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A METHOD TO DETECT SUDDEN CHANGES IN SIGNALS

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Abstract – Measurements may contain disturbances like transients or abrupt level changes. Although the duration of these effects is often short, the computed results derived from the measurements may be heavily corrupted. In order to be able to analyse only the non-disturbed region of the signal, the detection of anomalies is needed. In this paper we introduce a method for detecting sudden changes applying windowed moving variance computation technique.

Keywords: signal processing, sudden changes, moving variance

1. INTRODUCTION

Measurements are sometimes disturbed by phenomena that cause the measured values to be inconsistent with the main part of data. These show up either short duration impulsive disturbances like transients or abrupt level changes generating sets of measurements lying distinctly apart from each other. Despite the short duration of these effects, the computed results derived from the measurements may be heavily corrupted. If no correction of the erroneous values is needed, the detection of errors is the only concern. In this paper we introduce a method applying windowed moving variance computation technique to the detection of both inconsistent signal values and abrupt level changes.

2. DETECTION OF VARIANCE CHANGES

The main principle of the method applying the computation of moving variance related test value is introduced in [1]. This algorithm applies standard deviation based limits in detecting short duration disturbances. However, the dynamic properties of signals are not taken into account properly enough in that procedure. To do this, the bandwidth of signals is estimated and the variance estimate itself is applied. The developed version of the algorithm is presented in the following.

The local variance is estimated by a windowed moving variance technique. In order to speed up calculations, the sequences of cumulative sum $s_\mathrm{c}[t]=\sum_{i=1}^{t}x[i]$ and cumulative square sum $q_\mathrm{c}[t]=\sum_{i=1}^{t}x^2[i]$ of signal x are computed. $t=1,\ldots,N$, N denotes the number of data values in signal x and $s_\mathrm{c}[0]=q_\mathrm{c}[0]=0$. The mean values and the mean square values for the intervals $(i,i+w]$, where $i=0\ldots N-w$ and w denotes the width of the moving window, are computed as

$$\mu_\mathrm{c}(i,i+w]=\frac{s_\mathrm{c}[i+w]-s_\mathrm{c}[i]}{w} \tag{1}$$

and

$$\psi_\mathrm{c}^2(i,i+w]=\frac{q_\mathrm{c}[i+w]-q_\mathrm{c}[i]}{w}. \tag{2}$$

The moving variance $\sigma_\mathrm{c}^2(i,i+w]$ is computed as

$$\sigma_\mathrm{c}^2(i,i+w]=\psi_\mathrm{c}^2(i,i+w]-\mu_\mathrm{c}^2(i,i+w]. \tag{3}$$

The values of the sequence σ_c^2 are expected to be χ_n^2-distributed, where $n=f_\mathrm{bw}\cdot(w-1)$ denotes the degrees of freedom of the distribution, and f_bw denotes the relative bandwidth of the signal x. If the bandwidth is not known, it has to be estimated, e.g. based on the PSD estimate of the signal (see section 2.1. as an example).

The data point $x_{\lceil i+w/2\rceil}$ is expected to be abnormal if

$$\sigma_\mathrm{c}^2(i,i+w]>T_\alpha, \tag{4}$$

where T_α is a threshold value and α denotes the level of significance of the distribution, i.e. [2]:

$$\Pr\left[nT_\alpha/E\left(\sigma_\mathrm{c}^2\right)>\chi_{n,\alpha}^2\right]=\alpha. \tag{5}$$

