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A COMPARATIVE EVALUATION OF SOME LMS-BASED ALGORITHMS FOR CALCULATING OF IMPEDANCE COMPONENTS IN THE SAMPLING SENSOR INSTRUMENT

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Abstract – The properties of some algorithms based on digital signal processing for the impedance components evaluation in circuits with sampling sensor have been analysed. It is supposed that the voltage and current are sampled synchronously to the fundamental frequency of the generated sinusoidal signal. Two groups of fitting sine wave algorithms, which are based on the least mean square (LMS) technique, have been described. The first one reconstructs indirect measurement method. The second group of algorithms estimates the unknown impedance components by direct method. In all these algorithms to simplify the calculations one can use different form of input matrix. The uncertainty's propagation by described algorithms can be analysed by means of covariance matrix. In order to verify the performance of the considered algorithms (e.g., accuracy, estimator bias and convergence) the Monte Carlo simulations are realised in MATLAB. It is shown that those algorithms provide minimisation of uncertainty for selected number of samples and phase angles. The influence of the quantization error of the AD converters and jitters of the sampling time upon the uncertainty of the processing results of described algorithms have been carried out.

Keywords impedance measurements, LMS method, and uncertainty analysis.

1. INTRODUCTION

For many years, a lot of papers [1-5] have presented various types of circuits, techniques, and instruments for impedance measurement. In many circuits high accuracy is obtained using sampling sensor and digital algorithm for measuring unknown impedance. In these circuits the components of measuring impedance are calculated on the grounds of values of voltage drop *u(t)* and current *i(t)* flowing into the unknown impedance Z_x, which are simultaneously sampled by the dual channel data acquisition system. Some of used algorithms are based on the least mean square (LMS) method [1,3,5].

In this paper two groups of fitting sine wave algorithms, which are based on the LMS technique, have been analysed. The propagation of uncertainty throughout the algorithm is studied in some detail. The key parameters that affect the performance of the algorithm for short records of samples are derived.

2. LEAST SQUARE FIT TO SINE WAVE ALGORITHMS

Both sampled signals *u(t)* and *i(t)* can be written in the time domain as

$$u(t) = U_m \sin(\omega t + \psi_u),$$
$$i(t) = I_m \sin(\omega t + \psi_i), \tag{1}$$

where U_m, ψ_u, and I_m, ψ_i indicate the unknown amplitude and phase angle of the first and the second signal, respectively. It is supposed that signals described in (1) are sampled with sampling frequency f_s synchronously to the fundamental frequency f_g of the generated sinusoidal signal. Assume that the data record $\{u(n)\}$ and $\{i(n)\}$ contain the sequence of N samples for each signal, taken at time instants nT_s (n=0,1,...,N-1). It is further assumed that both signals can be modelled by

$$y(n) = X_c \sin \omega n T_s + X_s \cos \omega n T_s, \tag{2}$$

where X_c and X_s are unknown constants [5-6].
Expression (2) can be written in the matrix form as follows

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(n) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \sin \omega T_s & \cos \omega T_s \\ \vdots & \vdots \\ \sin \omega n T_s & \cos \omega n T_s \\ \vdots & \vdots \\ \sin \omega (N-1) T_s & \cos \omega (N-1) T_s \end{bmatrix} \begin{bmatrix} X_c \\ X_s \end{bmatrix} = \mathbf{AX}. \tag{3}$$

Estimates of the unknown parameters in **X** are obtained by the least mean squares method. Equation (3) is an overdetermined (if N>2) set of linear equations, with the LMS solution given by

$$\hat{\mathbf{X}} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{y}. \tag{4}$$

