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DYNAMIC ERROR CORRECTION FOR MEASUREMENT SYSTEM WITH DIFFERENTIATING SENSORS

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Abstract - The dynamic measurement is considered. The measurement system with differentiating sensor is taken into account. The 'blind' method has been applied to solve the dynamic error correction. Discrete time corrector has been adapted. Simulation technique has been used to verify the usefulness of the correction.

Keywords: discrete-time model, dynamic error, correction

1. INTRODUCTION

The correct dynamic measurement means the dynamic error of the measurement tends to zero. We would like to improve the dynamic characteristics of the measurement system and construct an 'ideal' system i.e. with linear characteristics and constant transfer function (k). Such a goal could be achieved with use of different methods, this work, however, develops the methodology called the 'blind' correction. The concept of the 'blind' correction is widely discussed in [1].
Fig.1. presents the functional block diagram of the method. The measurement system involves two parallel channels. They both consist of a sensor, an analog-to-digital (A/D) converter and a serial corrector in algorithm form. The analog outputs x1(t) and x2(t) from the sensors are converted into digital form x1(n) and x2(n). The conversion is done with sampling period T_S . The DSP processes these sampled signals using properly chosen correction algorithm. In general, the conception of the 'blind correction' consists in defining such a correcting algorithm, that the signals y1(n) and y2(n) reconstruct the measured transient signal u(t) at time instants $t=nT_S$. Obviously, the correctors stand for discrete-time objects.
We introduce some assumptions concerning the sensors.
1^o The measurement system with sensors of differentiating feature is taken into account. In practise, the measurement system of high voltage surge is a good example of such a system. The unknown parasitic serial capacitance is the

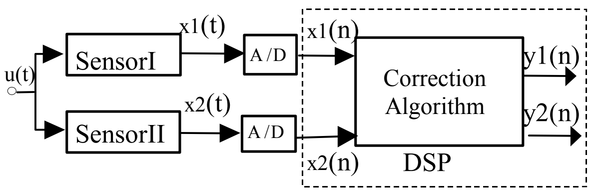

Fig.1. Block diagram of the system

cause of feed-forward action resulting in differentiating action of voltage divider.
2^o The static parameters of the sensors are known, and they are the same. However, we don't know sensor's dynamic parameters. We assume they are quite different. On the other hand the reconstruction of the measured signal u(t) and consequently the correction algorithm depends on these parameters. Since, similar to adaptive systems the problem of tuning the parameters must be solved.
Certain numerical procedures of the optimization are needed in order to solve the tuning problem. Some of them are considered during computer simulation.
Unknown parameters of the correction algorithm are well-defined when y1(n)=y2(n) for any n [1]. Because of numerical errors calculated signals y1(n) and y2(n) are not quite the same, therefore the average value of y1(n) and y2(n) i.e. y(n) = (y1(n)+y2(n))/2 is taken into account as a final result of the correction.
Furthermore, the dynamic behaviour of the whole measurement system is examined by computer simulation.

2. MATHEMATICAL MODEL OF MEASUREMENT SYSTEM COMPONENTS

To perform a computer simulation of the measurement system the mathematical model of its components must be constituted. Both sensors are modeled as a linear system with transfer function of the form:

$$G(s) = \frac{k(1+Ts)}{(1+kTs)} \tag{1}$$

where k - gain factor k<<1 (k is known)
kT - time constant of inertia
T - dominant time constant of the differentiating dynamics.

According to the rule of 'blind' correction the continuous-time corrector should be modeled as the inverse dynamics of the transfer function (1) i.e. $G^{-1}(s)$. The model of $G^{-1}(s)$ as a discrete-time model can be realized in different ways, directly in complex domain or indirectly in time domain. There are many transformations that can be used either in time or complex domain. Generally, discrete-time system is defined by its structure, order and parameters. The linear discrete-time system which approximates the continuous-time system might be described in the form (2):

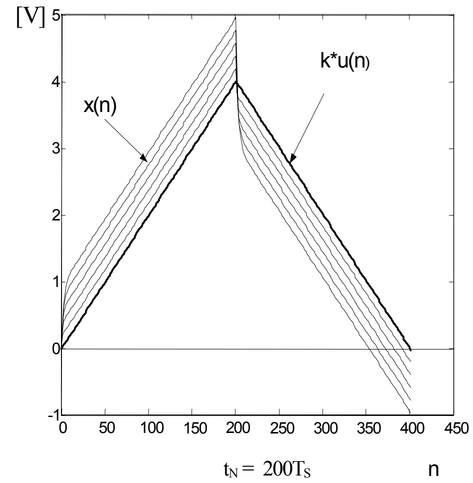

Fig.2. The discrete-time signal ku(n) and x(n) for different t_c.

