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DIGITAL MEASUREMENT OF PHASE DIFFERENCE OF LF SIGNALS – A COMPARISON OF DSP ALGORITHMS

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Abstract – The paper compares six methods of measurement of the phase difference of low frequency distorted sinusoidal digitized signals corrupted by additive noise. Influence of SNR, THD, signal DC offset, ADC resolution, ADC range use, and number of samples per signal period can be investigated and presented in graphical form by means of the program written in MATLAB environment. The investigated methods are two modifications of classical zero-crossing based measurement, DFT based measurement, usage of phase-sensitive rectifier and two modifications of sine-wave fit algorithm. Selected results of both simulations and measurements are presented.

Keywords: DSP in phase measurement, phase measurement accuracy, phase difference of power-line signals.

1. INTRODUCTION

The aim of this contribution is to compare different digital algorithms used for measurement of phase difference of LF (low-frequency) digital signals. Algorithms are compared under identical disturbing conditions (values of signal total harmonic distortion THD, external additive noise characterized by signal-to-noise ratio SNR, and DC offset) and algorithm parameters (ADC quantization noise, number of samples per signal period). Numerical values of all the above mentioned parameters can be chosen. Methods are programmed in MATLAB environment, and the same MATLAB files are used for simulations and for measurement on physical signals. The only difference between simulations and measurements is the way of getting the signal samples. In case of simulations signals are generated by MATLAB files, and in case of measurement they are gained by sampling physical signals by a PC DAQ plug-in board controlled by Data Acquisition Toolbox of MATLAB. Both simulation and measurements were performed for signals with power-line frequency, but the algorithms can be used for any frequency in simulation and for frequencies corresponding to the frequency band of the used DAQ plug-in board in case of measurements. The six methods to be compared are briefly described below.

2. OVERVIEW OF THE INVESTIGATED METHODS

Classical phase difference measurement of two sinusoidal signals of the same frequency is based on converting signals into square waves and measuring the time difference of adjacent positive zero-crossings or of adjacent pulse centres of these waves two waves [1], [2]. This difference divided by signal period (found also as the time of neighbouring positive zero crossings, but this time for the same wave) is equal to the phase difference (in degrees if multiplied by 360 or in radians if multiplied by 2π). When using digital signals the same principle can be used, but it is not necessary to convert signal into square waves [3]. Local replacement of signal in the neighbourhood of zero crossings by straight line (using either linear interpolation or linear regression [4]) can decrease measurement uncertainty. The method is denoted here as **ZCR** (Zero CRossing) method. If the signal is corrupted by additive noise or by heavy harmonic distortion causing additional zero crossings , linear interpolation alone fails to give the phase value of the signal, and some preliminary filtration of the signal has to be used. For the case of using a LP digital filter (FIR or IIR filter), moving average or signal integration with signal centering after integration (before zero crossing finding) the method is denoted as **ZCRF** (Zero CRossing with Filtration) method. The abbreviation **ZCRR** (Zero CRossing with Regression) method is used for zero-crossings finding after liner regression applied to suitable group of samples around zero crossing. A moving average filtration was used in our case before computation of regression straight line.

Phase can be measured digitally also by means of virtual phase sensitive rectifier. Contrary to classical hardware phase sensitive rectifiers, reference signal here is not a rectangular pulse train, but sinusoid of the same frequency as the signal. Phase shift of the reference is considered to be zero phase reference for both measured signals. Here we compute mean values of two point-by-point products of measured signals and (two) reference sinusoids. All signals have the same frequency and the phase shift between the two reference signals is $\pi/2$. The phase angle between the two measured signals is found as the difference between phase shifts of both signals with reference to the reference signals [4] (phase shift of each signal is found as the arctangent of ratio of imaginary and real parts real of signal phasor). Since also magnitudes of both measured signals can be found here, the method could be used for measurement of both amplitude and phase frequency characteristics of electric circuits. This method is denoted here as the **PSR** (Phase Sensitive Rectifier) method.

The same principle of phase estimation (computing arctangent of ratio of imaginary and real parts of the signal fundamental spectrum component) is in fact used in the next method. DFT (Discrete Fourier Transform) is applied to both signals, and phases of the fundamental harmonic components are found [5], [6]. Difference of these phase shifts is the measured phase difference. Interpolation and windowing can be used for leakage reduction by non-synchronous sampling. This method can also be used for frequency response measurement. It is denoted as **DFT** method. (Walsh spectrum can be used instead of Fourier spectrum as well [7].)

