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RESONANCE MEASUREMENT OF THE INDUCTANCE Q-FACTOR IN ULTRA-ACOUSTIC FREQUENCY RANGE UTILIZING IMPEDANCE ANALYSER

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Abstract − Assignment of the inductance Q-factor is given by the determination of equivalent circuit parameters of the measured inductor or the Q-standard. In case of serial equivalent circuit, values of the inductance L_S and the resistance R_S have to be evaluated. Conventional precious bridges routine used for inductance and resistance measurements typically work up to some hundreds of kHz. For higher frequency ranges these methods are not available to apply for finding both parameters inductance L_S and resistance R_S.

This paper describes a method based on utilizing resonance technique for inductance Q-factor measurement. The measured inductor and auxiliary capacitor form the resonance circuit. Impedance of this circuit is measured by means of an impedance analyser in suitable wide frequency range around the desired frequency. Subsequently mathematical data processing is applied to improve results obtained from the impedance analyser. This method has been developed for calibration of Q-standards designed for ultra-acoustic frequency range (typically hundreds of kHz up to tens of MHz).

Keywords: inductance Q-factor, impedance analyser

1. INTRODUCTION

Resonance methods for inductance measurements have been already described in many articles [1][2][3]. The same methods could be used also for measuring inductance Q-factor as well as for measuring Q-factor of Q-standard. One of many possible principles of this well known technique is presented in Fig. 1.

A harmonic generator over-tuned in the required frequency range supplies a serial resonance circuit consisting of measured inductor and an auxiliary variable capacitor C_P. According to Fig. 1 the measured inductor is illustrated by parameters L_X, R_X and C_X. The inductance L_X is self-inductance of the measured inductor, the resistance R_X involves together DC resistance of the inductor and loss resistances caused by eddy currents and skin effect acting at the given frequency. The capacitor C_X includes all parasitic capacitances of the inductor. The auxiliary capacitor C_P enables to attain the desired resonance state. A resistance divider (R_1, R_2) serves to match the generator impedance to those of the serial resonance circuit (suppose the components L_X and C_P). In case of achievement of resonance state, the following equation is valid for inductance Q-factor calculation:

$$Q = \frac{\omega_R L_X}{R_X} = \frac{U_V}{U_G} \frac{R_1 + R_2}{R_2} \qquad (1)$$

where U_V means the voltage across the capacitance C_P, U_G expresses the generator output voltage and ω_R is the angular resonance frequency.

Now it should be noted, that this method has a number of restrictions, if reliable measurement results should be obtained.

Firstly, the resistance R_X of measured inductor has to be much larger than used resistor R_2, in this case 0.1 Ω. Secondly, the input impedance of the AC-voltmeter has to be also much larger than the resistance R_P including loss resistances of the auxiliary capacitor C_P. In spite of actual modern instruments and devices for AC-voltage measurements, this condition is very difficult to perform. It is given by their relative low input impedances, which depend indirectly on the input voltage frequency and also on used measurement range [4]. Therefore, this method is not appropriate for measurements in ultra-acoustic frequency range as for higher frequency range. These reasons and the necessity to calibrate Q-standards in ultra-acoustic frequency range led us to the development of some new method applicable for inductance Q-factor measurement, respectively for calibration of Q-standards.

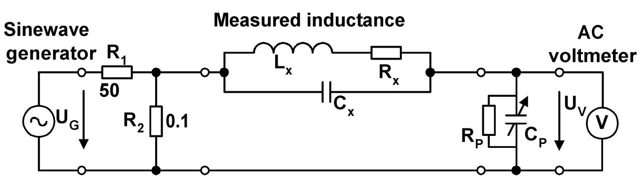

Fig. 1. Resonance based method for Q-factor measurement

2. EXPERIMENT

The principle of the method and configuration of measurement system designed for Q-factor measurement are illustrated in Fig. 2. A parallel connection of the measured inductor or Q-standard (represented by parameters L_X, R_X and C_X) and an auxiliary capacitor C_P (with loss resistor R_P) form a resonance circuit. This component combination is connected to a commercial impedance analyser in order to carry out measurements of resonance circuit impedance