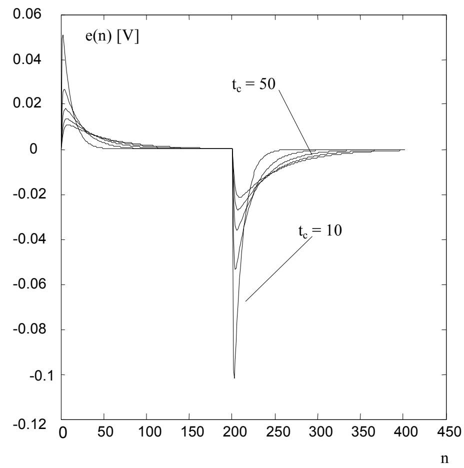

Fig.3. The error signal e(n) in a single channel for Euler's method

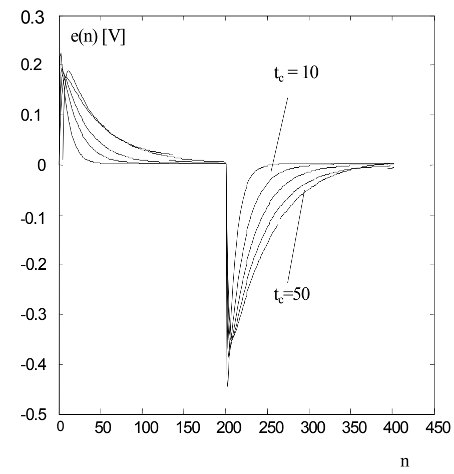

Fig.4. The error signal e(n) in a single channel for Tustin's method.

$$y(n) = \sum_{i=1}^{r} a(i)y(n-i) + T_S \sum_{i=0/1}^{r} b(i)x(n-i) \qquad (2)$$

where: x(n), y(n) are shown in fig.1.
a(i), b(i) – coefficients of the model
r – system order and T_S as defined above.

The coefficients of the model are selected (as a special task) before the system simulation starts.

The measured signal u(t) is assumed to be a triangular signal. The u(t) has got a discontinuous derivative at the point $t=t_N$. It can be written as:

$$u(t) = At \qquad \text{for } t \in [0, t_N] \qquad (3)$$

$$u(t) = -A(t-t_N) + At_N \qquad \text{for } t \in [t_N, 2*t_N]$$

where: t_N - rise time, A – a slope of a triangle

For such input signal the output of the sensor x(t) can be analytically determined including initial conditions for the variables u(t) and x(t) at the characteristic time instants t=0 and $t=t_N$. The obtained solution x(t) is transformed into a new scale of the time and the amplitude. The new scale is introduced in order to fit output range of sensors to the input range of the A/D converters. After transformation the signal x(t) is written at sampling instants as:

$$x(n) = \frac{u_m}{N}(n + (1-k)t_c(1 - e^{\frac{-n}{kt_c}})) \qquad \text{for } n<N \qquad (4)$$

$$x(n) = -\frac{u_m}{N}(n + (1-k)t_c(1 - e^{\frac{-n}{kt_c}}) + u_m(1 - e^{\frac{-n}{kt_c}}) +$$

$$+ x(t_N)e^{\frac{-n}{kt_c}} \qquad \text{for } N<n<2N+1$$

where: N – the number of samples in a certain time interval
$t_c = T/T_S$ - time constant of the sensor related to unit T_S
u_{max} - voltage range of the A/D converter
$x(n) \in [-u_{max}, u_{max}]$, $u_m = 0.8\, u_{max}$
n - current discrete time .

