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# INTERPOLATION BASED ON WAVELET COEFFICIENTS FOR FREQUENCY MEASUREMENT

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**Abstract** – Presented paper concerns the problem of signal period estimation improvement by means of the wavelet coefficients. In particular, the interpolation based on the wavelet coefficient has been proposed for this aim. The method does not require virtually any additional computational power of measurement device, under assumption that the wavelet transform has been implemented for waveform analysis. In the paper results of experimental research have been presented and its merits as well as shortcomings have been discussed.

Keywords: interpolation, wavelet coefficients

# 1. INTRODUCTION

One of the basic tasks during voltage parameters measurement is its period estimation. It has utmost importance for measurement of great deal of electrical power parameters. Conventional method of analysed signal period estimation is evaluation of time between its two zerocrossings. A simple method of the period measurement is detection of zero-crossing moment of analysed signal by analysis of sample polarisation changes. The accuracy of this method heavily depends on sampling frequency. For improving performance of the method, real zero-crossing moments are to estimate by means of interpolation. In this way a better accuracy could be achieved. Accordingly, a number of samples taken into account during a period of analysed signal has been equal to non-integer value (*N*-I)+ $\Delta N$ , where *N* means number of samples taken into consideration during a period of analysed signal, while  $\Delta N$  can be defined as below [1]:

$$\Delta N = \left| \frac{s_0}{s_0 - s_{-1}} - \frac{s_{N-1}}{s_N - s_{N-1}} \right| \tag{1}$$

where:  $s_{-1}$  – the last sample taken in previous period,

 $s_0$  – the first sample taken in analysed period,  $s_{N-I}$  – the last sample taken in analysed period

 $s_N$  – the first sample taken in next period.

v – the first sample taken in next period.

Obviously, the proper period estimation influences results of different parameter measurements. But for this paper purpose the frequency has been chosen for the method evaluation. Taking into account the formula (1), a frequency can be calculated as follows:

$$f = \frac{f_s}{(N-1) + \Delta N} \tag{2}$$

where:  $f_s$  – sampling frequency.

A significance of interpolation for frequency measurement is easily discernible in Fig. 1 [1].



Fig.1. Standard deviation  $\sigma$  of frequency measurement error  $\Delta f$  of sinusoidal signal for different sampling frequencies and different number of bits of AD converter (results with i mark – with interpolation method and with mark w – without interpolation method): a) simulation results, b) experimental results.

The method leads to proper results only if one assumes purely sinusoidal character of analysed signal. Harmonics and noise can corrupt the results of frequency measurement. These limitations of the method are easily discernible in Fig. 1, namely the standard deviations of frequency measurement increase if number of bits of AD converter decreases. It is due to quantization noise increasing. This same problem occurs for signals with low signal to other noises ratio.

There are quite a few methods of solving the problem of proper frequency estimation in presence of harmonics as well as noise. Different algorithms have been presented in many papers, e.g. [2], [3], [4]. These algorithms are to render proper results in case of high harmonics and noise values and even multiple zero-crossings. However, if one assumes that multiple zero-crossings are not so common and takes simple precautions (e.g. proper sampling frequency, limits of possible frequency changes), these methods have one obvious downside. Namely they lead to increase the need for computational power of measurement device. At the same time the constant progress in development of mathematical tools for different frequency components analysis has been observed. One of these tools is Digital Wavelet Transform DWT. Applying these tools for discussed aim leads to fast and reliable period (frequency) estimation in common cases.

# 2. WAVELET COEFFICIENTS FOR FREQUENCY MEASUREMENT

Nowadays the Digital Wavelet Transform is widely implemented in many electrical power engineering applications. The analysis filter banks of DWT divides the signal into logarithmically spaced frequency bands as shown in Fig 2 [5].



Fig. 2. Wavelet Decomposition; a) sample of analysis filter banks:  $h_l$  – low-pass filter coefficients,  $g_l$  – high-pass filter coefficients,  $c_{j-1,k}$ ,  $d_{j-1,k}$  – wavelet coefficients for first layer of decomposition, j – whole number of decomposition layers, L – number of filter coefficients, b) related frequency bandwidths,  $f_0$  – fundamental frequency

Since DWT divides the original signal into separated frequency bands, it is possible to analyse higher frequency components for each band independently. At the same time one should remember that wavelet coefficients of low-pass filters of analysis filter bank  $(d_{j-1,k})$  are smoothed version of original signal. In particular, the influence of the noise is reduced. Then, it is possible to apply interpolation to appropriate wavelet coefficients for frequency measurement efficiency improvement. The proposed algorithm consists in using in formula (1) wavelet coefficients for first layer of decomposition instead of signal samples, like in following equation:

$$\Delta N = \left| \frac{c_{j-1,0}}{c_{j-1,0} - c_{j-1,-1}} - \frac{c_{j-1,N-1}}{c_{j-1,N} - c_{j-1,N-1}} \right|$$
(3)

The first layer has been chosen for reasons explained in the 2.2 section.

#### 2.1. Conditions of experimental research

An extensive experiment has been carried out for checking out applicability of the proposed algorithm. The appropriate software for frequency measurement has been developed in assembler and it has been implemented on digital signal processor ADSP-21061 of Analog Devices Sharc family. The block diagram of research set has been depicted in Fig. 3. The results of measurement have been stored in RAM memory of the processor and afterwards transferred to PC computer by RS 232 interface.