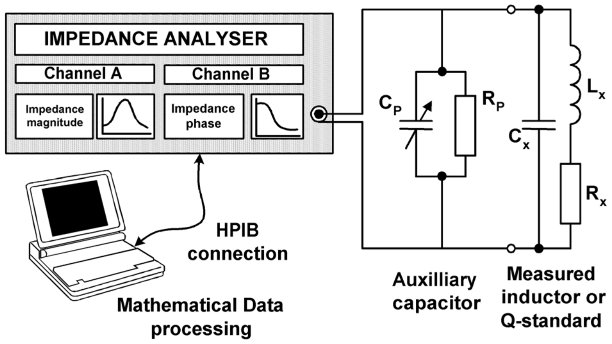

Fig. 2. Experiment configuration

(magnitude as well as phase of impedance). These measurements are carried out in the frequency range around the required frequency. The change of the resonance frequency is made by the change of variable capacitance C_P (we have used an air plate capacitor). It is clear that parasitic capacitance C_X is connected in parallel with capacitance C_P and thus it has an influence only on the resonance frequency of the parallel combination and therefore the Q-factor is not affected by this capacitance. The value of C_X is limiting for the achievement of a maximum resonance frequency. After tuning resonance circuit on desired frequency and carrying out all impedance measurements, data from the instrument are transferred via HPIB interface into the PC for the following data processing. This operation gives us a possibility of the next improving measured data, which are loaded especially by different level of noise.

The data processing is based on the assignment of parameters L_X, R_X, C_S and R_S of supposed mathematical model of impedance of the measured parallel circuit (see Fig. 3). The capacitance C_S involves the capacitance C_P as well as the capacitance C_X and together with the resistor R_S represent equivalent serial parameters of circuit in Fig.3 on the left.

At the beginning all parameters L_X, R_X, C_S and R_S are estimated very roughly. On the base of mathematical model knowledge, the impedance of the measured circuit is calculated and the fitting algorithm enumerates these parameters so that deviations between the measured resonance curve and the calculated curve will be minimised for both magnitude and phase of impedance.

After establishment of the parameters L_X, R_X, C_S and R_S, the resonance frequency of mathematical model is calculated according to (2). Now, if the resonance frequency is known, the Q-factor of inductor or Q-standard is calculated according to (1).

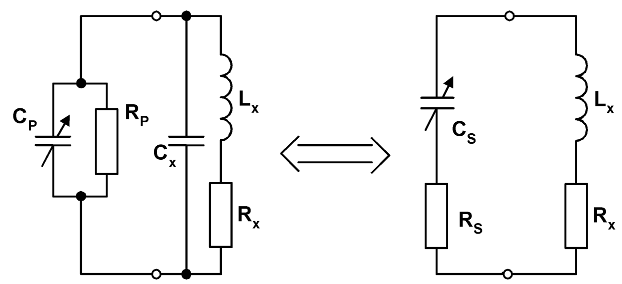

Fig. 3. Supposed equivalent circuit for inductance Q-measurement and its mathematical model

$$f_r = \frac{1}{2\pi.\sqrt{L_X C_S}} \sqrt{\frac{R_X{}^2 - \frac{L_X}{C_S}}{R_S{}^2 - \frac{L_X}{C_S}}} \tag{2}$$

3. DEVICE UNDER TEST

Two different types of Q-standards from non-referred manufacturer have been evaluated, type 513 and type 518-A. Each one of them is calibrated from manufactory for 3 different frequencies (type 513 for 0.5, 1 and 1.5 MHz, type 518-A for 15, 30 and 45 MHz). As it is seen (see Fig. 4), these Q-standards have banana connectors. Therefore we

Fig. 4. Tested Q-standard, type 518 A

had to make special adaptor, which enables a connection of the used air plate capacitor with the impedance analyser having another type of connector, type N. This adaptor does not have any shielding, but all these measurements have been made in the shielded room.

4. IMPEDANCE ANALYSER

As mentioned above, the impedance analyser is used for magnitude and phase impedance measurement. A very important thing is setting used impedance analyser. The most of commercial available analysers enable a user to change parameters as bandwidth, central frequency, number of points, number of averages, SPAN (frequency bandwidth) and especially bandwidth of the IF filter. Of course, all of these parameters affect impedance measurements. In our case, the following parameters appeared as optimal choice:

number of points = 400,
number of averages = 0,
bandwidth of IF filter - see Tab. I,
central frequency = resonance frequency,
SPAN (frequency bandwidth) = 3 up to 5 times great than Δf, where Δf is defined as:

$$\Delta f = \frac{f_R}{Q} \tag{3}$$

where f_R means the resonance frequency and Q is Q-factor of resonance circuit. The bandwidth Δf is limited by a decrease about –3dB from the maximum value of impedance.

