XVII IMEKO World Congress
Metrology in the 3rd Millennium
June 22–27, 2003, Dubrovnik, Croatia

THE SENSITIVITY OF THE PROBABILITY DENSITY FUNCTION OF THE QUANTIZATION ERROR TO THE FLUCTUATION OF THE AMPLITUDE OF A SINUSOIDAL SIGNAL UNDERGOING ANALOG-TO-DIGITAL CONVERSION

Anna Domańska[1], *Adam Derkowski*[2]

[1]Institute of Electronics and Telekommunications, Poznań University of Technology, Poznań, Poland
[2]Department of Wood-Based Materials, Agricultural University of Poznań, Poznań, Poland

Abstract – The characteristics of analog-to-digital converters (A/D converters) are established and assessed based on the results of the conversion of a sinusoidal signal. This method of measurement has been selected because it is relatively easy to generate sinusoidal signals with the required good parameters. Among the criteria of the measurement of an actual a-d conversion is the probability density function (PDF) of the quantization error. Unfortunately, the distribution of the PDF of the error of the quantization of a sinusoidal input signal is not rectangular (because sinusoidal signals do not comply with Widrow's quantization theorem), and moreover the form of the function is highly sensitive to slight offset errors or gain errors appearing in the part of the signal path preceding the A/D converter. The paper discusses sensitivity of the PDF of the quantization error on the variations of the amplitude of the signal and the possibilities of remedying this situation by means of quantization with dither signal.

Keywords: analog-digital conversion, dithering, quantization error.

1. THE RELATIONSHIP BETWEEN THE PROBABILISTIC MODEL OF THE QUANTIZATION ERROR AND THE CONVERTED SIGNAL

The probabilistic model of quantization consists of the probabilistic model of the quantization error and the probabilistic model of the relationship between the signal before quantization and the signal after quantization. Among the criteria of the evaluating of an actual a-d conversion are the PDF of the processed signal and the PDF of the quantization error. The form of the former PDF is researched in order to determine the values of the code transition levels, the DNL, the INL, the gain error and the offset error, while the form of the latter, to determine the quantization error.

The form of the PDF of the quantization error depends on the form of the processed signal. It is also significantly affected by the presence on an additive component (an effect of the offset error) in the sinusoidal signal, and by the variations in the amplitude of the signal (due to the gain error). A slight additive component having a value from the range of $(-0.5q; 0.5q)$ or a slight variation in the amplitude of the signal, falling in the range of $(-0.5q; 0.5q)$, significantly change the form of the PDF of the quantization error. The sensitivity of the form of the PDF to these factors diminishes the practical value of the function as a descriptor of the characteristics of A/D converters. The sensitivity may be reduced by modifying the process of quantization through applying quantization with an appropriate dither signal. Wagdy and Ng [1] have investigated the effect of the dither signal on making the PDF of the quantization error independent of the form of the processed signal. Hejn and Pacut [2] have studied the effect of the dither signal on making the PDF of the quantization error independent of the additive component appearing in the processed signal, its value being comparable with the width of the range of quantization.

The paper discusses the sensitivity of the PDF of the quantization error to slight variations in the amplitude of the processed signal, and the possibilities of reducing this sensitivity by means of quantization with dither signal. It is assumed that quantization is ideal and that the quantizer has "round off" characteristics. The sinusoidal signal is assumed to be free of the constant component, and the variations in the amplitude, to be deterministic. Because of the local symmetry of the sawtooth curve of the error of the operation of quantization relative to the points $+/- kq$, the variations in the amplitude taken into consideration are limited to the range of $[0, q/2]$.

2. THE EFFECT OF THE FLUCTUATIONS OF THE AMPLITUDE OF THE SINUSOIDAL SIGNAL ON THE FORM OF THE *PDF* OF THE QUANTIZATION ERROR

The variations in the amplitude of the sinusoidal signal at the input of the investigated A/D converter may be caused by the following factors:
- the limited accuracy of the signal source,
- the gain error of the analog part of the signal path preceding the A/D converter.

