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DETERMINATION OF EQUIVALENT PARAMETERS FOR INDUCTANCE STANDARDS

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Abstract − The confidence in self inductance base can be obtained by regular comparisons between particular metrological laboratories and by mutual comparisons of inductance standards within own laboratory.

Standard methods of inductance measurements in its applications don't give explicitly all of parameters of standard substitute circuit nor the accuracy which is requested by fundamental principles of inductance standard calibration. In order to ensure the adequate low measurement uncertainty, which suits these requirements, it is necessary to elaborate the procedure of standard calibration.

Keywords: inductance standards, calibration

1. INTRODUCTION

The coil inductance L is calculated from known coil resistance R and impedance Z, which is measured by U-I method at AC current of low frequency. Because of phenomenon of skin-effect, the wire resistance is bigger than at DC current, so it is necessary to verify its influence, in spite of low frequency.

In principle, the coil self-capacitance C can be determinated using the known coil geometry, but, as regards to enclosed construction of standards, the specification for that estimation is not accessible.

Hence the coil capacitance is determined by combining the method of resonant frequency measurement and iterative estimating from previous measured results of resistance and inductance.

2. COIL RESISTANCE MEASUREMENT

Coil resistance measurement is performed by U-I method at DC current in diagram according to Fig.1. The voltage on coil is measured by HP 3458A digital voltmeter, and Fluke 5700A calibrator is the current source in measurement circuit. In order to reduce the interference on voltage measurement, with known approximate resistance of coil, the current I should be chosen, such that the working power on coil is less than 1 mW. The shunting effect of the voltmeter can be neglected, since its internal resistance R_V is greater than 10 GΩ on DC ranges.

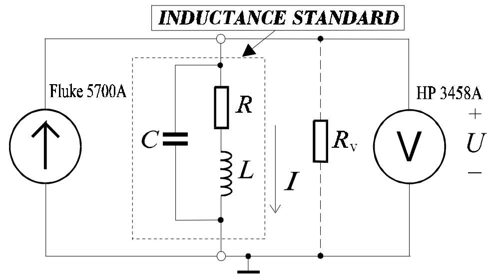

Fig. 1. Coil resistance measurement

The voltmeter shows arithmetic mean value and standard deviation of series with n measurements. The uncertainty of the measured resistance is determined by uncertainty u_U of measured voltage and uncertainty u_I of adjusted current:

$$u_R^2 = \sum_{i=1}^{N} \left\{ \frac{\partial R}{\partial X_i} \right\}^2 \cdot u_{X_i}^2 = \left\{ \frac{\partial \left(\frac{U}{I} \right)}{\partial U} \right\}^2 \cdot u_U^2 + \left\{ \frac{\partial \left(\frac{U}{I} \right)}{\partial I} \right\}^2 \cdot u_I^2 \quad (1)$$

Measurement uncertainty u_U consists of two components; uncertainty u_A (which is equal to *unreliability* $s_{\bar{U}}$ *of arithmetic mean value* of series with n measurement) and u_B (which is derived from absolute limits of error G_U of voltmeter and submitted to rectangular distribution):

$$u_A = s_{\bar{U}} = \frac{t \cdot s_U}{\sqrt{n}} \qquad u_B = \frac{G_U}{\sqrt{3}} \quad (2)$$

Measurement uncertainty of measured voltage is:

$$u_U = \sqrt{u_A^2(U) + u_B^2(U)} \quad (3)$$

Measurement uncertainty of adjusted current derived from *statistic limits of error* G_I of calibrator which is submitted to the Gauss distribution. With G_I the manufacturer gives its confidence level P, while the factor k was already taken into account of G_I (k = 2, by P = 95 %):

$$u_I = G_I \quad (4)$$

Now, the expression of complex measurement uncertainty assumes a form:

$$u_R = \sqrt{\frac{1}{I^2}\cdot\left[\left\{\frac{t\cdot s_U}{\sqrt{n}}\right\}^2 + \left\{\frac{G_U}{\sqrt{3}}\right\}^2\right] + \frac{U^2}{I^4}\cdot G_I^{\,2}} \quad (5)$$