Another method of phase estimation is based on various sine wave fit algorithms [8, 9]. Using some of these algorithms, three or four sine wave parameters can be found by means of least-square algorithm. Of these parameters only phase shift (and possibly DC offset) is of interest here. We shall call this category of methods **SWF** *(Sine-Wave-Fit)* method.

Computation can be performed using formulae derived directly for sinusoidal signal [9] or using another mathematical mean of minimization of function of several variables.

MATLAB has *fminsearch* built-in function usable for the task. (Function fminsearch finds the minimum of a scalar function of several variables, starting at an initial estimate.) The minimized function here is the mean square error between the signal samples sequence and the ideal sinusoid, parameters of which are iteratively changed starting from the initial estimate. The modification of SWF method using MATLAB fminsearch function will be denoted **SWFF** (*Sine-Wave-Fit Fminsearch*) method.

The SWF method modification described in [9] by R. Micheletti will be denoted as **SWFM (**Sine-Wave-Fit Micheletti) method. Here the method of least squares is applied to find values of four auxiliary parameters used for phase shifts of signals finding so that the total square error with respect to each of parameters is minimised.

The PSR and the DFT methods are in fact identical ones. In both cases four sums of point-by-point multiplication of measured digital signal with sinusoid and cosinusoid with zero phase shift are found and used for phase difference of measured signals estimation. In case of PSR method it is multiplication of measured signals with reference signals, and sum is found as arithmetic mean, i.e. divided by number of samples. Since signal phase shifts are found as arctangent of ratio of two such means, ratios have the same values as ratios of sums as found in the DFT method for signal fundamental harmonic component by coherent sampling of integer number of periods of signal. The SWFM method uses for phase difference computation very similar expressions as the PSR and DFT method, but modified by the use of the least-squares algorithm. That is the cause why numerical results of simulations and measurement presented below are identical for PSR and DFT methods and practically identical to those of the SWFM method. The SWFM method has an advantage that less then one signal period need to be sampled to find the phase shift (half of period is satisfactory and even a quarter of period is enough is higher phase difference bias is accepted [9]). For coherent sampling of integer number of periods all three methods give identical values of phase difference

3. PROGRAM STRUCTURE

The developed program uses MATLAB, Signal Processing Toolbox for use with MATLAB, and in case of measurements also Data Acquisition Toolbox for use with MATLAB. It is composed of the main MATLAB script and number of functions in some cases nested to the level of three. Values of phase shifts of both signals are given in case of simulations, but not in case of measurement. Number of sampled periods, percentage of ADC range use, number of samples per signal period, and values of SNR, THD and ADC resolution can be chosen. Signal frequency is either given or found by integrated zero crossing method [10].

Harmonic distortion (requested THD value) is achieved with harmonic components amplitude ratios according to international LF electromagnetic compatibility standard [11]. Number of harmonics taken into account can be selected to up to 50 ,but 10 harmonics were used for most of our simulations.

Program can be modified to run in chosen number of cycles for selected values of different parameters (e.g. for selected set of values of THD, SNR or ADC bits).

Inside these cycles input signals are either generated (for simulations) or measured (for physical signal measurement). Afterwards all examined algorithms are subsequently applied on samples of both signals and mean values and standard deviations of these means are computed. Various functions are used in individual algorithms, both MATLAB functions and functions written by the author.

After the chosen number of cycles is passed, results are presented in 2-D figures. MATLAB command *sprintf* serving to printing strings into files is extensively used in printing titles and legends in figures. In this way correct figures description is provided automatically and human mistakes by figures labelling are to a large extent excluded.

Graphical user interface is being prepared for the phase difference measurements and simulations.

4. SIMULATION AND MEASUREMENT RESULTS

Both simulations and measurements are based on program described in part 3. Signals samples are in case of measurement gaine by a PC plug-in board controlled by means of MATLAB Data Acquisition Toolbox commands. Since we wanted to know the phase difference of the physical signals to be measured, we have used the HP324A Universal Source. This two-channel generator allows the phase shift between generated sinusoidal signals to be set numerically with resolution 0.001 degree, but its uncertainty is not given in the Users manual. Sinusoidal signals with frequency under 3 kHz have THD<-56dB (i.e. approx. 0.16 %) and SNR<-62dB (i.e. noise RMS value is about 0,08% of signal RMS value).