Having estimated the vectors of orthogonal components of voltage and current, then the components of impedance (for series equivalent of the impedance Z_x.) can be obtained as

$$\hat{R}_x = \frac{\hat{U}_c \hat{I}_c + \hat{U}_s \hat{I}_s}{\hat{I}_c^2 + \hat{I}_s^2}, \qquad \hat{X}_x = \frac{\hat{U}_s \hat{I}_c - \hat{U}_c \hat{I}_s}{\hat{I}_c^2 + \hat{I}_s^2}, \tag{5}$$

where $\hat{U}_c$, $\hat{I}_c$ and $\hat{U}_s$, $\hat{I}_s$ indicate the estimates of real and imaginary components of voltage and current, respectively. The number N of recorded samples per channel is connected to the number M of samples per period T_g by

$$T_m = NT_s = \frac{N}{M} T_g \ , \qquad (6)$$

where T_m indicates the acquisition time, N and M are natural numbers.

It is possible to start the sampling process with other initial conditions than showed by (3). For arbitrary fixed initial phase angle α a new input matrix $\mathbf{A}_\alpha$ can be written as follows

$$\mathbf{A}_\alpha = \mathbf{A}\mathbf{T}^\mathbf{T} , \qquad (7)$$

where $\mathbf{T}$ is the transformation matrix [7].

However, the LMS solution of the unknown parameters in $\mathbf{X}$ is still obtained by (3), putting $\mathbf{A}$=$\mathbf{A}_\alpha$. Algorithm described above reconstructs indirect measurement method.

On the other hand, the voltage $u(t)$ and the current $i(t)$ can be described in terms of equation as follows

$$u(n) = |Z_x| I_m \sin(\omega n T_s + \psi_i + \varphi), \qquad (8)$$

where $|Z_x| = \sqrt{R_x^2 + X_x^2}$, $\varphi = arctan \frac{X_x}{R_x} = \psi_u - \psi_i$.

Expression (8) can be expanded similar to (2) as

$$u(n) = R_x I_m \sin(\omega n T_s + \psi_i) + X_x I_m \cos(\omega n T_s + \psi_i) . \qquad (9)$$

Considering that the sequence of current samples $\{i(n)\}$ satisfies (1), equation (9) can be written as

$$u(n) = R_x i(n) + X_x i\left(n + \frac{M}{4}\right). \qquad (10)$$

In this case, to create the input matrix $\mathbf{A}$, the sequence of current samples $\{i(n)\}$ and the sequence shifted by $M/4$ corresponding to it, can be used

$$\mathbf{A}_I = \begin{bmatrix} i(\psi_i) & i\left(\psi_i + \frac{\pi}{4}\right) \\ i(\omega T_s + \psi_i) & i\left(\omega T_s + \psi_i + \frac{\pi}{4}\right) \\ \vdots & \vdots \\ i(n\omega T_s + \psi_i) & i\left(\omega n T_s + \psi_i + \frac{\pi}{4}\right) \\ \vdots & \vdots \\ i(\omega(N-1)T_s + \psi_i) & i\left(\omega(N-1)T_s + \psi_i + \frac{\pi}{4}\right) \end{bmatrix} . \qquad (11)$$

The values of impedance components can be calculated directly from (4). In this case the input matrix is undetermined; the error matrix and the input matrix $\mathbf{A}_I$ are correlated. Thus, by a LMS method biased estimates of the impedance components are obtained. Estimator obtained on its basis is biased due to the noise that distorts the results of the measurement.

In order to eliminate the bias of estimated components of impedance, the instrumental variable method (IVM) is proposed [9]. On account of application this method, the vector of estimates $\hat{\mathbf{X}}$ can be written as follows

$$\hat{\mathbf{X}} = (\mathbf{Z}^\mathbf{T}\mathbf{A})^{-1}\mathbf{Z}^\mathbf{T}\mathbf{y} \qquad (12)$$

where $\mathbf{Z}$ is a Nx2 dimensional matrix, whose elements are uncorrelated with the error matrix and strong correlated (asymptotically) with the sequence of current samples in matrix $\mathbf{A}_I$. If the assumptions presented above are satisfied, the estimator obtained by (12) is consistent. There are some methods that optimise the selection of elements values of the instrumental variable matrix. They provide the minimisation of the covariance matrix [9]. Matrix $\mathbf{A}_I$ contains in its columns the sequences of current samples in time quadrature. Therefore two kinds of matrixes $\mathbf{Z}$ containing orthogonal elements in its columns have been proposed. The first one contains values of sine and cosine function at successive time instants nT_s, as in the input matrix in (3). In the second one - the first order Walsh's functions are used respectively [10]