3. COIL SELF-CAPACITANCE MEASUREMENT

Accurate determination of coil inductance is possible only if we know its capacitance. On coil substitute diagram (Fig. 2.) the coil capacitance is located in parallel to coil inductance, i.e. the diagram represents a parallel resonance circuit with self resonant frequency:

$$\omega_0 = \sqrt{\frac{L - CR^2}{L^2C}} \quad (6)$$

In principle, the resonant frequency can be determinated by seeking for maximal voltage on voltmeter (Fig. 3.). In this case the coil capacitance C, increased by voltmeter input capacitance $C_{V,}$ will be determined, and uncertainty of this capacitance will be several times bigger than the voltmeter capacitance uncertainty. Therefore, the coil capacitance C is measured by circuit according to Fig. 2. The resonant frequency is founded by sweeping the frequency of calibrator voltage, and by looking for zero of phase shift reading between coil current and coil voltage (by means of digital oscilloscope).

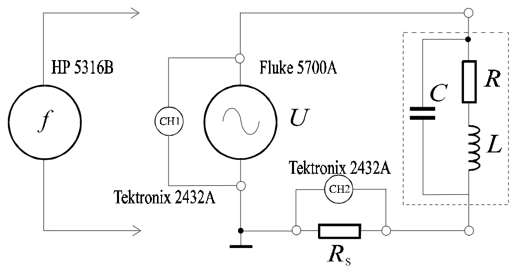

Fig. 2. Measurement of coil resonant frequency

The influences of input impedances of first channel (CH1) Tektronix 2432A digital oscilloscope (R_0 = 1 MΩ, C_0 = 15 pF) and HP 5316B counter (R_C = 1 MΩ, C_C = 40 pF), which are connected to voltage generator (Fluke 5700A calibrator), are irrelevant, while the input impedance of the second channel (CH2) is located in parallel to shunt R_S = 1 kΩ (this is optimum value with regard to oscilloscope sensitivity and influence to measurement result). The impedance $Z_r = \{(L - CR^2)/CR\}$ of coil is very large at the resonance point (in the order of 100 MΩ), so series impedance, which is consisted of R_0, C_0 and R_S in parallel, has neglected influence to current and its phase shift. In first estimate, the coil capacitance is given by:

$$C = \frac{L}{\omega_0^{\,2}L^2 + R^2} \quad (7),$$

where L is nominal value L_n of inductance standard, while the R is coil resistance R_f at resonant frequency ω_0. Measurement uncertainty of this capacitance follows from uncertainty of L_n and R_f . The value of inductance, which will (according to (10)) derive from these data, will give capacitance which is supposed to be quite accurate value of coil self-capacitance C .

Since the resonant frequency ω_0 of inductance standards has value in the order of 10^6 Hz, and the wire (of which the coil is made) has relatively spacious cross-section area, significantly increase of coil resistance, as a consequence of skin-effect, is present. However, on this frequencies the R_f is typically 10^4 times lesser than inductive reactance $X_L = \omega_0 L$, so it is discernible from (7) that R_f has negligibly influence on calculated C and associated u_C.

Uncertainty u_f of resonance frequency $\omega_0 = 2\pi f_0$, which is measured by HP 5316B precision counter, has value in the order of 10^{-2} Hz, but the component $(\partial C/\partial f)\cdot u_f$ is negligible compared to $(\partial C/\partial L)\cdot u_L$, which is simple to verify. This is true even in first estimate where u_L represents indicated uncertainty of nominal inductance. Therefore, the uncertainty u_C of calculated capacitance follows from expression:

$$u_C = \sqrt{\left(\frac{\partial C}{\partial L}\right)^2 u_L^{\,2} + \left(\frac{\partial C}{\partial f}\right)^2 u_f^{\,2} + \left(\frac{\partial C}{\partial R}\right)^2 u_R^{\,2}} \approx \left(\frac{\partial C}{\partial L}\right)\cdot u_L \quad (8)$$