We have simulated separately influence of ADC resolution (see Fig.1 to Fig.4), THD (Fig.5 and Fig.6) and SNR (Fig.7 to Fig.11) for all the methods described in part 2. Various filters were used in the ZCRF method: IIR low-pass (LP) Chebyshev1, order N =3, corner frequency *f*c=55 Hz, FIR LP designed by windowing and *f*c=55 Hz, filter length L=16. Also numerical integration with subsequent mean subtraction and moving average of length equal to a quarter of number of samples per signal period were tested.

Both simulation and measurements (Fig.12 to Fig.15) reported below were performed for phase difference equal to 50°. It was found that neither mean nor standard deviation of phase difference depends substantially on values of individual signals phase shifts, only difference of those phase shifts is of importance. Parameters of simulation or measurement are given in figure titles. Angular degrees were chosen as units instead of radians in accordance with common practice.

4.1. Simulation examples – influence of ADC resolution

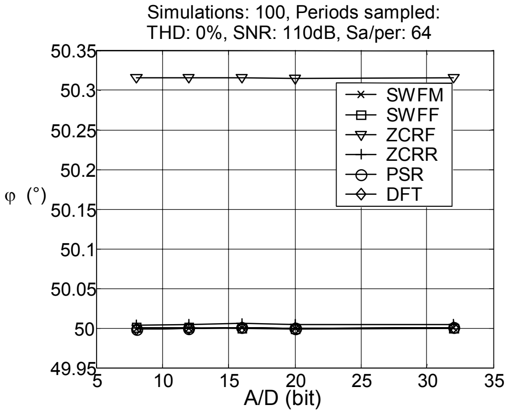

Fig. 1. Dependence of phase difference on ADC resolution, ZCRF: FIR LP, pure sinusoid

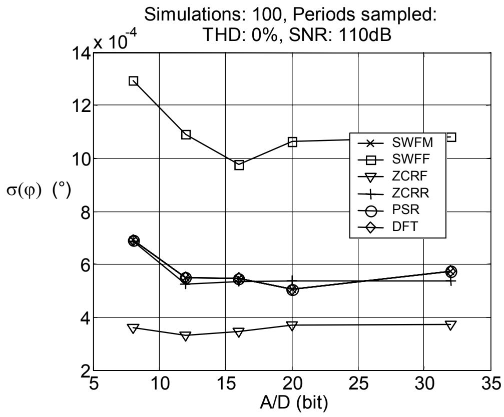

Fig. 2. Dependence of phase difference standard deviation on ADC resolution, ZCRF: FIR LP, pure sinusoid

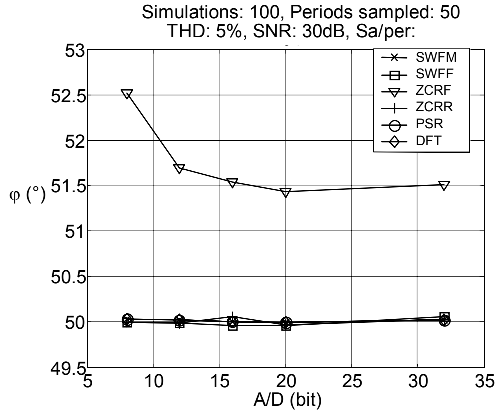

Fig 3. Dependence of phase difference on ADC resolution, ZCRF: FIR LP, noise and distortion included, compare with Fig.1

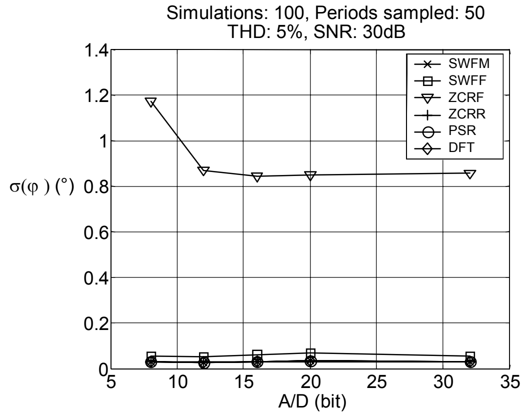

Fig. 4. Dependence of phase difference standard deviation on ADC resolution, ZCRF: FIR LP, noise and distortion included, compare with Fig.2

