XVII IMEKO World Congress
Metrology in the 3rd Millennium
June 22–27, 2003, Dubrovnik, Croatia

RENEWAL OF INPUT SIGNALS OF NONLINEAR MEASURING CONVERTERS BY FOURIER-INTEGRAL METHOD

Oleksandr B. Mokin, Boris I. Mokin

Vinnytsia State Technical University, Vinnytsia, Ukraine

Abstract – The present paper presents the results of solution of the task of renewing nonlinear measuring converter input signals at their fixed output signals with the application of the author's method called Fourier-integral.

Keywords: renewal, signal, converter.

1. INTRODUCTION

It is known [1] that any measuring converter even with nonlinear characteristic represents a dynamic system, the outcoming coordinate *y(t)* of which renews *x(t),* making always some dynamic error. So, the task of incoming signals renewing at their fixed dynamically distorted reactions on these signals, has always been urgent for measuring converters.

In work [2] the senior author succeeded in composing an effective method of incoming signals renewing of measuring converters with linear characteristics, called Fourier-integral.

The author had also realized not entirely successful attempt to generalize Fourier-integral method on nonlinear measuring converters, the renewing of incoming signals of which is far more difficult task.

In present work the junior author under the supervision of the senior author developed a specific variant of an algorithm of renewing of nonlinear measuring converter incoming signals by Fourier-integral method, which completes the research of the work [4].

2. RECEIVING OF BASIC CALCULATION CORRELATIONS

Let us assume that the nonlinear measuring converter can be represented, as shown on the figure, in the form of series connection of linear link with the weight characteristic *g(t)* and nonlinear with the characteristic $f_k(y)$ [1] of the following type

$$f_k(y) = \nu_1 y + \nu_2 y^2 + \cdots + \nu_k y^k. \tag{1}$$

In this case, as it is shown in work [5], an outcoming signal *z(t)* can be represented in the form of

$$z(t) = \sum_{i=1}^{k} \nu_i \cdot \underbrace{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}_{i} x(t-\tau_1)\ldots x(t-\tau_i) \cdot g(\tau_1,\ldots,\tau_i)\, d\tau_1 \ldots d\tau_i. \tag{2}$$

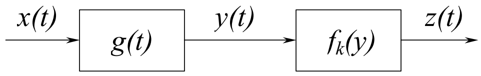

Fig. Structural diagram of nonlinear measuring converter

In case if incoming signal *x(t)* is the chopped Fourier series in form of

$$x(t) = \sum_{n=-N}^{N} \gamma_n \cdot e^{jn\omega_1 t}, \tag{3}$$

where $\omega_1 = \frac{2 \cdot \pi}{T}$ – the first harmonic frequency, T – its period, and

$$\gamma_n = \frac{1}{T} \int_0^T x(t) \cdot e^{-jn\omega_1 t} dt, \tag{4}$$

and also in case if the weight characteristic *g(t)* relates to the class of separables, or it is true for it that

$$g(t_1, t_2, \ldots, t_k) = g(t_1) \cdot g(t_2) \cdots g(t_k), \tag{5}$$

— under these conditions the outcoming signal *z(t)* (as it is shown in work [4]) can be represented in the form of

$$\begin{aligned} z(t) = \sum_{i=1}^{k} \nu_i \cdot (\sum_{\lambda=0}^{i} c_i^{\lambda} \cdot \left[\gamma_{-N} \cdot W(-jN\omega_1) \cdot e^{-jN\omega_1 t}\right]^{i-\lambda} \cdot \\ \cdot (\sum_{p=0}^{\lambda} c_{\lambda}^{p} \cdot \left[\gamma_{-(N-1)} \cdot W(-j(N-1)\omega_1) \cdot e^{-j(N-1)\omega_1 t}\right]^{\lambda-p} \cdot \\ \cdot (\cdots(\sum_{\theta=0}^{\psi} c_{\psi}^{\theta} \cdot \left[\gamma_{N-1} \cdot W(j(N-1)\omega_1) \cdot e^{j(N-1)\omega_1 t}\right]^{\psi-\theta} \cdot \\ \cdot \left[\gamma_N \cdot W(jN\omega_1) \cdot e^{jN\omega_1 t}\right]^{\theta}))\cdots), \end{aligned} \tag{6}$$