4. COIL IMPEDANCE MEASUREMENT AND CALCULATION OF INDUCTANCE

Coil resistance measurement is performed by *U-I* method at AC current in diagram according to Fig. 3. The Fluke 5700A calibrator is the source of AC current in measurement circuit, and the voltage on coil is measured by HP 3458A digital voltmeter. The frequency of current must be chosen in range 30 Hz – 120 Hz, because of capacitance influences at higher frequencies, as well as voltmeter and calibrator errors at too low frequencies. Before the impedance measurement starts, it is necessary to perform the measurement of adjusted frequency by HP 5316B precision counter. At that point, the calibrator output is loaded by 10 Ω ballast resistor, and through this, the counter influence to impedance measurement is eliminated. The stability of adjusted frequency is better than 10^{-5}, so the possibility of errors, because of inaccuracy of frequency, is negligible.

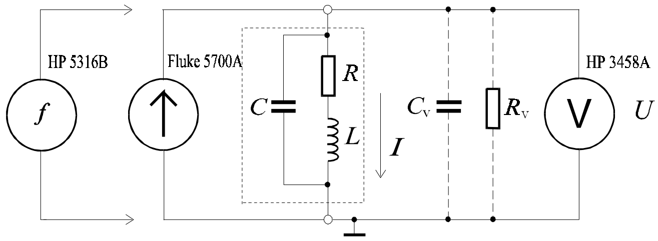

Fig. 3. Coil impedance measurement

Input impedance of HP 3458A voltmeter on AC range is represented with parallel combination of R_V = 1 MΩ and C_V ≈ 256 pF. Associated uncertainties of these values are u_R/R_V

$\approx 10^{-3}$ and $u_C/C_V \approx 10^{-1}$. Likewise, the coil capacitance uncertainty u_C, which is calculated in Chapter 3, is relatively wide. However, this isn't critical, since the addition parallel impedance of C, C_V and R_V is many times larger than impedance of R and L in series. The impedance, measured in circuit according to Fig. 3, is:

$$Z = \frac{U}{I} = \frac{\dfrac{R_V}{\sqrt{[\omega R_V (C_V + C)]^2 + 1}} \cdot \sqrt{(\omega L)^2 + R^2}}{\dfrac{R_V}{\sqrt{[\omega R_V (C_V + C)]^2 + 1}} + \sqrt{(\omega L)^2 + R^2}} \qquad (9)$$

The expression of coil inductance is:

$$L = \frac{1}{2\pi f} \cdot \sqrt{\left(\frac{Z R_V}{R_V - Z\sqrt{[\omega R_V (C_V + C)]^2 + 1}} \right)^2 - R^2} \qquad (10)$$

If the capacitance ($C_V + C$) is mark as C_S, then the complex measurement uncertainty of inductance is given by:

$$u_L^2 = \sum_{i=1}^{N} \left(\frac{\partial L}{\partial X_i} \right)^2 u_{X_i}^2 = \left(\frac{\partial L}{\partial Z} \right)^2 u_Z^2 + \left(\frac{\partial L}{\partial R_V} \right)^2 u_{R_V}^2 + \left(\frac{\partial L}{\partial C_S} \right)^2 u_{C_S}^2 + \left(\frac{\partial L}{\partial R} \right)^2 u_R^2 + \left(\frac{\partial L}{\partial f} \right)^2 u_f^2 \quad (11),$$

and partial derivations are following from (10):

$$\frac{\partial L}{\partial Z} = \frac{1}{\omega^2 L} \cdot \frac{Z R_V^{\,3}}{\left(R_V - Z\sqrt{(\omega R_V C_S)^2 + 1} \right)^3} \approx \frac{Z}{\omega^2 L} \qquad (12)$$