Fig. 3. Block diagram of research set; FG – function generator (Agilent 33120A); AF – antialising filter; ADC – analog to digital converter (ΣΔ 16 bits); DSP – digital signal processor (ADSP-21061 of Analog Devices); PC – PC computer

The research has consisted of measurement of signal frequency by means of different wavelets. In particular, the Daubechies wavelets have been applied as well as symmlets Vaidyanathan and Smith&Barnwell. All measurements have been carried out for three frequencies of original signal (49,925Hz, 50Hz and 50,075 Hz) and sampling frequency 16 kHz. Then different  $\Delta N$  values occurred. For example,  $\Delta N$  equals to 0,48 for 49,925 Hz, 0 for 50 Hz and 0,52 for 50,075 Hz. Standard deviations have been calculated for 3760 measurements. That exact value has been imposed by attainable RAM memory of used processor. A number of AD converter bits has been changed by software way. The maximum rms value of input signal is equal to 2 V.

### 2.2. Results of experimental research

As it has been mentioned above, the different kind and length of analysis filters have been taken into account during experimental research. But the kind of filter and its length has any importance for the method of frequency measurement. The exemplary results of measurement of sinusoidal signal frequency with amplitude of 0,1V by means of 16 bits AD converter as well as different analysis filters have been presented in fig. 4.



Fig. 4. Standard deviations  $\sigma$  of frequency measurement error  $\Delta f$  of sinusoidal signal for different analysis filters; Daubechies analysis filters of length 8 (D6) and 20 (D20) as well as symmlets (S8),

Smith&Barnwell (S&B8) and Vaidyanathan (V8) analysis filters of length 8

Eventually, in the paper only chosen results for Daubechies analysis filter of length 8 due to minor influence of the kind and length of filter for whole analysis has been shown. In the following examples, results of experimental research marked as "samples" are based on the formula (1), whereas results marked as "wavelet c." are based on the formula (3).

Standard deviations  $\sigma$  of frequency measurement error  $\Delta f$  of sinusoidal signal (amplitude equals to 1V) for different number of bits of AD converter have been shown in Fig. 4.



Fig. 4. Standard deviation  $\sigma$  of frequency measurement error  $\Delta f$  of sinusoidal signal for different number of bits of AD converter

It is easy perceptible that the standard deviations  $\sigma$  of frequency measurement error  $\Delta f$  have been significantly reduced for the interpolation based on wavelet coefficients, especially for 10 and 12 bits converters. The effect has been observed mainly for these converters with relatively high values of quantisation noise. So similar effect can be

observed when amplitude of measured signal decreases. Standard deviations  $\sigma$  of frequency measurement error  $\Delta f$  of sinusoidal signal for different amplitudes of analysed signal and 16-bit AD converter have been presented in fig. 5.



Fig. 5. Standard deviation  $\sigma$  of frequency measurement error  $\Delta f$  of sinusoidal signal for different amplitude of analysed signal and 16-bit AD converter

One more advantage of the proposed method has to be pointed out. It is less sensitive to multiple zero-crossings due to elimination of the higher frequency components and downsampling processes.

Finally some limits to the proposed method should be mentioned. It is sensitive for signal with higher frequency component and small noise values. During research the values of standard deviations  $\sigma$  of frequency measurement error  $\Delta f$  assume greater values for the proposed method if the analysed signal contains only harmonic 20-th or higher order and virtually any noise (16 bit AD converter and harmonic content 1%). These results have been for analysis based on the first layer of analysis filter bank. It has been main reason for previously mentioned choice of the wavelet coefficients of the first layer analysis for frequency measurement. But it should be underlined that in real electrical power engineering systems such high harmonics are uncommon. For the most cases it is sufficient to consider harmonic range from 2 to 20 [6].

# 2.3. Time of real measurement

One of the most important criteria of an algorithm evaluation is time required for whole analysis. In case of the proposed method the most time consuming task is reciprocal estimation in formulas (2) and (3). It requires 13 cycles for the processor used in the research. But it is easy to note that one needs only two such operations if measured periods succeed without any breaks. Then, only one reciprocal is required for  $\Delta N$  calculation. In case of software developed for this paper purpose, duration of whole frequency estimation equals to 1,3 µs. This time includes calculation of expressions (2) and (3). So, the method proves very fast.

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### 3. CONCLUSIONS

Careful analysis of whole research leads to a few conclusions:

- the improvement of measurement results (decreased standard deviations  $\sigma$  of frequency measurement error  $\Delta f$ ) have been observed for interpolation based on wavelet coefficient in comparison to interpolation based on samples of original signal,
- the kind of wavelet as well as filter length have any significant influence for achieved results,
- if wavelet transform is to implement for transient analysis, this improvement could be attained virtually without any need of additional computational power of measurement device,
- only limitation of proposed method is due to signal which contain significant harmonics of high order and concurrently do not contain considerable noise.

Summing up, the method should be very useful in common cases of the voltage quality analysis in electrical power engineering systems, assuming an implementation of Digital Wavelet Transform for waveform analysis. Taking into account range and values of harmonics occurred in real systems, it enables fast achieving satisfactory results as well as reduces risk of corruption of measurement results by signal multiple zero-crossings.

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