where

$$c_i^{\lambda} = \frac{i!}{\lambda! \cdot (i-\lambda)!} \text{ —} \tag{7}$$

binomial coefficient, and

$$W(j\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t}\, dt = \int_{0}^{\infty} g(t) \cdot e^{-j\omega t}\, dt \text{ —} \tag{8}$$

gain-phase frequency characteristic (GPFC) of linear part of nonlinear measuring converter. Let's remind that the following comes true for the weight characteristic *g(t):*

$$g(t) = \begin{cases} g(t), & npu \;\; t \ge 0, \\ 0, & npu \;\; t < 0. \end{cases} \tag{9}$$

The given work presents the developed algorithm for opening the brackets in correlation [6] with the help of which this correlation can be represented in the form of

$$\begin{aligned} z(t) = {} & \varphi_{-kN}(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) \cdot e^{-jkN\omega_1 t} + \\ & + \varphi_{-(kN-1)}(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) \cdot e^{-j(kN-1)\omega_1 t} + \cdots \\ & \cdots + \varphi_{-1}(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) \cdot e^{-j\omega_1 t} + \\ & + \varphi_0(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) + \\ & + \varphi_1(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) \cdot e^{j\omega_1 t} + \cdots + \\ & + \varphi_{kN-1}(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) \cdot e^{j(kN-1)\omega_1 t} + \\ & + \varphi_{kN}(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) \cdot e^{jkN\omega_1 t}. \end{aligned} \tag{10}$$

Having developed the outcoming signal *z(t)* of nonlinear measuring converter into chopped series

$$z(t) = \sum_{n=-M}^{M} q_n \cdot e^{jn\omega_1 t}, \tag{11}$$

having set these series into correlation (10) and compared Fourier coefficients at the same frequency harmonics on the left and on the right of an equals sign we receive combined *(2N + 1)* equations

$$\begin{cases} \varphi_{-kN}(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) = q_{-kN}, \\ \varphi_{-(kN-1)}(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) = q_{-(kN-1)}, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \varphi_0(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) = q_0, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \varphi_{kN-1}(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) = q_{kN-1}, \\ \varphi_{kN}(\nu_{(\cdot)}, \gamma_{(\cdot)}, W(\cdot)) = q_{kN}, \end{cases} \tag{12}$$

which connects the multitude of Fourier coefficients $q_{(\bullet)}$ of outcoming signal *z(t)* of nonlinear measuring converter with the multitude of Fourier coefficients $\gamma_{(\bullet)}$ of its incoming signal *x(t)*, multitude of values $W(\bullet)$ of GPFC of linear part of converter on correspondent to harmonics frequencies and multitude $\nu_{(\bullet)}$ of parameters of nonlinear characteristic $f_k(y)$ of converter.

Solving recurrently the combined equations (12) relatively to Fourier coefficients $\gamma_{(\bullet)}$ of unknown signal *x(t)* by known Fourier coefficients $q_{(\bullet)}$ of outcoming signal *z(t),* known parameters $\nu_{(\bullet)}$ of nonlinearity and known values $W(\bullet)$ of GPFC of converter at correspondent to harmonics frequencies ω we can renew an incoming signal *x(t)* of nonlinear measuring converter of any complexity.

The work presents the developed correct recurrent procedure of solving the combined equations (12) and programme for its realization in Delphi programming language in Windows environment.

3. EXAMPLES

To receive the design relationship suitable for qualitative analysis, let's specify the number of the members *N* of the series (3) and the value of the index *k* of the multinomial (1).