4.2. Simulation examples – influence of harmonic distortion (negligible additional noise)

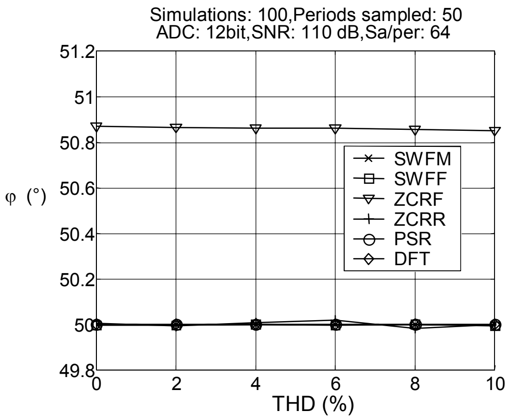

Fig. 5. Dependence of phase difference on signal THD, ZCRF: LP Chebyshev1

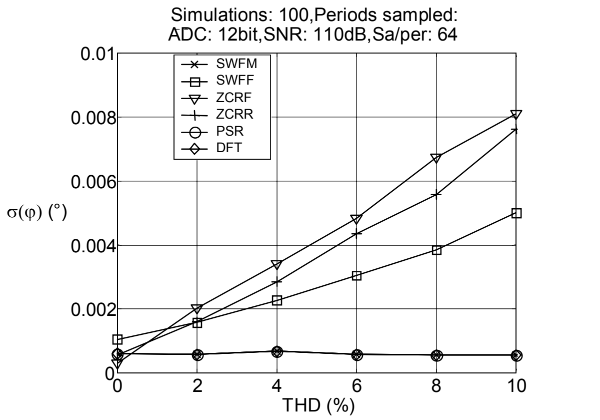

Fig. 6. Dependence of phase difference standard deviation on THD, ZCRF: LP, Chebyshev1, no external noise

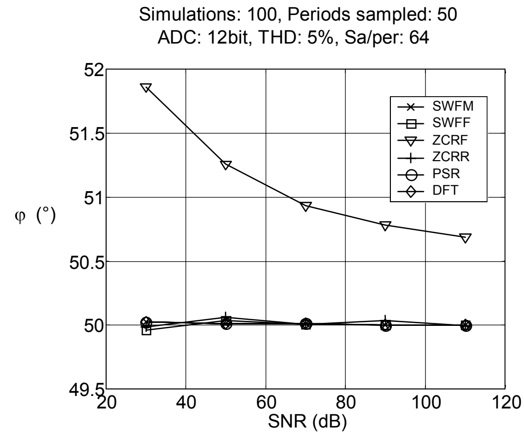

Fig. 9. Dependence of phase difference on SNR, ZCRF: FIR LP, signal distortion included, compare with Fig.7

4.3. Simulation examples – influence of additive noise

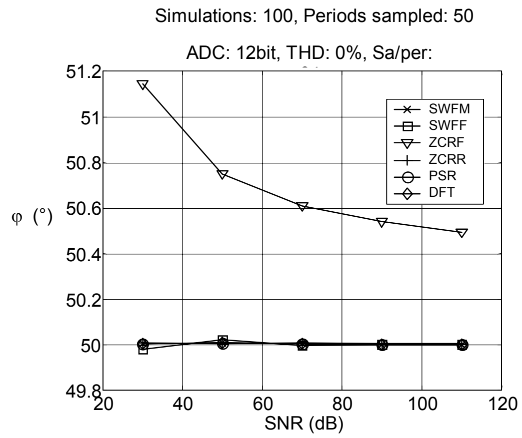

Fig. 7. Dependence of phase difference on SNR, ZCRF: FIR LP, zero harmonic distortion

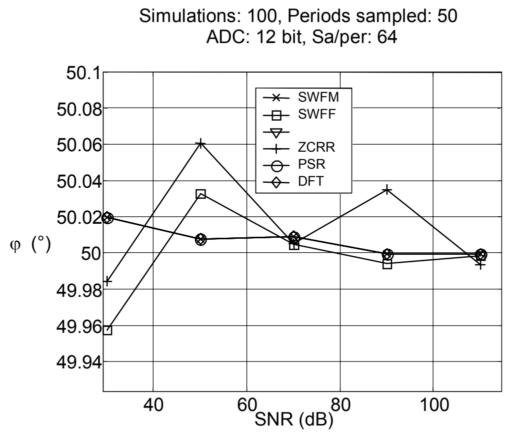

Fig.10. Detail of Fig.9, ZCRF graph is left out)