$$\frac{\partial L}{\partial R_V} = \frac{1}{\omega L} \cdot \frac{\dfrac{Z^3 R_V}{\sqrt{(\omega R_V C_S)^2 + 1}}}{\left(R_V - Z\sqrt{(\omega R_V C_S)^2 + 1} \right)^3} \approx \frac{1}{\omega L} \cdot \frac{Z^3}{R_V^{\,2}} \qquad (13)$$

$$\frac{\partial L}{\partial C_S} = \frac{1}{L} \cdot \frac{\dfrac{Z^3 R_V^{\,4} C_S}{\sqrt{(\omega R_V C_S)^2 + 1}}}{\left(R_V - Z\sqrt{(\omega R_V C_S)^2 + 1} \right)^3} \approx \frac{1}{L} \cdot Z^3 R_V C_S \qquad (14)$$

$$\frac{\partial L}{\partial R} = \frac{1}{\omega} \cdot \frac{1}{\sqrt{\left(\dfrac{Z R_V}{R_V - Z\sqrt{(\omega R_V C_S)^2 + 1}} \right)^2 - R^2}} \cdot (-R) = -\frac{R}{\omega^2 L} \qquad (15)$$

$$\frac{\partial L}{\partial f} = \frac{1}{f} \cdot \left(\frac{Z^3 R_V^{\,4} C_S^{\,2}}{L\left(R_V - Z\sqrt{(\omega R_V C_S)^2 + 1} \right)^3 \cdot \sqrt{(\omega R_V C_S)^2 + 1}} - L \right) \approx -\frac{L}{f} \qquad (16)$$

The measurement uncertainty of coil impedance Z is the identical as for coil resistance:

$$u_Z = \sqrt{\frac{1}{I^2} \cdot \left[\left\{ \frac{t \cdot s_U}{\sqrt{n}} \right\}^2 + \left\{ \frac{G_U}{\sqrt{3}} \right\}^2 \right] + \frac{U^2}{I^4} \cdot G_I^{\,2}} \qquad (17)$$

Uncertainty of parallel combination $C_S = (C_V + C)$ is equal to:

$$u_{C_S} = \sqrt{u_{C_V}^{\,2} + u_C^{\,2}} \qquad (18)$$

5. MEASUREMENT RESULTS

A) Inductance standards data:

Standard	*1**	*2***
Nominal inductance L_n / H	0.1	0.01
Nominal resistance R_n / Ω	16	1.3
Accuracy class	0.1	0.1

Table 1. Inductance standards data

* *КАТУШКА ИНДУКТИВНОСТИ* P547, *N* 1588, CCCP
** *КАТУШКА ИНДУКТИВНОСТИ* P547, *N* 1287, CCCP

B) Coil resistance measurements:

Standard	*1*	*2*
I / A	$5\cdot10^{-3}$	$25\cdot10^{-3}$
U / V	$81.19553\cdot10^{-3}$	$33.80429\cdot10^{-3}$
R / Ω	**16.239**	**1.352**
s_U / V	$0.51\cdot10^{-6}$	$0.59\cdot10^{-6}$
G_U / V	$1.73\cdot10^{-6}$	$1.30\cdot10^{-6}$
u_U / V	$1.03\cdot10^{-6}$	$8.02\cdot10^{-7}$
G_I / A	$3.3\cdot10^{-7}$	$2.3\cdot10^{-6}$
u_R / Ω	**$1.09\cdot10^{-3}$**	**$1.28\cdot10^{-4}$**
u_R / ppm	**67**	**95**

Table 2. Results of coil resistance measurements

C) Coil capacitance in the first estimation:

Standard	*1*	*2*
L_n / H	0.1	0.01
R /Ω	16.239	1.352
f_0 / Hz	$85.56\cdot10^{3}$	$238.2\cdot10^{3}$
C' / F	**$34.6\cdot10^{-12}$**	**$44.6\cdot10^{-12}$**
u_L' / H	10^{-4}	10^{-5}
u_C' / F	**$3.5\cdot10^{-14}$**	**$4.5\cdot10^{-14}$**