The scale factor of the amplitude is defined by the relation $Ak = u_m/t_N$, whereas the scale factor of the time is determined by the time interval $t_N = NT_S$ of the measurement process. The discrete-time signal x(n) and u(n) are presented in fig.2.

3. CHOICE OF THE DISCRETE–TIME CORRECTOR MODEL

The discrete-time corrector has been chosen after a series of simulation experiments, performed for a single measurement channel. The Euler's, Gear's and Tustin's transformations has been tested. The parameter (τ) of the corrector model has been exactly tuned to the time constant of the sensor (t_c) . Then the correction error e(n)=u(n) –y(n) has been verified for several values of the parameter (t_c). The signals of e(n) for different transformations and various (t_c) in the same channel are shown in fig. 3 and 4. It can be seen that the Tustin's method gives the best results in all cases. Further the transformation of Tustin is taken into consideration. This transformation yields the following representation of the corrector model (2) for every one of channels and r equals 1:

$$a(1) = (2\tau-1)/(1+2\tau) \quad (5)$$
$$b(0) = (1/k+2\tau)/(1+2\tau)$$
$$b(1) = (1/k-2\tau)/(1+2\tau)$$

τ- parameter of the corrector

This algorithm has a large stability domain and very good properties in the frequency domain.
The computer simulation has been performed for different number of bits of the A/D converter as well. The results obtained for the 8-bit A/D converter are not acceptable, but for 12-bit are good enough.

4. COMPUTER SIMULATION FOR TWO-CHANNEL'S MEASUERMNT SYSTEM

The main goal of the computer simulation is the evaluation of usefulness of the 2-channel's system with the 'blind' corrector for the case when differentiating sensors could be used. From here additional assumptions concerning simulation have to be specified. They are:

1° Criteria of the parameters selection $\tau1$ and $\tau2$ that are selected as the vector norms as follows:

$$MAX = \max\{abs(y1(n)-y2(n))\} \quad n \in [0, 2N+1] \quad (6)$$

$$ABS = \sum_{n=0}^{2N+1} abs(y1(n) - y2(n))$$

$$SQR = \sum_{n=0}^{2N+1} abs(y1(n) - y2(n))^2$$

2° The minimization problem of the objective functions (6) ought to be solved by means of certain numerical procedure because of its discrete character. There are many optimization algorithms that deal with this problem. We use the simplest methods like simplex of Nelder and Mead (without constraints) and quasi-Newton's method with minimization along preferential direction (with constraints).

3° The effectiveness of the correction is evaluated by index **Q**. **Q** shows to what extent the dynamic error of measurements could be reduced as a consequence of using the 'blind' correction. The correction error is calculated from expression:

$$emax = \max |u(n) - y(n)| \text{ with respect to } (n)$$

4° The simulation starts from zero initial conditions for all system variables. The measurement system with following parameters of all components has been examined :

sensors: $k1 = k2 = 0,05$

$t_c1 \in [0, 30]$ $t_c2 \in [0, 30]$

$t_c1 \neq t_c2$ according to [1]

A/D – converters: 12-bit converters , $u_m = 4[V]$

the whole time of simulation: $2t_N = 400T_S$, $N = 200$

criteria of the selection ($\tau1$ and $\tau2$) as in (6)

Simulation studies have been carried out for each tuning criterion, separately. Furthermore, the large number of experiments have been performed for various t_c1 and t_c2. Results of the simulation are presented in fig.5 and 6, and also in the table 1.

TABLE 1. The results of numerical algorithms

SIMPLEX	ABS	SQR	MAX
Starting point (2,4)			
number of iterations	161	136	205
Q	302,2731	315,2512	182,3369
emax	0,05041	0,04833	0,08357
Starting point (3,3)			
number of iterations	220	139	193
Q	302,2731	315,2512	182,3369
emax	0,05041	0,04833	0,08357
NEWTON	**ABS**	**SQR**	**MAX**
Starting point (2,4)			
number of iterations	32	12	60
Q	302,2685	315,2979	182,3586
emax	0.05041	0.04833	0.08356
Starting point (3,3)			
number of iterations	28	15	45
Q	302,2734	315,1686	238,9796
emax	0.05041	0.04835	0.06376