Let's assume that *N* = 1, and *k* = 3, that is

$$x(t) = \sum_{n=-1}^{1} \gamma_n \cdot e^{jn\omega_1 t}, \tag{13}$$

$$f_3(y) = \nu_1 y + \nu_2 y^2 + \nu_3 y^3. \tag{14}$$

For this case the equation (10) can be rewritten as follows:

$$\begin{aligned} z(t) = {} & \varphi_{-3}(\nu_3, \gamma_{-1}, W(-j\omega_1)) \cdot e^{-j3\omega_1 t} + \\ & + \varphi_{-2}(\nu_3, \nu_2, \gamma_{-1}, \gamma_0, W(-j\omega_1), W(0)) \cdot e^{-j2\omega_1 t} + \\ & + \varphi_{-1}(\nu_3, \nu_2, \nu_1, \gamma_{-1}, \gamma_0, \gamma_1, W(-j\omega_1), W(0), W(j\omega_1)) \times \\ & \times e^{-j\omega_1 t} + \\ & + \varphi_0(\nu_3, \nu_2, \nu_1, \gamma_{-1}, \gamma_0, \gamma_1, W(-j\omega_1), W(0), W(j\omega_1)) + \\ & + \varphi_1(\nu_1, \nu_2, \nu_3, \gamma_{-1}, \gamma_0, \gamma_1, W(-j\omega_1), W(0), W(j\omega_1)) \times \\ & \times e^{j\omega_1 t} + \\ & + \varphi_2(\nu_2, \nu_3, \gamma_0, \gamma_1, W(0), W(j\omega_1)) \cdot e^{j2\omega_1 t} + \\ & + \varphi_3(\nu_3, \gamma_1, W(j\omega_1)) \cdot e^{j3\omega_1 t}. \end{aligned} \tag{15}$$

The expression (15) shows that the signal *z(t)*, apart from the first harmonica and the constant component, stipulated by the signal *x(t)*, set up by the cut series (3), contains the second and the third harmonicas, which are stipulated by nonlinearity (1).

So, having the realization of the output signal *z(t)* on the segment *T*, we set it up as the cut Furies series, which contains the constant component and the three first harmonicas, that is

$$z(t)=q_{-3}\cdot e^{-j3\omega_1 t}+q_{-2}\cdot e^{-j2\omega_1 t}+q_{-1}\cdot e^{-j\omega_1 t}+ \\ +q_0+q_1\cdot e^{j\omega_1 t}+q_2\cdot e^{j2\omega_1 t}+q_3\cdot e^{j3\omega_1 t}, \quad (16)$$

the Furies factors $q_n,\ n=(-3,-2,-1,\ 0,\ 1,\ 2,\ 3)$, calculated according to the formula (6).

$$q_n=\frac{1}{T}\int_0^T z(t)\cdot e^{-jn\omega_1 t}dt. \quad (17)$$

For *x(t), f₃(y), z(t)* set up by expressions (13), (14) and (16), the system of equations (12) will look like the following

$$\begin{cases}\varphi_{-3}(\nu_3,\gamma_{-1},W(-j\omega_1))=q_{-3},\\ \varphi_{-2}(\nu_3,\nu_2,\gamma_{-1},\gamma_0,W(-j\omega_1),W(0))=q_{-2},\\ \varphi_{-1}(\nu_3,\nu_2,\nu_1,\gamma_{-1},\gamma_0,\gamma_1,W(-j\omega_1),W(0),W(j\omega_1))=q_{-1},\\ \varphi_0(\nu_3,\nu_2,\nu_1,\gamma_{-1},\gamma_0,\gamma_1,W(-j\omega_1),W(0),W(j\omega_1))=q_0,\\ \varphi_1(\nu_1,\nu_2,\nu_3,\gamma_{-1},\gamma_0,\gamma_1,W(-j\omega_1),W(0),W(j\omega_1))=q_1,\\ \varphi_2(\nu_2,\nu_3,\gamma_0,\gamma_1,W(0),W(j\omega_1))=q_2,\\ \varphi_3(\nu_3,\gamma_1,W(j\omega_1))=q_3.\end{cases} \quad (18)$$