TABLE I. Setting bandwidth of IF filter of the imp. analyser

Resonance Frequency [MHz]	Bandwidth of IF [Hz]	Sweep time [s]
0.5	100	approx. 23
1	100	approx. 23
1.5	100	approx. 23
15	300	approx. 23

The setting of bandwidth of IF filter of impedance analyser has been chose as the compromise between level of noise included in the measured resonance curve and time of measurement. The level of noise is indirectly dependent on the bandwidth of IF filter, as well as on time of measurement.

5. DATA PROCESSING

A fitting algorithm is applied to improve measured data from the impedance analyser. The algorithm is based on generalized method of the least squares. The algorithm finds the minimum of the following function:

$$E = \sum_{i=1}^{n} \left| Z_i - Zm_i \right|^K \tag{4}$$

where Z_i is the measured impedance at the frequency i and Zm_i is the impedance of selected model at the same frequency. N is number of points and K is a constant depending particularly on the level of noise in the measured resonance curves (we experimentally used $K = 1$ for the data containing a lot of noise, $K = 2$ for the data containing a little of noise, compare Fig. 6 and Fig. 9).

In Fig. 5 the main window of the application written in MATLAB for data processing from analyser is illustrated. The application allows loading data from a text file, it employs the algorithm for the best fitting of measured resonance curve and it enables a calculation of the resonance frequency and the Q-factor. All calculated results can be saved into text file.

At work with application, firstly the parameters L_X, R_X, C_S and R_S must be entered and then it is possible to compare both measured and calculated resonance curve. After running iteration algorithm for best fitting of measured curve, the resonance frequency of circuit, respectively inductance Q-factor is calculated according to (2), respectively (1). Sometimes it happens that the iteration procedure fails due to wrong estimated initial parameters. In this case new parameters have to be entered. The success of iteration procedure is determined by following criteria:

$$C = \frac{\sum_i \left| Z_i - ZD_i \right|}{\sum_i Z_i} \tag{5}$$

where Z_i and ZD_i express respectively calculated impedance and measured impedance. In case of ideal matching both resonance curves satisfy condition the criteria C = 0.