The threshold value T_α can be expressed with the help of the estimate of the expected value $E\left(\sigma_\mathrm{c}^2\right)$ as

$$T_\alpha=c_{n,\alpha}\cdot\hat{E}\left(\sigma_\mathrm{c}^2\right), \tag{6}$$

where the coefficient $c_{n,\alpha}$ is

$$c_{n,\alpha}=\frac{\chi_{n,\alpha}^2}{n}. \tag{7}$$

The chi-square value $\chi_{n,\alpha}^2$ for the specific values of n and α may be computed with the inverse chi-square probability density function. $\hat{E}\left(\sigma_\mathrm{c}^2\right)$ can not be directly estimated from the sequence due to the abnormal values that can bias the estimate. A more robust way is to apply the fact that the expected value $E\left(\chi_n^2\right)=n$ exists at a certain probability level $\Pr\left[n>\chi_n^2\right]=p_{E\left(\chi_n^2\right)}$. The probability $p_{E\left(\chi_n^2\right)}$ is a function of degrees of freedom and it can be estimated via an incomplete Gamma function as [3]

$$p_{E\left(\chi_n^2\right)}=Q\left(\frac{n}{2},\frac{n}{2}\right). \quad (8)$$

The expected value $\hat{E}\left(\sigma_c^2\right)$ is then estimated as the $\left(p_{E\left(\chi_n^2\right)}\cdot N_{\sigma_c^2}\right)$th sample of the sequence σ_c^2 sorted in ascended order. $N_{\sigma_c^2}$ denotes the number of valid samples of the sequence σ_c^2. Some of the original samples can be excluded from the valid samples on the grounds of external rules, e.g. based on á priori knowledge about process breaks, zero variance etc. This is needed in order to make the practical distribution meet the theoretical one.

2.1. Estimating the bandwidth of the signal

As noted earlier, the bandwidth f_{bw} of the signal affects the degrees of freedom *n* of the χ_n^2-distributed variance sequence σ_c^2, see (3). Because the measurements may contain different types of non-stationarities corrupting e.g. the PSD estimates, the estimation of the bandwidth has to be done without these effects. The noise structure of the signal is estimated based on the residual signal generated by removing the piecewise linear fit from the original signal. The generation of the piecewise linear curve is presented in section 2.1.1. As an example, an original signal, a fitted piecewise linear curve, and a residual signal are presented in Fig. 1. The bandwidth is estimated based on the PSD estimate of the residual signal. In the current implementation the spectrum is computed with WOSA method. In order to eliminate the effect of narrow-band (e.g. periodic) components in the spectrum the largest 2% of the components are not taken into account while estimating the maximum noise level S_m of the spectrum. The bandwidth f_{bw} is composed of all the frequency components being larger than $0,2\cdot S_m$. This is illustrated in Fig. 2.

2.1.1. Signal representation with piecewise linear curve

The fitting of piecewise linear curve is initialised by introducing evenly spaced break points in the signal and adjusting them by minimizing the square sum of the fitting error. In addition to the cumulative sums $s_c[t]$ and $q_c[t]$, the algorithm applies also the cumulative sum of time multiplied signal $u_c[t]=\sum_{i=1}^{t} ix[i]$. These sums are computed just once at the beginning of the function.

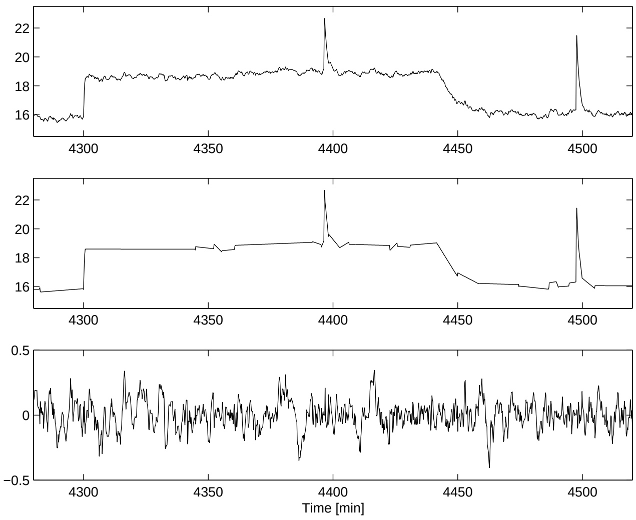

Fig. 1. The original signal (upper), fitted piecewise linear curve (middle), and the residual signal (lower).