Even if the value of the variations in the amplitude does not exceed the width of the range of quantization, the form

of the PDF of the quantization error changes. Fig. 1 illustrates this sensitivity exemplified by two amplitudes with significantly different nominal values A.

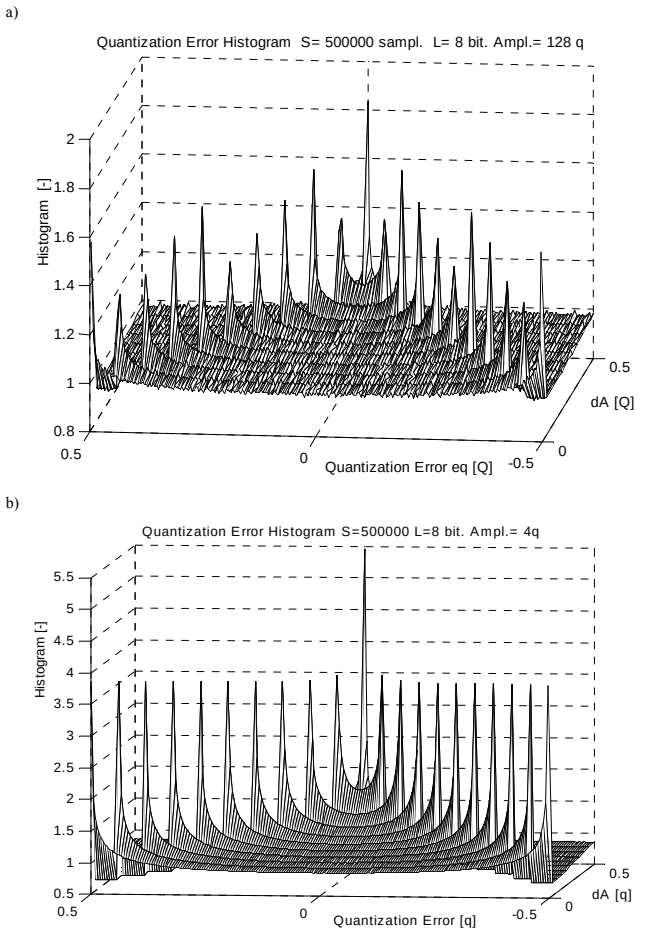


Fig. 1. Histograms of the error of the quantization of a sinusoidal signal with an amplitude fluctuating within the range of *[0, q/2]*: a) nominal amplitude $A=2^7q$, b) nominal amplitude $A=2^2q$

The PDF of the error of the quantization of a certain code word at the output of the A/D converter may vary in accordance with the position of the amplitude relative to the limits of the range of quantization.

The PDF of the quantization error is of the same nature for both large and small amplitudes. Differences appear only in the extreme values, which are proportionally larger for smaller amplitudes.

The sensitivity may be reduced by applying quantization with dither signal with the average value amounting to zero. Our experiments have tested the effects of a dither signal with a rectangular PDF of a width varying from +/– 0.05q (Fig. 2) to +/– 0.45q (Fig. 3) and of a dither signal with a normal PDF and a variance from 0.05q (Fig. 4) to 0.45q (Fig. 5). In each case, the variations in the PDF of the quantization error have been evaluated for sinusoidal signals with both large and small amplitudes.