$$\mathbf{Z}_{Walsh} = \begin{bmatrix} 0 & 1 \\ \mathrm{sal}_1\, \omega T_s & \mathrm{cal}_1\, \omega T_s \\ \vdots & \vdots \\ \mathrm{sal}_1\, \omega n T_s & \mathrm{cal}_1\, \omega n T_s \\ \vdots & \vdots \\ \mathrm{sal}_1\, \omega(N-1)T_s & \mathrm{cal}_1\, \omega(N-1)T_s \end{bmatrix} , \qquad (13)$$

where $\mathrm{sal}_1(t)$ and $\mathrm{cal}_1(t)$ indicate uneven and even first order Walsh's functions (with the period T_g).

The instrumental variable matrix $\mathbf{Z}$ can be described as (2) or (14), independently of the value of initial phase of current, because the change in order or other non-singular linear transformation of the instrumental variable has no influence on the value of estimator $\hat{\mathbf{X}}$ [10].

The application of the Walsh's functions enables significant simplification of calculations, which are necessary to determine the estimates of impedance components. Most of multiplication operations occurring in algorithm (13) can be replaced by summing of successive samples of voltage and current with an adequate sign dependently on the value of the Walsh's function

$$\mathbf{Z}_{Walsh}^\mathbf{T}\mathbf{A} = \begin{bmatrix} \sum_{n=0}^{N-1} \mathrm{sal}_1\, \omega n T_p\ i(n) & \sum_{n=0}^{N-1} \mathrm{sal}_1\, \omega n T_p\ i\left(n + \frac{M}{4}\right) \\ \sum_{n=0}^{N-1} \mathrm{cal}_1\, \omega n T_p\ i(n) & \sum_{n=0}^{N-1} \mathrm{cal}_1\, \omega n T_p\ i\left(n + \frac{M}{4}\right) \end{bmatrix} , \qquad (14)$$

$$\mathbf{Z}_{Walsh}^\mathbf{T}\mathbf{y} = \begin{bmatrix} \sum_{n=0}^{N-1} \mathrm{sal}_1\, \omega n T_p\ u(n) \\ \sum_{n=0}^{N-1} \mathrm{cal}_1\, \omega n T_p\ u(n) \end{bmatrix} . \qquad (15$$

3. NUMERICAL SIMULATIONS

In order to verify the performance of the considered algorithms (e.g., accuracy, estimator bias and convergence) the Monte Carlo simulations in MATLAB have been realised. Simulations of the influence of the quantization error of the AD converter and jitters of the sampling time upon the uncertainty of the processing results in presented algorithms have been carried out. The values of parameters given in [1,2,5] were used to test the proposed algorithms. Thus, the frequency of the sinusoidal forcing signal was assumed equal to f_g=1kHz and the maximum value of this voltage was set to 20V. For the simulation, according to [1,5], the 16-bit AD converters have been set up. Additionally, in order to simplify the considerations, it was established that independently of the values of measured impedance, the ranges of the AD converters were adequate for the amplitudes of the sampled signals. Influence of jitters of sampling time of measured signals has been taken into consideration due to adding random (uniformly distributed) noise. The amplitude of this noise was fixed and set to

$\pm 0{,}001\% T_s$. The sampling frequency was set to f_s=48kHz, as given in [1,2,5], with the length of data window $T_m=(4....48)T_s$. The investigations have been carried out with data window that cannot exceed one cycle of the processed signals. Simulations were based on 1000 independent realisations.

The effect of the number of samples N on the estimates of impedance components has been tested for two different impedance, which are defined by the impedance phase angle. The first one is inductive impedance with equal value of both components; the second one is a capacitive impedance with strongly prevailing reactance.