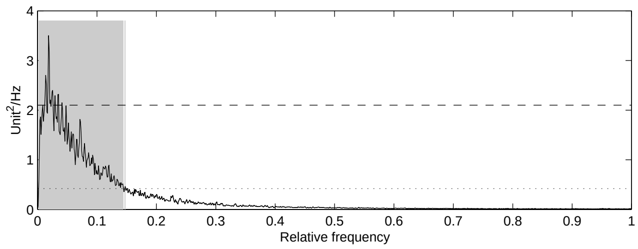

Fig. 2. The PSD estimate of the residual signal. The maximum noise level S_m (– –), the threshold level $0,2\cdot S_m$ ($\cdots$) and the estimated bandwidth f_{bw} (grey zone) are illustrated.

Three successive break points *i*, *j*, and *k*, where $i<j<k$, define two successive intervals [*i*, *j*) and [*j*, *k*). The total fitting error of the two intervals between [*i*, *k*) is calculated as

$$Q_{j,[i,k)}=\sum_{[i,j)}\left(x-\left(a_{[i,j)}t+b_{[i,j)}\right)\right)^2 + \sum_{[j,k)}\left(x-\left(a_{[j,k)}t+b_{[j,k)}\right)\right)^2, \quad (9)$$

where *j* varies in the interval [*i*+1, *k*–1), and $a_{[i,j)}$ or $b_{[i,j)}$ and $a_{[j,k)}$ or $b_{[j,k)}$ denote the first or zeroth order polynomial coefficients of the lines fitted to the intervals [*i*, *j*) and [*j*, *k*), respectively. The new adjusted break point is the point *j* producing the minimum value of $Q_{j,[i,k)}$. The fitting error, e.g. for the interval $Q_{[i,j)}$, is computed as

$$\begin{aligned} Q_{[i,j)} &= \sum_{[i,j)} \left[x^2[t] - 2x[t]\left(a_{[i,j)}t + b_{[i,j)}\right) + a_{[i,j)}^2 t^2 + 2a_{[i,j)}b_{[i,j)}t + b_{[i,j)}^2 \right] \\ &= q_c[j-1] - q_c[i-1] - 2\left[a_{[i,j)}\left(u_c[j-1] - u_c[i-1]\right) + b_{[i,j)}\left(s_c[j-1] - s_c[i-1]\right)\right] \\ &\quad + a_{[i,j)}^2 \frac{j}{6}\left(2j^2 + 3j + 1\right) + a_{[i,j)}b_{[i,j)}j(j+1) + b_{[i,j)}^2 j - a_{[i,j)}^2 \frac{i}{6}\left(2i^2 + 3i + 1\right) - a_{[i,j)}b_{[i,j)}i(i+1) - b_{[i,j)}^2 i. \end{aligned} \tag{10}$$

The latter term applies the sums of powers equations [4] that are defined for the first and second order cases as $\sum t[j] = j(j+1)/2$ and $\sum t^2[j] = j(j+1)(2j+1)/6$, respectively. Parameters $a_{[i,j)}$ and $b_{[i,j)}$ are solved by computing the partial derivatives of the equation (10) in respect to parameters in question, and setting the derivatives to zero. This results in the parameters $a_{[i,j)}$ and $b_{[i,j)}$ as follows:

$$a_{[i,j)} = \frac{4(j-i)\left(u_c[j-1] - u_c[i-1]\right) - 2\left(j(j+1) - i(i+1)\right)\left(s_c[j-1] - s_c[i-1]\right)}{2(j-i)\left(\frac{j}{3}\left(2j^2 + 3j + 1\right) - \frac{i}{3}\left(2i^2 + 3i + 1\right)\right) - \left(j(j+1) - i(i+1)\right)^2}, \tag{11}$$

$$b_{[i,j)} = \frac{1}{j-i}\left(s_c[j-1] - s_c[i-1] - \frac{a_{[i,j)}}{2}\left(j(j+1) - i(i+1)\right)\right). \tag{12}$$