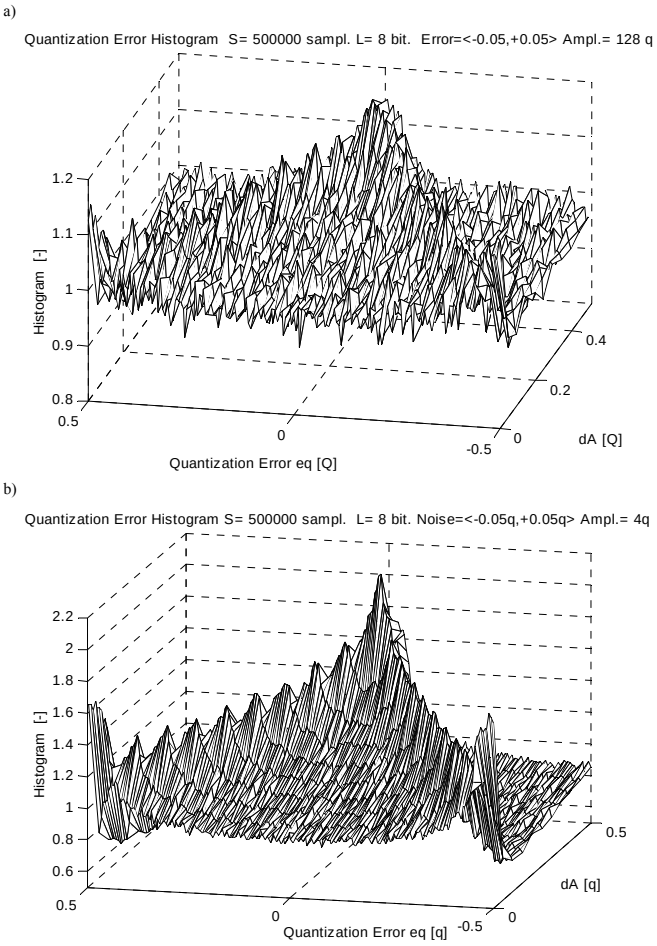


Fig. 2. Histograms of the error of the quantization of a sinusoidal signal with an amplitude fluctuating within the range of [0, q/2], with a dither signal with a rectangular PDF of the width of *+/– 0.05q*: a) nominal amplitude $A=2^7q$, b) nominal amplitude $A=2^2q$

When a dither signal with a rectangular PDF is applied, the width of its PDF strongly affects the reduction of the sensitivity of the PDF of the quantization error to the variations in the amplitude. The larger the width, the more efficient the reduction becomes, approximating a half of the width of the range of quantization. This relationship applies to sinusoidal signals with both large and small amplitudes.

When a dither signal with a normal PDF is applied, the value of its variance hardly affects the reduction of the sensitivity of the PDF of the quantization error to the variations in the amplitude. A marked reduction is registered at practically any value of variance within the investigated range. This relationship applies to sinusoidal signals with large amplitudes.