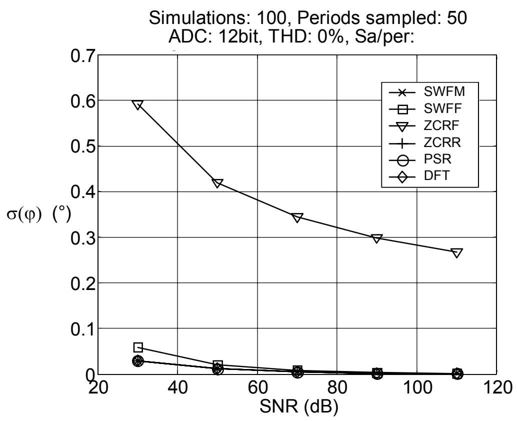

Fig. 8. Dependence of phase difference standard deviation on SNR, ZCRF: FIR LP, zero harmonic distortion

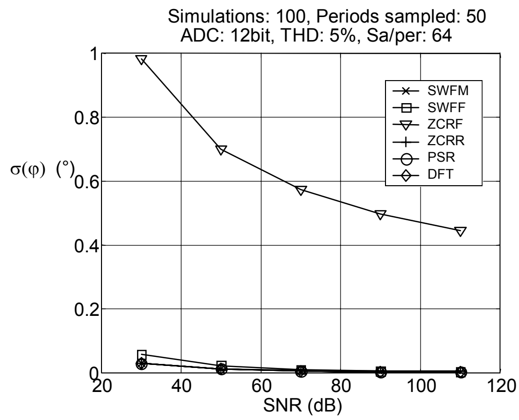

Fig.11. Phase difference standard deviation on SNR, ZCRF: FIR LP, signal distortion included, compare with Fig.8

4.4. Measurement result examples

In case of measurement signals were sampled and quantized by a low-cost National Instruments DAQ (Data Acquisition) plug-in board NI 6023E. It is a DAQ board with 12-bit ADC and multiplexer in front of the ADC. Therefore the two signals are not sampled simultaneously. We have used sampling frequency equal twice the desired one and used even samples in one channel and odd samples for the other one.

Since local linear interpolation is used in methods ZCRF and ZCRR, interleaving the two sample sequences should not increase uncertainty here.

The signal generator (HP 324A Universal Source) generates very clean sinusoidal signals. Noise was added to the measured signal during numerical processing of samples gained from the DAQ board so that the desired SNR values be achieved. Values of phase difference and its standard deviation for SNR=110 dB in Fig.12 to Fig. 15 correspond to the signals supplied by the generator – practically no additional noise was added by software to the measured signal this case. The influence of signal DC offset was also investigated – see Fig.14 and Fig.15.

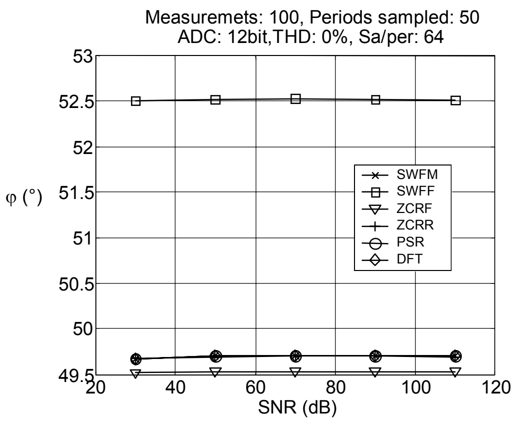

Fig. 12. Phase difference as function of SNR, ZCRF: Chebyshev1 LP

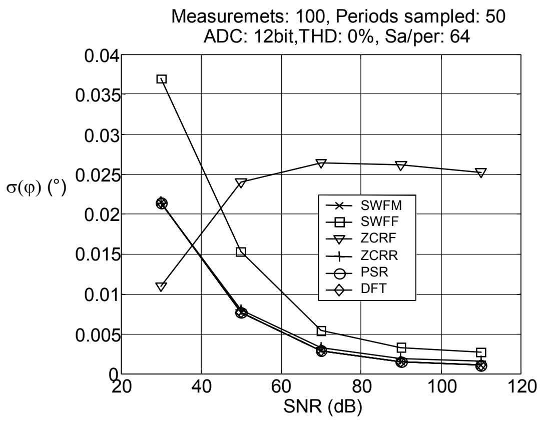

Fig. 13. Phase difference standard deviation as function of SNR, ZCRF: Chebyshev1 LP