The parameters for the interval $[j,k)$ are computed correspondingly. After these first adjustments the algorithm continues by iteratively trying to find out the optimum number of intervals with proper break point locations. This is done by merging intervals and inserting new break points when needed. For the merging of intervals the sum of the fitting errors of two successive intervals $[i, j)$ and $[j, k)$ is compared with the fitting error of a combined interval $[i, k)$ of these two intervals. The merging is done if the sum of the errors of the intervals is only slightly smaller than the error of the combined interval satisfying the following equation [5]:

$$\frac{Q_{[i,k)} - \left(Q_{[i,j)} + Q_{[j,k)}\right)}{\sum_{i=1}^{N_i-1} Q_{[i,j)}} < \frac{F_{\text{merge}}}{N - N_i}. \tag{13}$$

F_{merge} is a value of $F(1,\infty)$-distribution, N denotes the number of samples, N_i denotes the number of intervals, $N - N_i$ denotes the number of degrees of freedom for the testing, and $Q_{[\cdot)}$:s are the fitting errors of the intervals, see (10).

New break points are inserted if the fitting error of an interval is large, relative to the sum of the fitting errors of the intervals. If the following equation becomes true, a new break point between the points i and j is inserted:

$$\frac{Q_{[i,j)}}{\sum_{i=1}^{N_i-1} Q_{[i,j)}} > \frac{F_{\text{insert}}}{N - N_i}. \tag{14}$$

F_{insert} is an experimental value. After insertion the new break point is adjusted by minimizing the square sum of the fitting error. As a result a piecewise linear curve (see Fig. 1) is produced being divided into N_i intervals by the break points found.

2.2. *Manipulation of the detected samples*

As a result of the detection procedure introduced so far the abnormal samples are detected based on (4). However, there may be some inconsistencies like one or two undetected samples between detected abnormal regions. In order to fulfil the demands of practical applications, further manipulation of results is needed. The samples of the signal are classified to belong to two categories, i.e. abnormal or accepted regions. Samples are classified as abnormal if they are statistically detected as anomalies or there are too few of them to be classified as accepted. Samples are accepted if

- they are not detected as anomalies,
- they are statistically detected as anomalies in the first place based on the variance changes but the amplitude change during the region composed of the samples in question is relatively small, or,
- they are located in the vicinity of a large amplitude anomaly and, due to the detection technique sometimes applying relatively long windows, are forced to be detected as anomalies in the first place composing a region containing the reason for the detection in the middle and the samples in question on one or both sides of the region without amplitude changes large enough actually to be classified as abnormal.

The procedures are introduced in the following. The regions having totally relatively small amplitude changes are classified as accepted if

$$\frac{\max\left(x\left[n,n+N_r\right)\right)-\min\left(x\left[n,n+N_r\right)\right)}{2}<k_{T_1}\cdot T_{\text{med}}\,, \quad (15)$$

where $\left[n,n+N_r\right)$ denotes the anomaly interval of length N_r and T_{med} is a threshold value, defined by the square root of the smallest median of the sequences σ^2_{c,N_w} computed at various values of N_w

$$T_{\text{med}}=\sqrt{\min_j\left(\text{med}\left[\sigma^2_{\text{c},N_{w_j}}\right]\right)}\,, \quad (16)$$

where $j=1\ldots M_{N_w}$. M_{N_w} denotes the number of different values of N_w used, and $\text{med}\left[\sigma^2_{\text{c},N_{w_j}}\right]$ denotes the median of the sequence $\sigma^2_{\text{c}}\left(i,i+N_{w_j}\right]$ of the jth window of length N_{w_j} and $i=0,\ldots,N-N_{w_j}$. T_{med} is a rough estimate for the standard deviation of the signal. k_{T_1} is a coefficient defining the coverage of the test. By default, $k_{T_1}=4$.