Fig. 5 and 6 illustrate the index **Q** as a function of two arguments t_c1 and t_c2. Parameters of the corrector $\tau1$ and $\tau2$ that have been just tuned by the minimization algorithm. These results are concerned with the case when criterion SQR is applied and numerical calculations are performed by both Simplex and Newton methods, for starting point $\tau1=0$, $\tau2=0$. The shape of **Q** for both methods is broken (narrow valley) for arguments which have the same value $t_c1 = t_c2$. The effectiveness of the correction for these arguments is extremely low. As pointed in [1] the solution does not exist for those cases. The surfaces **Q** in fig 5,6 are almost the same. It means that the numerical procedure gives solutions convergent to the same points.

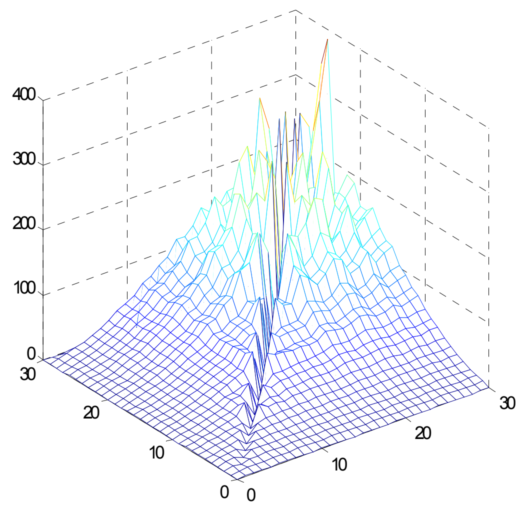

Fig.5. The graphic of effectiveness **Q(t_c1,t_c2)** for SQR criterion and Simplex method.

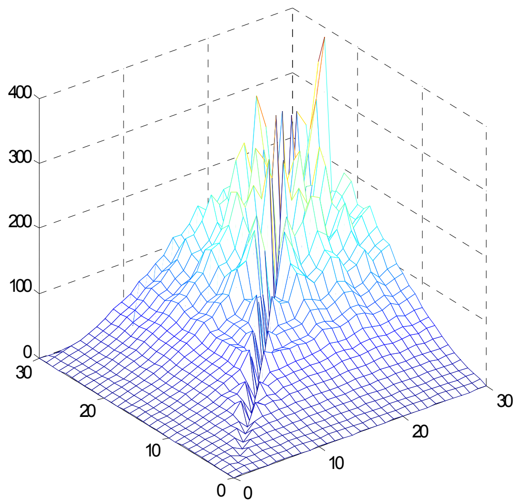

Fig.6. The graphic of effectiveness **Q(t_c1,t_c2)** for SQR criterion and Newton method.

The table 1. gives information about numerical algorithms. Both algorithms are always convergent to the accurate solutions. Simplex method is insensitive to the change of starting points. Newton methods result in turn, in a faster convergence. It can be seen that among different criteria ABS, SQR, MAX the best one is the SQR.

5. CONCLUSION

The suggested 'blind' method of the correction the dynamic error of measurements turns out to be efficient also for particular system with differentiating sensors. The verification of the method has been performed by simulation techniques. The corrector has been treated as a certain discrete-time object. The Tustin's method of discretization has been used. Corrector parameters for both channels have been tuned by means of optimization procedures.

The best correction i.e. the reconstruction of the transient signal u(t) has been achieved for the SQR criterion, larger t_c1 and t_c2 and the 12-bit A/D converter at least.

In general, the problem of finding the best correctors is well-conditioned from a numerical point of view. Very simple algorithms are convergent to the solutions in a very short time, about 20-iterations. However this method can't be used in real-time system because of a large amount of computation. It might work perfectly in off-line mode. The whole measurement system could be realized as a two hierarchical systems, the first one as a master for the proper correction and the second one as a slave for parameters tuning and working in the background.

Further, we are going to make practical experiment with implementation of the correction algorithm onto the DSP. Certain experiments with I-order inertial sensors have been finished successfully. Additional research for other type of differentiating sensors with more complex structures and described by several parameters are planned.

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