As the measure of expected value, the relative error of components estimation has been assumed

$$\delta_R = \frac{1}{1000}\frac{\sum_{m=1}^{1000}\hat{R}_m - R_x}{R_x} \quad , \quad \delta_X = \frac{1}{1000}\frac{\sum_{m=1}^{1000}\hat{X}_m - X_x}{X_x}, \quad (16)$$

where: $\hat{R}_m, \hat{X}_m$ denote the components estimates in the mth realisation, R_x, X_x indicate adequate values of components assumed for simulation of voltage and current waves.

The dispersion of the estimation results was evaluated by calculating the estimators of the relative standard deviation

$$s_R = \sqrt{\frac{\sum_{m=1}^{1000}\left(\hat{R}_m - R_x\right)^2}{999R_x^2}} \quad , \quad s_X = \sqrt{\frac{\sum_{m=1}^{1000}\left(\hat{X}_m - X_x\right)^2}{999X_x^2}} \quad . \quad (17)$$

For algorithm that calculates impedance components using LMS method (4) and two kinds of the instrumental variable methods (12), the characteristics of relative errors of components estimators δ_R, δ_X and relative standard deviations s_R, s_X as a function of relative length of data window T_m/T_g have been determined. In addition, it is necessary to consider, that in accordance with (1) the sampling begins with the following initial conditions

$$u(0) = U_m \sin\psi_u \,, \qquad i(0) = I_m \sin\psi_i \,. \quad (18)$$

As one can expect, the uncertainty of the processing results is dependent on the initial phase of the analysed signals.

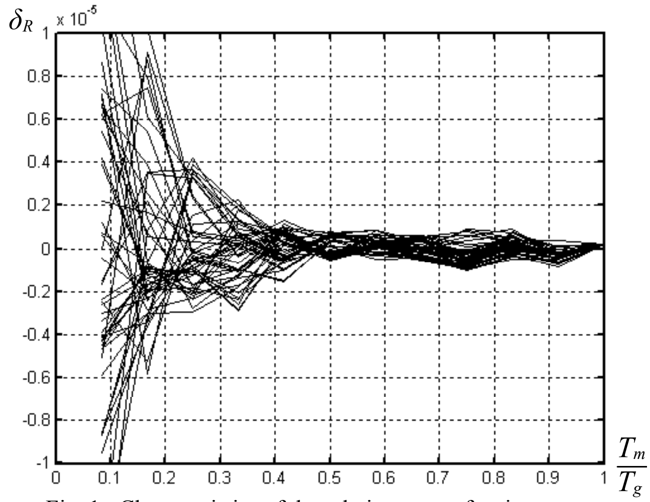

Fig. 1. Characteristics of the relative error of resistance as a function of relative length of data window T_m/T_g for ψ_i=0...2π, Z_x=(1000+j1000)Ω and LMS method

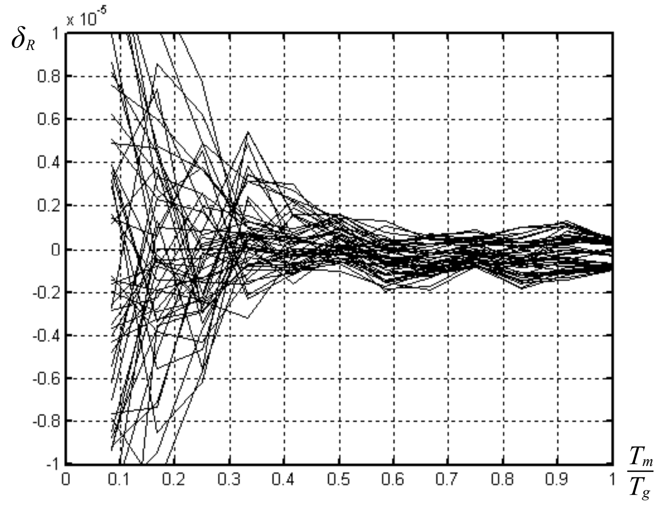

Fig. 2. Characteristics of the relative error of resistance as a function of relative length of data window T_m/T_g for ψ_i=0...2π, Z_x=(1000+j1000)Ω and IVM with the Walsh's functions