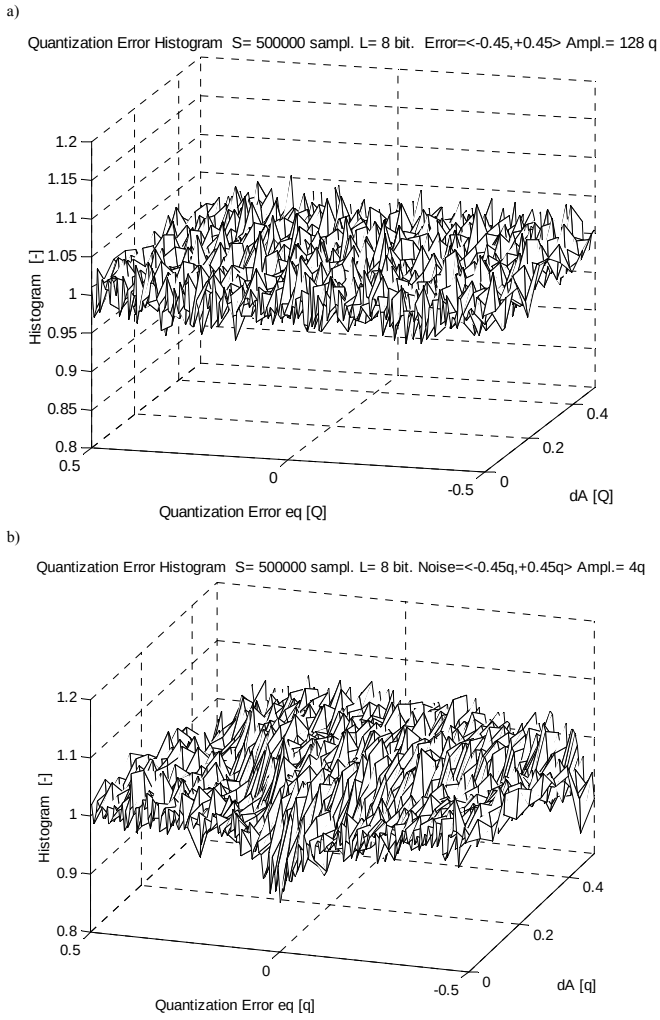
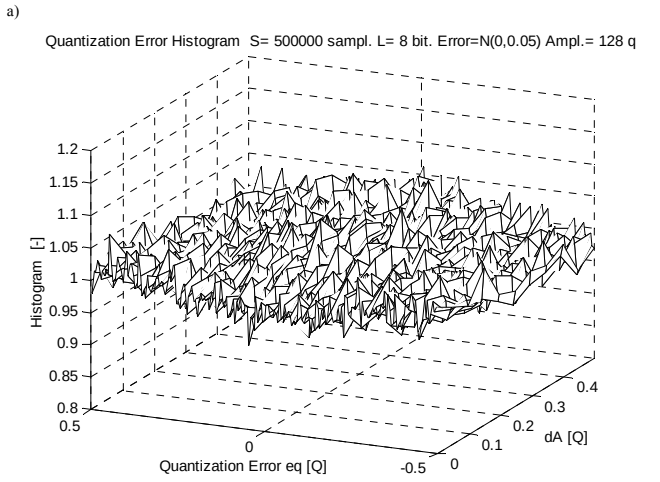


Fig. 3. Histograms of the error of the quantization of a sinusoidal signal with an amplitude fluctuating within the range of [0, q/2], with a dither signal with a rectangular PDF of the width of *+/– 0.45q*: a) nominal amplitude $A=2^7q$, b) nominal amplitude $A=2^2q$



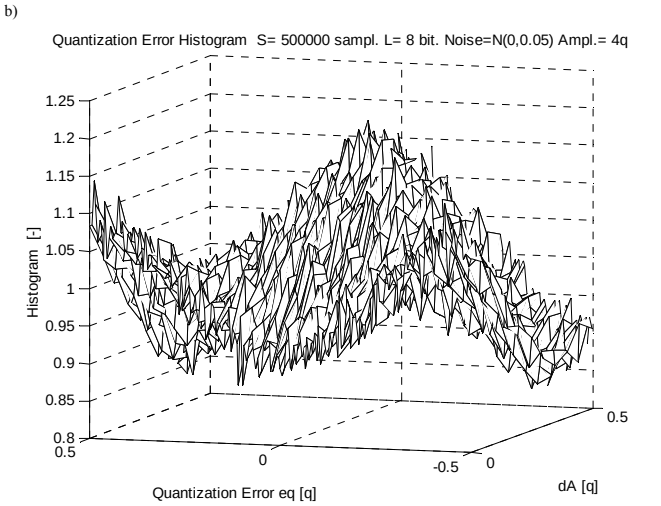


Fig. 4. Histograms of the error of the quantization of a sinusoidal signal with an amplitude fluctuating within the range of [0, q/2], with a dither signal with a normal PDF and the variance of *0.05q*: a) nominal amplitude $A=2^7q$, b) nominal amplitude $A=2^2q$