Table 3. The coil capacitance estimated from nominal value of L_n and accurate value of R

D) Coil inductance measurements:

Standard	*1*	*2*
I / A	$5\cdot10^{-3}$	$25\cdot10^{-3}$
U / V	$354.9835\cdot10^{-3}$	$100.19439\cdot10^{-3}$
F / Hz	109.9973	59.9985
Z / Ω	70.9967	4.00778
L / mH	**100.0094**	**10.00792**
s_U / V	$2.77\cdot10^{-6}$	$2.25\cdot10^{-6}$
G_U / V	$4.48\cdot10^{-6}$	$2.70\cdot10^{-6}$
u_U / V	$2.89\cdot10^{-6}$	$1.88\cdot10^{-6}$
G_I / A	$1.05\cdot10^{-6}$	$7.0\cdot10^{-6}$
u_f / Hz	$1.40\cdot10^{-5}$	$5.08\cdot10^{-6}$
u_L / H	**$2.28\cdot10^{-5}$**	**$3.19\cdot10^{-6}$**
u_L / ppm	**228**	**319**

Table 4. Results of coil inductance measurements

E) Determining of coil capacitance by iteration:

Standard	*1*	*2*
L / mH	100.0094	10.00792
R / Ω	16.239	1.352
f_0 / Hz	$85.5579 \cdot 10^3$	$238.1943 \cdot 10^3$
C / F	**$34.6 \cdot 10^{-12}$**	**$44.6 \cdot 10^{-12}$**
u_L / H	$2.28 \cdot 10^{-5}$	$3.19 \cdot 10^{-6}$
u_C / F	$8 \cdot 10^{-15}$	$1.5 \cdot 10^{-14}$
u_C / ppm	**231**	**336**

Table 5. The coil capacitance C derived from accurate values of L and R

The voltage U in tables represents the arithmetic mean value of series with 20 measurement results; s_U is standard deviation of U, G_U and G_I are absolute limits of errors defined by equipment manufacturer, and u_U is voltage uncertainty at confidence level of P = 95,4 % (2σ; t = 2,09). The uncertainty u_f of frequency (which is measured by counter) is derived from absolute limits of errors G_f (submitted to rectangular distribution), consists of two components: resolution and time base error, both given from manufacturer.

6. CONCLUSION

Measured inductance of standards 0.1 H and 0.01 H, differ from nominal values by + 9.4 μH and +7.9 μH respectively, and both results are within limits of error that is defined by accuracy class 0.1.
The measurement results with associated limit uncertainties, in comparison with nominal values and its limits of error, are graphically given in Fig. 4 and Fig. 5.

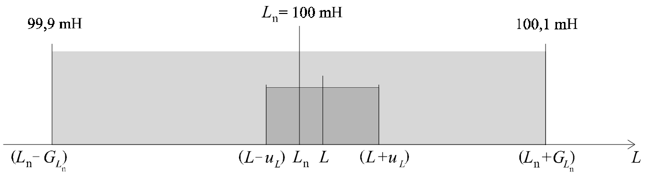

Fig. 4. Measurement results for 0.1 H inductance standard

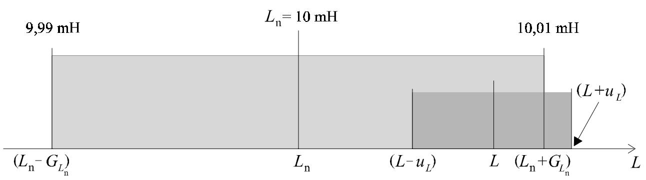

Fig. 5. Measurement results for 0.01 H inductance standard

Determination of standard self-capacitance by iteration demonstrates that it is quite enough to take its nominal value of inductance L_n into account, since the result is the same as in the first step of iteration with measured L.
In order to get measurement results of the highest confidence and to verify its repeatability, it is necessary to revise all measurements within specific time interval. The obtained measurement results will indicate a certain dispersion, while the measurement uncertainty will not change essentially.

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