Specifying the functions $\varphi_{-3}(\cdot),\ \varphi_{-2}(\cdot),\ \varphi_{-1}(\cdot),\ \varphi_0(\cdot),\ \varphi_1(\cdot),\ \varphi_2(\cdot),\ \varphi_3(\cdot)$, instead of the non-determined equation system (18) we receive the equations system

$$\begin{cases}\nu_3\gamma_{-1}^3 W^3(-j\omega_1)=q_{-3},\\ \nu_2\gamma_{-1}^2W^2(-j\omega_1)+3\nu_3\gamma_{-1}^2\gamma_0W^2(-j\omega_1)\cdot W(0)=q_{-2},\\ \nu_1\gamma_{-1}W(-j\omega_1)+2\nu_2\gamma_{-1}\gamma_0W(-j\omega_1)W(0)+\\ +3\nu_3\gamma_{-1}^2\gamma_1W^2(-j\omega_1)W(j\omega_1)+\\ +3\nu_3\gamma_{-1}\gamma_0^2W(-j\omega_1)W^2(0)=q_{-1},\\ \nu_1\gamma_0W(0)+2\nu_2\gamma_{-1}\gamma_1W(-j\omega_1)W(j\omega_1)+\\ +\nu_2\gamma_0^2W^2(0)+6\nu_3\gamma_{-1}\gamma_0\gamma_1W(-j\omega_1)W(0)W(j\omega_1)+\\ +\nu_3\gamma_0^3W^3(0)=q_0,\\ \nu_1\gamma_1W(j\omega_1)+2\nu_2\gamma_0\gamma_1W(0)W(j\omega_1)+\\ +3\nu_3\gamma_{-1}\gamma_1^2W(-j\omega_1)W^2(j\omega_1)+\\ +3\nu_3\gamma_0^2\gamma_1W^2(0)W(j\omega_1)=q_1,\\ \nu_2\gamma_1^2W^2(j\omega_1)+3\nu_3\gamma_0\gamma_1^2W(0)W^2(j\omega_1)=q_2,\\ \nu_3\gamma_1^3W^3(j\omega_1)=q_3,\end{cases} \quad (19)$$

which can be used to conduct the specific calculations when performing the task of renewing its input signal.

Let us now consider the other example.

Let's assume that $N=2$, and $k=2$, that is

$$x(t)=\sum_{n=-2}^{2}\gamma_n\cdot e^{jn\omega_1 t}, \quad (20)$$

$$f_2(y)=\nu_1y+\nu_2y^2. \quad (21)$$

Performing the same succession of actions, as in the first example, we receive the following equation system, which connects the Furies factors of the input and output signals of the nonlinear dynamic system with their amplitude phase-response characteristic values on the corresponding harmonicas frequency signals:

$$\begin{cases}\nu_2\gamma_{-2}^2W^2(-j2\omega_1)=q_{-4},\\ 2\nu_2\gamma_{-1}\gamma_{-2}W(-j\omega_1)W(-j2\omega_1)=q_{-3},\\ 2\nu_2\gamma_0\gamma_{-2}W(0)W(-j2\omega_1)+\nu_2\gamma_{-1}^2W^2(-j\omega_1)+\\ +\nu_1\gamma_{-2}W(-j2\omega_1)=q_{-2},\\ 2\nu_2\gamma_1\gamma_{-2}W(j\omega_1)W(-j2\omega_1)+\\ +2\nu_2\gamma_0\gamma_{-1}W(0)W(-j\omega_1)+\nu_1\gamma_{-1}W(-j\omega_1)=q_{-1},\\ 2\nu_2\gamma_{-2}\gamma_2W(-j2\omega_1)W(j2\omega_1)+\\ +2\nu_2\gamma_{-1}\gamma_1W(-j\omega_1)W(j\omega_1)+\\ +\nu_2\gamma_0^2W^2(0)+\nu_1\gamma_0W(0)=q_0,\\ 2\nu_2\gamma_{-1}\gamma_2W(-j\omega_1)W(j2\omega_1)+\\ +2\nu_2\gamma_0\gamma_1W(0)W(j\omega_1)+\nu_1\gamma_1W(j\omega_1)=q_1,\\ 2\nu_2\gamma_0\gamma_2W(0)W(j2\omega_1)+\nu_2\gamma_1^2W^2(j\omega_1)+\\ +\nu_1\gamma_2W(j2\omega_1)=q_2,\\ 2\nu_2\gamma_1\gamma_2W(j\omega_1)W(j2\omega_1)=q_3,\\ \nu_2\gamma_2^2W^2(j2\omega_1)=q_4.\end{cases} \quad (22)$$

4. ANALYSIS OF THE RESULTS OBTAINED

Considering the equation systems (19) and (20), we can assert the following:

1. Even though the input signal *x(t)* doesn't contain the constant component, that is $\gamma_0=0$, this means that if nonlinear characteristics $f_k(y)$ has paired power (in our case ν_2y^2), in the output signal of the measuring converter *z(t)* there appears the constant component.