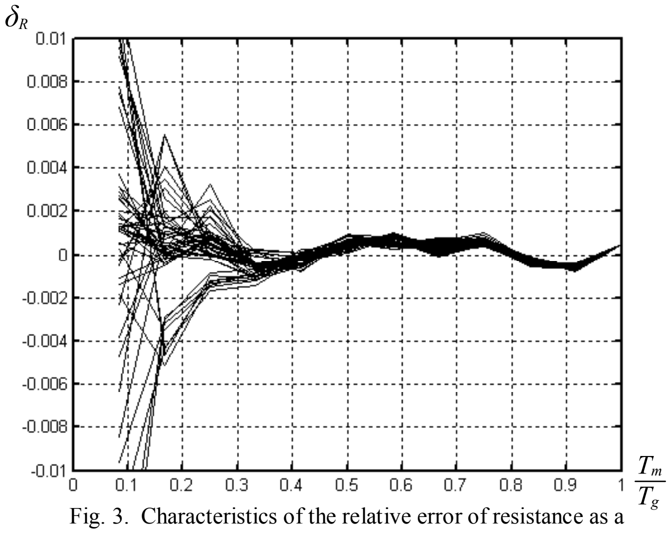

Fig. 3. Characteristics of the relative error of resistance as a function of relative length of data window T_m/T_g for ψ_i=0...2π, Z_x=(1-j1000)Ω and LMS method

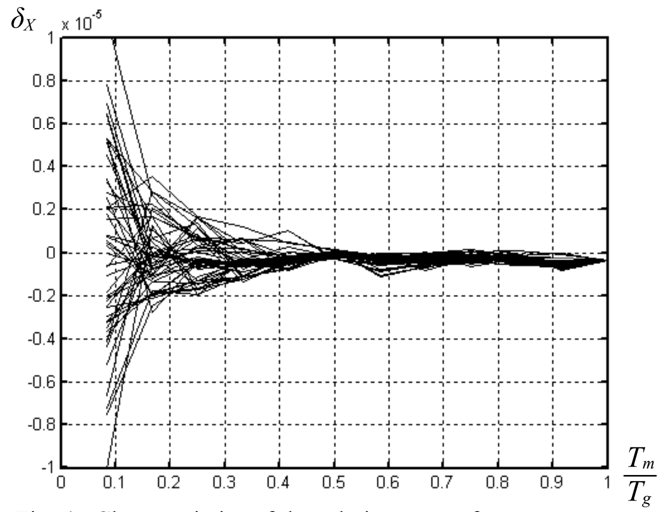

Fig. 4. Characteristics of the relative error of reactance as a function of relative length of data window T_m/T_g for ψ_i=0...2π, Z_x=(1-j1000)Ω and LMS method

Therefore, the value of initial phase was changed in the range of round angle. Figs.1-5 show the values of relative errors of the resistance estimator as a function of T_m/T_g for 48 values of initial phase ψ_i (0...2π) and algorithms: LMS, and IVM with Walsh's functions. Characteristics for algorithm IVM with trigonometric functions and algorithm that realised LMS method behave in the similar way. For impedance with strongly prevailing reactance the characteristics for prevailing component confirm the outcome of the simulations for impedance with equal value of both components.