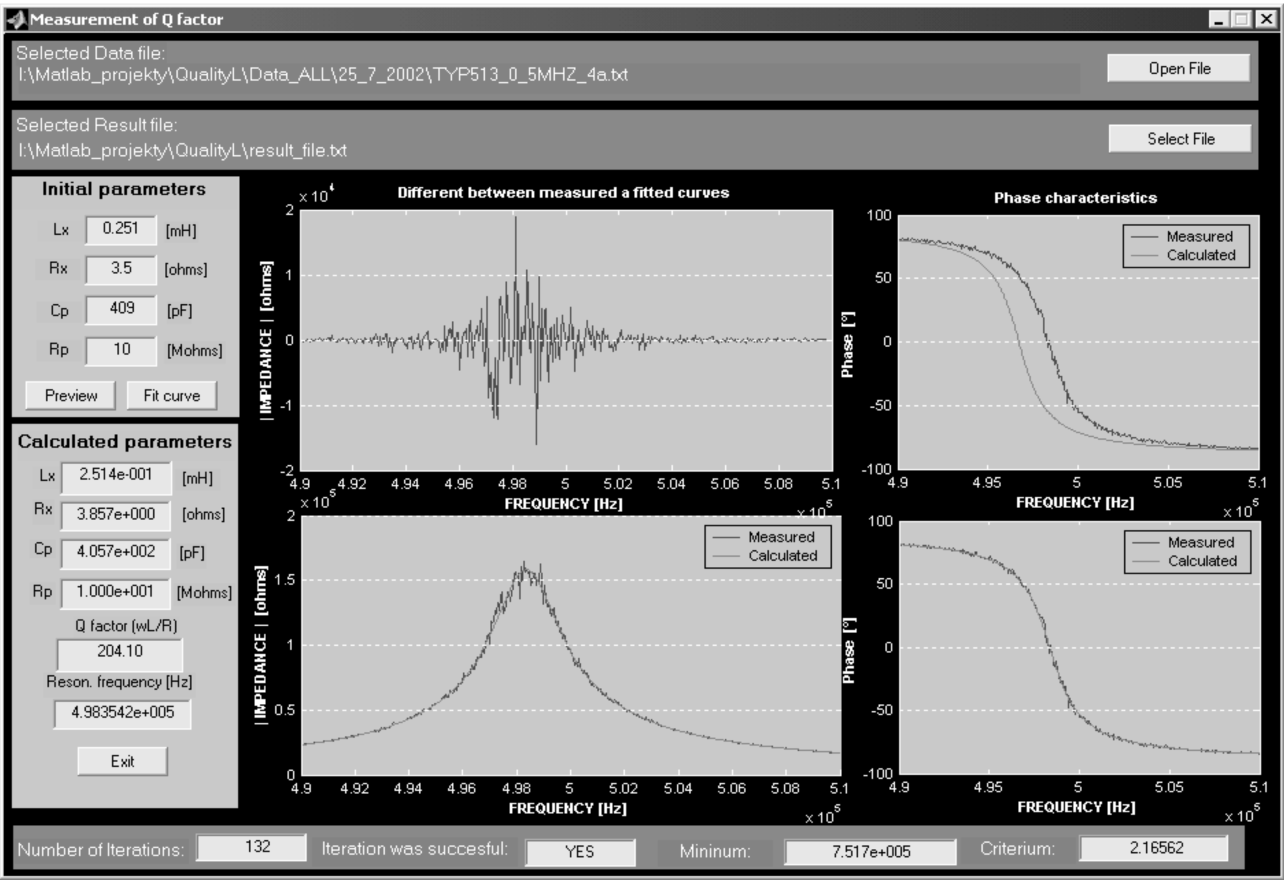

Fig. 5. Main application window for fitting resonance curve and calculation of Q-factor

6. RESULTS AND CONCLUSION

The proposed paper describes a method for Q-factor measurement of inductor or Q-standard in ultra-acoustic frequency range (typically hundreds of kHz up to tens of MHz). The measured inductor or Q-standard and an auxiliary variable capacitor form a resonance circuit, whose impedance is measured by means of an impedance analyser. Subsequently data processing is applied to improve results from impedance analyser. The algorithm is based on the assignment of the best fitting resonance curve.

The method has been tested for two different types of Q-standard. Fig.6 and Fig. 9 show one of many measurements of resonance curve obtained by means of the impedance analyser. Graphs in Fig. 7 and Fig.8, respectively Fig. 10 and Fig. 11 illustrate an impedance magnitude and phase differences between the measured and calculated resonance curve of both tested Q-standards – type 513, respectively type 518 A. The measurement results are given in Tab. II. Values of the Q-factor have been calculated as the average of the five measurements. The relative error does not exceed 3 % for type 513 and 0.18 % for type 518 A

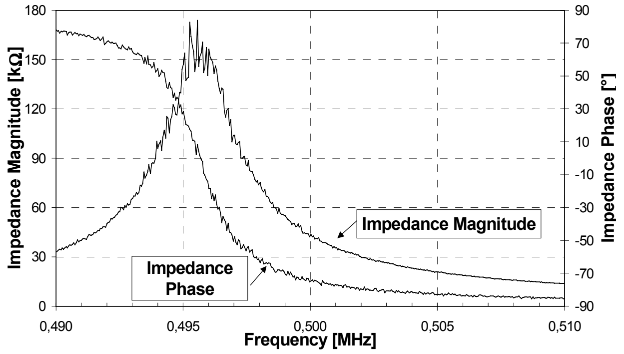

Fig. 6. Magnitude and phase characteristics measured by impedance analyser for the Q-standard - type 513, nominal resonance frequency f_R=0.5 MHz

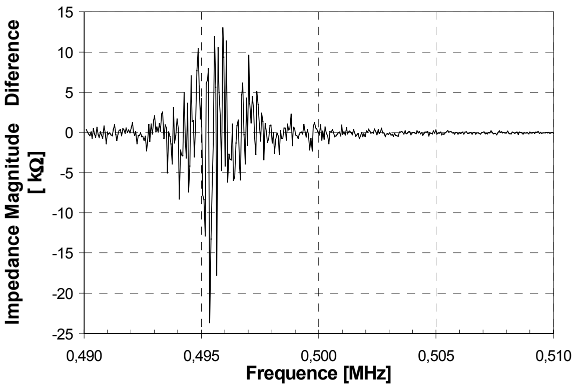

Fig. 7. Impedance magnitude difference between measured and calculated resonance curve (Q-standard - type 513, nominal resonance frequency f_R=0.5 MHz,K=1)