2. If the input signal has *x(t) N* harmonicas and degree *k* of the multinomial $f_k(y)$, which describes the nonlinearity of the measuring converter, the number of harmonics *M* in its output signal *z(t)* equals kN, this means that the following dependence is correct

$$M=kN. \quad (22)$$

3. The equation systems (19), (20) and in the general form (12) allow, when the nonlinearity $f_k(y)$ is set up in the task of signal renewing on the input of measuring converter, to determine the parameters of the required mathematical models with the help of recurrent procedure, shifting from extreme equations of the systems (19), (22) or (12) to their middle.

As the number of the equations in the systems (19), (22) or (12) is larger than the number of unknown parameters of the mathematical models synthesized, it will be appropriate to use the specific part of them for the determination of these unknown parameters, and the other — like the correctness criteria of task solving.

For example, when solving the task of the renewing the input signal *x(t)* with the application of the equation system (22), first according to the known values $\nu_2, q_4, q_{-4}, W(j2\omega_1), W(-j2\omega_1)$, the Furies factors γ_2, γ_{-2} are determined from the extreme equations. Then the Furies factors γ_1, γ_{-1} are determined from the equations for q_{-3} and q_3. And then from the other equation for q_{-2} or q_2, q_{-1} or q_1 or q_0 the Furies factor γ_0 is determined.

By substitution of the determined values $\gamma_{-2}, \gamma_{-1}, \gamma_0, \gamma_1, \gamma_2$ into the equations, which were not used for their determination, the extent of accuracy of the task solution of the renewing of the input signal *x(t)* in the kind of (20) is determined by the identical equality.

It's necessary to note that the recurrent procedures of the evaluation of the parameters values of the required mathematical models eliminate the problem of the incorrectness of the measuring converter input signals renewing tasks.

The authors had developed the automated computer algorithm for composing the equation systems (12) for the measuring converter with the nonlinearity of any degree of complexity and input signals with any number of harmonic components, as well as correct procedure of the solution of this equation system with the program for its realization on the Delphi programming language for Windows.

5. CONCLUSIONS

There had been suggested an algorithm of renewing the incoming signals of nonlinear measuring converters at their fixed outcoming signal with an application of the developed by authors method called Fourier-integral. An algorithm was realized in Delphi programming language in Windows environment.

REFERENCES

[1] Vashny E. Dynamics of measuring circuits. — M.: Energy, 1969. — 288 p. (Russian)

[2] Boris I. Mokin. The question of restoring signals of restricted duration. Metrology, 1980. — №9 p. 3–6. (Russian)

[3] Jozef Korbicz, Borys I. Mokin. METODY MATEMATYCZNE W ZAGADNIENIACH KONTROLI I STEROWANIA W ENERGETYCE. — ZIELONA GÓRA: WYDAWNICTWO WYŻSZEJ SZKOLY INŻYNIERSKIEJ, 1990. — 158 s.

[4] Boris I. Mokin Oleksandr B. Mokin. Generalization of Fourier integral method of identification and renewing of input signals of nonlinear dynamic systems. // Visnyk VPI. — 2002. — № 4 p.104–107. (Ukrainian)

[5] Van-Tris G. Sinthesis of optimal nonlinear control systems. — M.: Mir, 1964. — 167 p. (Russian)

AUTHORS: Stud. Olexander B. MOKIN. Dr. Sc., Prof., Acad. of UAPS Boris I. MOKIN, Vinnytsia State Technical University, TU Vinnytsia, 21021, Vinnytsia, Ukraine, Phone Int. (38)(0432) 32-57-18, Fax Int. (38)(0432) 46-71-25, E-mail: abmokin@vstu.vinnica.ua