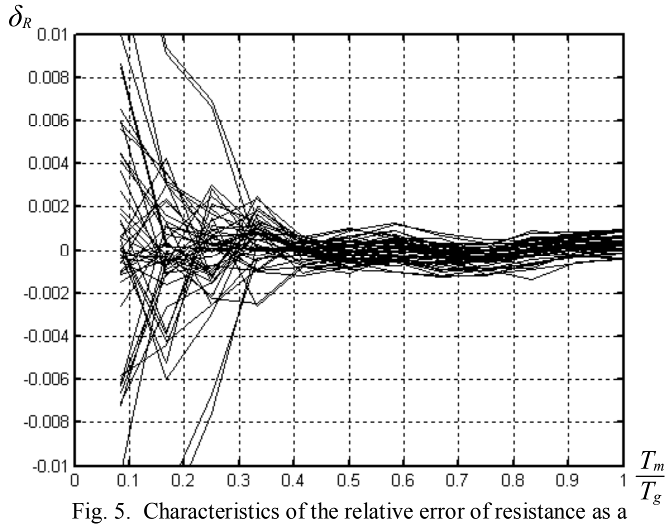

Fig. 5. Characteristics of the relative error of resistance as a function of relative length of data window T_m/T_g for ψ_i=0...2π, Z_x=(1-*j*1000)Ω and IVM with the Walsh's functions

The simulation results show that all analyzed algorithms provide convergence of impedance components estimates for the length of data window longer than 0,4 *Tg*. In this case algorithm IVM, based on the Walsh's functions, reveals only slightly greater estimation errors. The estimation errors of residual component increase in the same proportion as the values of impedance components remain. The estimation error of prevailing component does not change significantly. Figs.6-9 show the relative empirical standard deviation of impedance components as a function of relative length of data window T_m/T_g respectively.

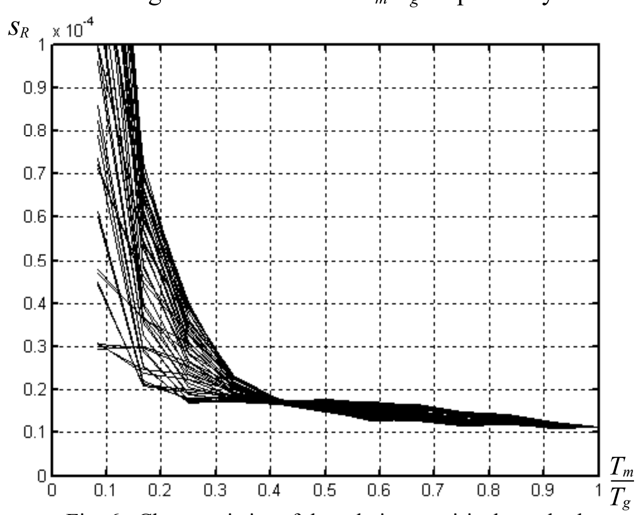

Fig. 6. Characteristics of the relative empirical standard deviation of resistance as a function of relative length of data window T_m/T_g for ψ_i=0...2π, Z_x=(1000+*j*1000)Ω and LMS method

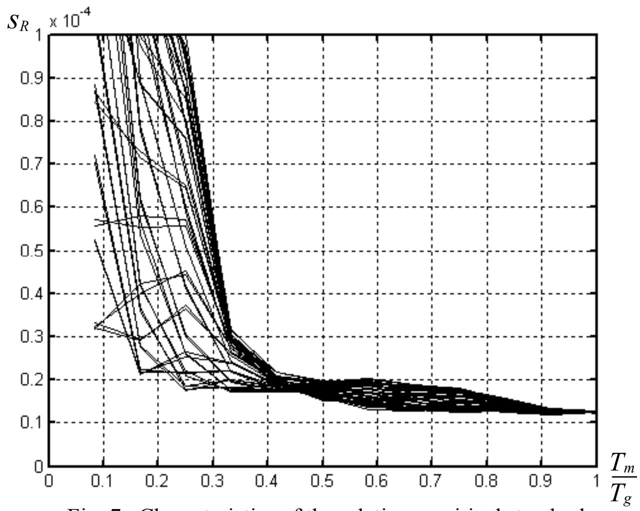

Fig. 7. Characteristics of the relative empirical standard deviation of resistance as a function of relative length of data window T_m/T_g for ψ_i=0...2π, Z_x=(1000+*j*1000)Ω and IVM with the Walsh's functions