Large amplitude anomalies cause small amplitude regions in the vicinity to be classified as abnormal. This region may be as large as half of the window length on both sides of an anomaly. These regions are classified as accepted if the amplitude changes fulfil the following conditions:

$$0,5\begin{pmatrix}\max\left(x\left[n,n+N_{r_1}\right)\right)\\ -\min\left(x\left[n,n+N_{r_1}\right)\right)\end{pmatrix}<k_{T_2}\cdot T_{\text{med}}\,, \quad (17)$$

$$0,5\begin{pmatrix}\max\left(x\left[n+N_r-N_{r_2},n+N_r\right)\right)\\ -\min\left(x\left[n+N_r-N_{r_2},n+N_r\right)\right)\end{pmatrix}<k_{T_2}\cdot T_{\text{med}}\,. \quad (18)$$

k_{T_2} is a coefficient defining the coverage of the test. By default, $k_{T_2}=3$. N_{r_1} and N_{r_2} denote the lengths of the non-anomaly intervals at the beginning and the end of the interval $\left[n,n+N_r\right)$.

3. EXAMPLE

In Fig. 3 an example of an original data sequence with three moving variance sequences are presented. Widths 2, 5 and 10 minutes are used for the moving windows, respectively. The threshold level for each processed subsequence is shown by dashed line. Fig. 4 presents the results of the detection procedure with the three different widths of moving windows. Especially the anomaly regions detected in the 10 min case contain small amplitude variations that are classified as accepted during the manipulation phase. The results after the manipulation phase are presented in Fig. 5. The small amplitude regions in the vicinity of anomalies are classified as accepted (light grey). The resulting abnormal samples are illustrated with dark grey.

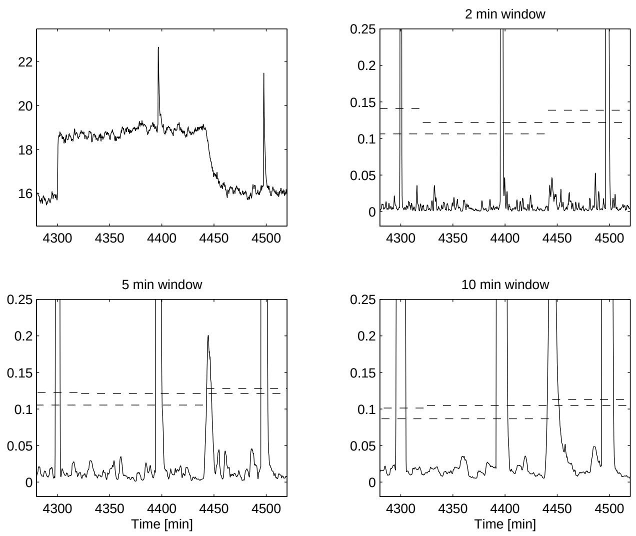

Fig. 3. Original data sequence with three moving variance sequences (2, 5 and 10 minutes windows). The threshold level for each processed subsequence is shown by dashed line.

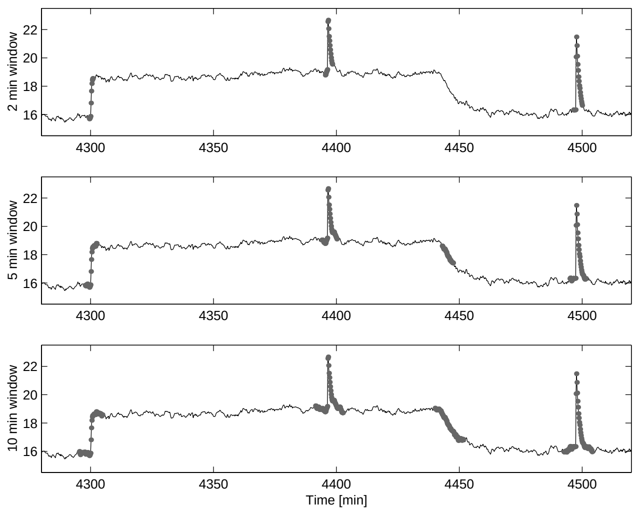

Fig. 4. The results after the detection procedure. The detected samples are illustrated with grey dots. Windows with widths of 2, 5 and 10 minutes were used.