Apart from bias of individual algorithms (difference between phase difference values found by simulation or measurement and the 50° "true value") and from standard deviations of the investigated algorithms given in the figures, also comparison of processing times of algorithms is of interest. MATLAB provides a suitable mean for such comparison – command *profile*, which presents both times in seconds and percentage of total computation time for all functions used in the program.

Using this command it was found that by far the most time-consuming algorithms are the SWFF and ZCRR algorithms, which consume each about 40 % of total computation time. Faster is the ZCRF method (about 13 % of total time), and the fastest are PSR, DFT and SWFM methods, which need less than 0.5 % of total computation time each. The computation speed of the last three methods is about the same.

The time comparison performed by *profile* command gives values valid only for programs written in MATLAB. Programs written in matrix form are especially fast in MATLAB, and method SWFM in [9] is described in matrix notation and therefore very fast when written in MATLAB. Using another language could lead to partially different conclusions.

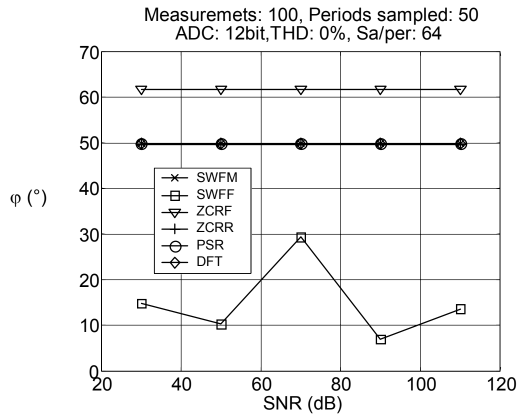

Fig. 14. Influence of signal DC offset, ZCRF: FIR LP, DC offset of one signal is 10% of amplitude

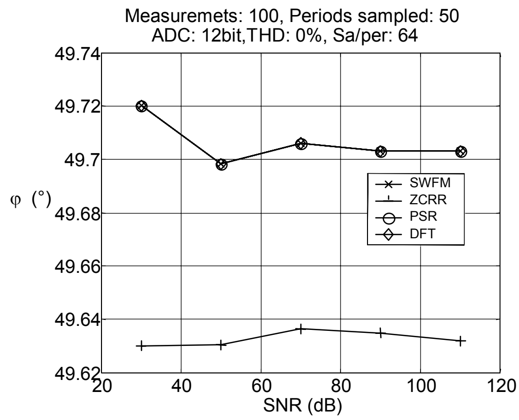

Fig. 15. Influence of signal DC offset – detail for methods least sensitive to offset, offset of one signal is 10% of signal amplitude

5. CONCLUSION

As can be seen from the figures above, methods PSR, DFT and SWFM, which give for the investigated conditions identical results, are the best ones both from the point of view of bias and standard deviations. It follows from Fig.8 and Fig.10 that phase difference standard uncertainty found by these algorithms could be about 0,03°, which corresponds 0,5 mrad. They are the best ones also from the point of computation time and they are because of their principle not sensitive to the DC offset of signals (see Fig.12 and Fig.15). The SWFF gives worse results especially in case of measurement, and is very time consuming. It was also very sensitive to signal DC offset. The ZCRR method gives good results, but is also very slow. The classical ZCR method is not usable if noise and signal distortion is not negligible. If modified by a sort of signal filtration before zero-crossing detections it can be used, but its bias is much higher then that of the PSR, DFT and SWFM methods and it is because of its principle sensitive to DC signal offset (see Fig.14). Contrary to the PSR and DFT methods, the SWFM method does not need that at least one complete signal period be sampled.

It should be noted here, that some capabilities of the program were not used yet – for example investigation of the influence of the percent use of ADC input range and of difference in signal amplitudes. Also results for various numbers of samples per signal period are not presented here. Some parts of the program work in the present form for coherent sampling only and comparison of the algorithms from the point of view of their sensitivity to non-coherent sampling might be of interest as well. These issues will be investigated in future.

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