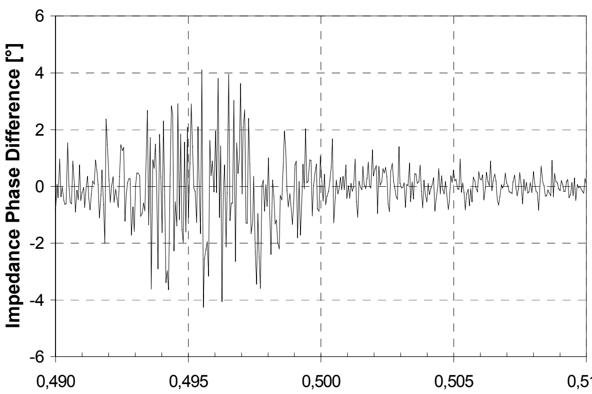

Fig. 8. Impedance phase difference between measured and calculated resonance curve (Q-standard - type 513, nominal resonance frequency f_R=0.5 MHz, K=1)

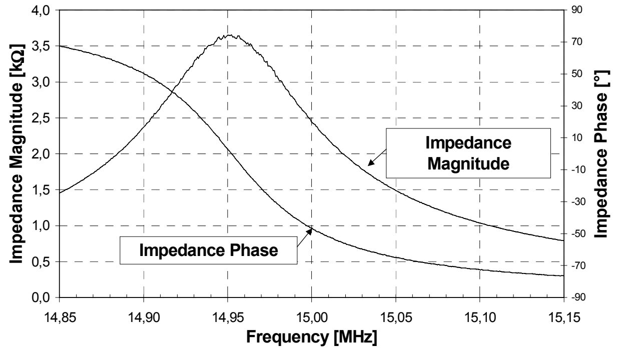

Fig. 9. Magnitude and phase characteristics measured by impedance analyser for the Q-standard - type 518 A, nominal resonance frequency f_R = 15 MHz

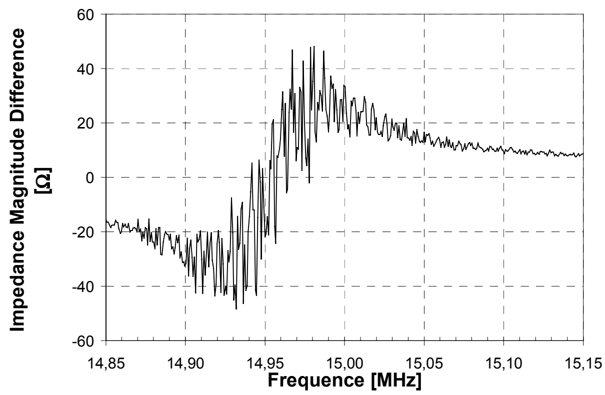

Fig. 10. Impedance magnitude difference between measured and calculated resonance curve (Q-standard - type 518 A, nominal resonance frequency f_R = 15 MHz, K=2)

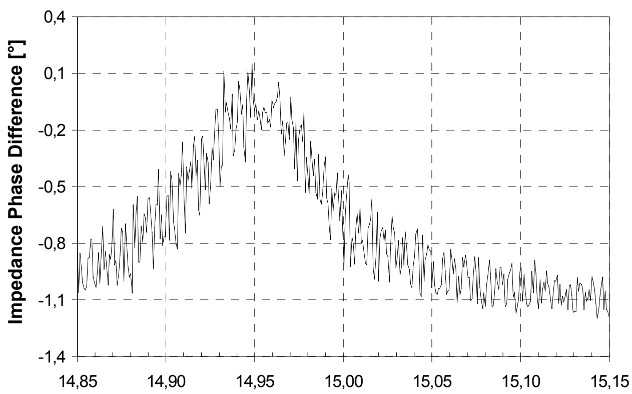

Fig. 11. Impedance phase difference between measured and calculated resonance curve (Q-standard - type 518 A, nominal resonance frequency f_R=15 MHz,K=2)

(the nominal values of Q-factor for each Q-standard has been taken from the calibration certificate).

The restriction of this method consists especially in the impossibility of measuring the Q-factor at the frequencies higher than 100 MHz, due to the self-resonance of the measured inductor or the Q-standard. Furthermore the impossibility to measure the Q-factor at desired frequency appears as the next disadvantage of this method (see Fig. 7 and Fig. 9). However the error arising due to this fact is negligible regarding the whole measurement error.

TABLE II. Results of two Q-standards measurements

TYPE 513				K=1
Frequency [MHz]	Nominal Q-factor [-]	Calculated Q-factor [-]	Relative error [%]	Standard deviation
0.5	205	203.93	-0.52	1.03
1.0	272	279.58	2.78	2.68
1.5	265	267.76	1.04	5.18
TYPE 518				K=2
Frequency [MHz]	Nominal Q-factor [-]	Calculated Q-factor [-]	Relative error [%]	Standard deviation
15	169	169.32	0.18	0.37

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