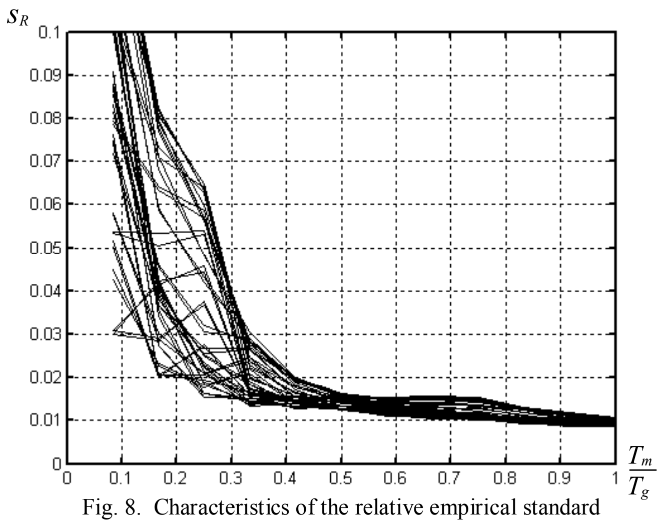

Fig. 8. Characteristics of the relative empirical standard deviation of resistance as a function of relative length of data window T_m/T_g for ψ_i=0...2π, Z_x=(1-*j*1000)Ω and IVM with the Walsh's functions

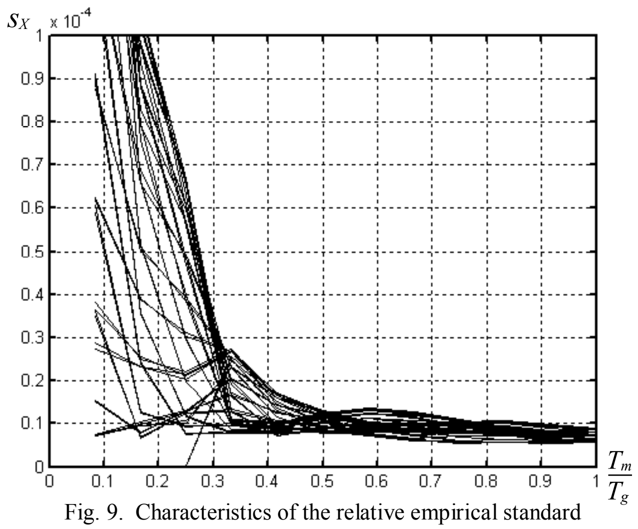

Fig. 9. Characteristics of the relative empirical standard deviation of reactance as a function of relative length of data window T_m/T_g for ψ_i=0...2π, Z_x=(1-*j*1000)Ω and IVM with the Walsh's functions

Results dispersion of components estimation for data windows longer than 0,4 *Tg* does not actually depend on the type of algorithm. The empirical standard deviation for residual component increased 1000 times, in the similar way as in case of the estimation errors. For some values of the initial phase, satisfactory dispersion and estimation errors for $T_m<0,4T_g$ can be obtained.

4. CONCLUSIONS

The comparison of performance (in terms of the impedance components estimation error) reveals that in several simulation conditions the algorithms, which used the instrumental variable method, are characterised by quite satisfactory properties. All algorithms have ensured the reduction of the measurement uncertainty for number of samples comparable with number of samples per period. For selected values of initial phase (different for each of impedance components), the uncertainty has been minimised. For impedance with known phase angle it is possible to select the value of initial phase to obtain minimum of uncertainty for data window shorter than $0,4T_g$. For impedance with strongly prevailing reactance, the estimation error and the empirical standard deviation for residual component increase in the same proportion as the values of impedance components remain. Essential advantage of the proposed algorithms is the simple form of expression, which describes the searched values of impedance components, especially when the instrumental variable method with the first order Walsh's functions in the matrix is used. Therefore, for an easy implementation, a digital signal processor can be applied.

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