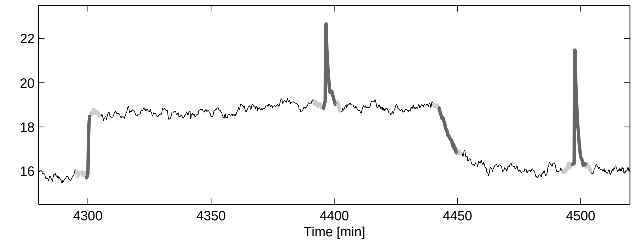

Fig. 5. Small amplitude regions (light grey) in the vicinity of large amplitude anomalies (dark grey) are classified as accepted

To illustrate the success of the estimation procedure for the bandwidth of the signal the histograms of the variance sequences σ^2_{c} produced with window widths of 2, 5, and 10 minutes are shown in Fig. 6. Due to the sampling period of

$\Delta t = 10\,\mathrm{s}$ the number of samples in a window equals to $N_{w_1} = 12$, $N_{w_2} = 30$, and $N_{w_3} = 60$, respectively. With $f_{\mathrm{bw}} = 0{,}1455$ the degrees of freedom result in $n_1 = 1{,}60$, $n_2 = 4{,}22$, and $n_3 = 8{,}58$, respectively. The theoretical χ_n^2 distributions are scaled so that their mean values are equal to the corresponding percentage levels of the histograms. These levels are illustrated with dashed lines.

The estimation of threshold value T_α with $\alpha = 10^{-6}$ is done based on (6) resulting in values $T_{\alpha,1} = 0{,}133$, $T_{\alpha,2} = 0{,}118$, and $T_{\alpha,3} = 0{,}102$, respectively. The threshold values produced by subsequential computation (see Fig. 3) match with these results. Without taking into account the effect of the bandwidth the theoretical χ_n^2-distribution with $n = N_w - 1$ is also shown in Fig. 6. The mean values of these distributions are illustrated with dotted lines. Applying the distributions with $\alpha = 10^{-6}$ the threshold values would be too small, i.e. $T_{\alpha,1} = 0{,}029$, $T_{\alpha,2} = 0{,}036$, and $T_{\alpha,3} = 0{,}040$, respectively. This would result in false identifications of abnormal samples that may be discovered based on Fig. 3.

4. CONCLUSIONS

A method for the detection of transients and sudden changes applying windowed moving variance computation technique is presented in this paper. Signal bandwidth estimation is applied in order to take into account the dynamic properties of the signals. In addition, an example of the detection procedure is given.

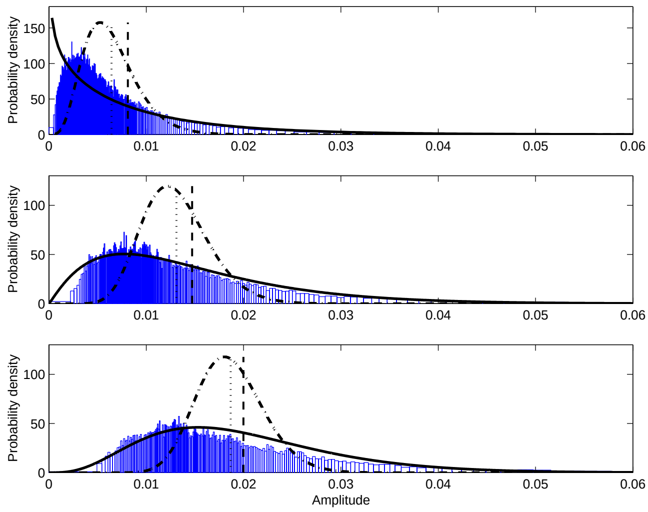

Figure 6. Histograms of variance sequences produced with window widths of 2 (upper), 5 (middle) and 10 (lower) minutes. Theoretical χ_n^2 amplitude distributions with degrees of freedom $n = f_{\mathrm{bw}} \cdot (N_w - 1)$ (—) and $n = N_w - 1$ (− ·) are shown with the corresponding mean values (– –) and (···), respectively.

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