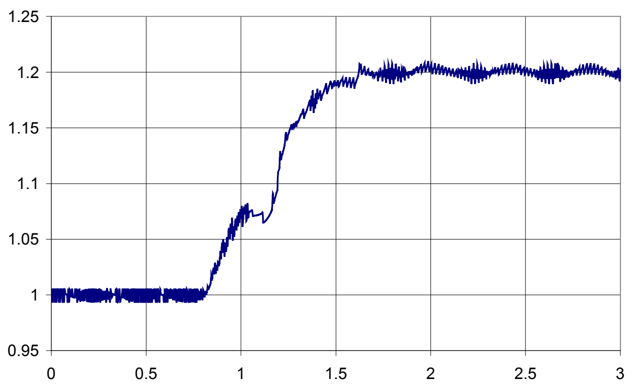

Fig 5 Response of the instantaneous frequency of the internal virtual signal to a step variation of the input frequency. The tables SIN and MULT are of 256 samples and the data taken are not interpolated.

The algorithm has been tested by simulation by means of a program built in Visual Basic and is now being implemented in an experimental prototype of a DSP embedded in a real time board. The table of the stored sine has a fixed length (from 64 to 1024 samples per period). By a proper choice of the feedback coefficients, the algorithm has shown the possibility to track frequency changes in wide range operative range (from 70% to 130% of the input frequency) in about one period of the input signal. Fig. 5 shows the response to a step transition of the frequency equal to 20% of the central frequency.

Due to the truncation of the phase the instantaneous frequency is not stable. However the stability of the frequency integrated over in one period of the input signal is about 0.01% even with a 1% gaussian noise superimposed to the input sinusoidal signal.

This algorithm has many advantages:

- It can operate in a sufficient wide frequency range (for example with an initial guessing about 30% or more apart from the frequency of the reference) and without corrections that cause a discontinuity in the tracking.
- The tracking is fast and it can follow very quick variations of the external frequency.
- The computation of the whole operation is made at each sample and requires only a few linear combinations (this operation is performed in one instruction in typical DSP used for signal processing).
- For the limited number of operations the same algorithm could be also easily integrated in very fast hardware digital circuit.

5. CONCLUSIONS

An algorithm suitable for tracking external sinusoidal signals suitable and for application in instruments and measurement devices has been experimented. The results obtained show that it can be used for fast variations of the parameters of an external signal. Furthermore, with suitable filters or post processing supplementary instructions, it can be also be employed for the precise tracking of both the frequency and the phase.

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