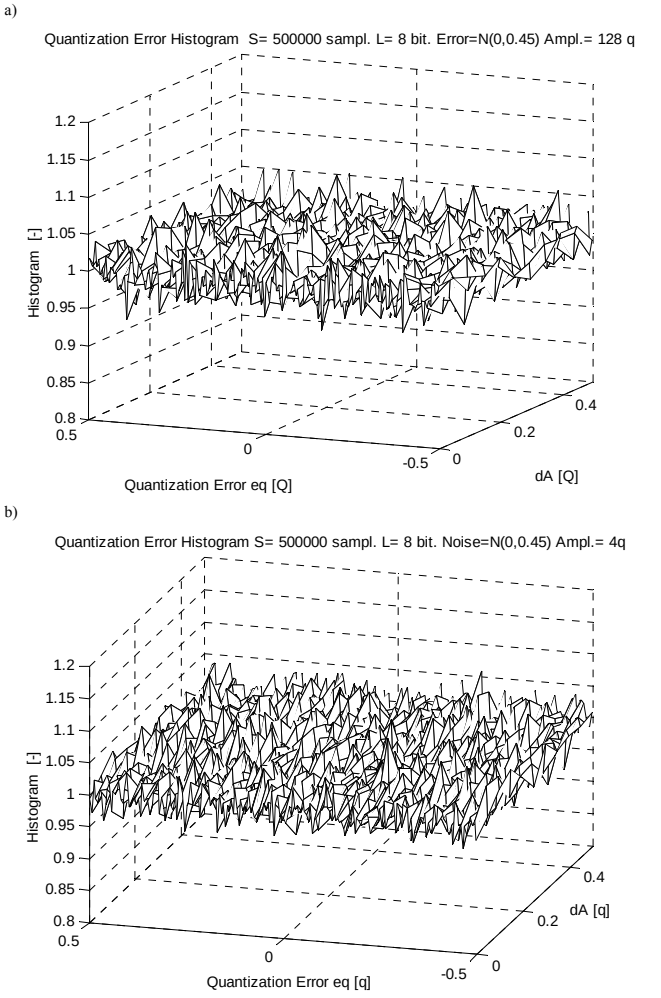


Fig. 5. Histograms of the error of the quantization of a sinusoidal signal with an amplitude fluctuating within the range of [0, q/2], with a dither signal with a normal PDF and the variance of *0.45q*: a) nominal amplitude $A=2^7q$, b) nominal amplitude $A=2^2q$

3. CONCLUSION

The form of the PDF of the quantization error is sensitive to deterministic variations in the amplitude of the signal undergoing a-d conversion, i.e. to variations which are comparable with the width of the range of quantization. This sensitivity may be reduced by applying quantization with dither signal with the average value amounting to zero.

The efficiency of the reduction depends on the form of the PDF of the dither signal. A dither signal with a normal PDF proves to be much more efficient. When a dither signal with a rectangular PDF is applied, the width of the function must be adjusted optimally to the width of the range of quantization.

The symmetry of the PDF of the dither signal is not sufficient on its own to affect the sensitivity of the PDF of the quantization error to the variations in the amplitude of the signal undergoing a-d conversion, although it is sufficient to reduce the sensitivity to the presence of a slight additive component in the signal.

REFERENCES

[1] M.F. Wagdy, W-M. Ng, “Validity of uniform quantization error model for sinusoidal signals without and with dither”, *IEEE Trans. Instrum. Meas*., vol. 38, no. 3, pp. 718-722, June 1989.

[2] Hejn K., Pacut A., “Generalized model of the quantization error-a unified approach”, *IEEE Trans. Instrum. Meas.,* vol. 45, no. 1, pp. 41-44, February 1996.

Author: Anna Domańska, Ph.D., D. Sc., Institute of Electronics and Telecommunications, Poznań University of Technology, ul. Piotrowo 3a, 60-965 Poznań, Poland, domanska@et.put.poznan.pl
Adam Derkowski, M.Sc., Department of Wood-Based Materials, Agricultural University of Poznań, ul. Wojska Polskiego 38/42, 60-627 Poznań, Poland, ADerkowski@woodcock.au.poznan.pl