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# **CROSSTALK BETWEEN TWISTED PAIRS WITH A MISMATCH OF THE PAIR EXTREMITIES**

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**Abstract** - A new crosstalk model between two twisted pairs when a mismatch of the wire pairs extremities exist, is proposed. The crosstalk model allows to predict the PSD at the near-end and far-end of the victim line if the PSD of the disturber is known. The model takes any mismatch into account which exists when a line is not terminated with its characteristic impedance. Using the same unbalance assumptions as other authors used with success in the past, analytical expressions are given to calculate the crosstalk.

**Keywords** transmission line, crosstalk, unbalance.

#### 1. INTRODUCTION

The crosstalk phenomenon was studied extensively in the past. Early work by Campbell [1] showed that crosstalk between adjacent wire-pairs could be predicted by expressing an unbalance between the wires. The unbalance function was found in terms of the mutual capacitance and inductances between the wires. Assuming that this unbalance function has a stochastic nature over the cable length, Cravis and Crater [2] derived expressions for the near-end crosstalk (NEXT) and far-end crosstalk (FEXT) power transfer functions. The simplified expressions of Cravis and Crater for NEXT and FEXT are frequently used to study the performance of telecommunication systems via simulation. Even nowadays, research about power back-off algorithms for VDSL and ADSL systems in order to reduce the crosstalk noise are using crosstalk models intensively [3]. Improvements of the current ADSL capabilities exploit crosstalk cancellation, leading to higher data rates [4][5]. Spectrum management of the access network of telecom operators rely on good crosstalk models. So, accurate crosstalk models are important.

The well known expressions, which give the Power Spectral Density (PSD) of the observed crosstalk voltage at the extremities of the victim line are used in a number of telecommunication standards, e.g. ADSL [6], SDSL en VDSL, and are the following:

$$
NEXT_{PSD} = |E|^2 \omega^2 K_n \frac{(1 - e^{-4\alpha L})}{4\alpha} \tag{1}
$$

$$
FEXT_{PSD} = |E|^2 \omega^2 e^{-2\alpha L} K_f L \tag{2}
$$

with E the source e.m.f.,  $\omega$  the angular frequency,  $K_n$  and  $K_f$  real constants,  $\alpha$  the line attenuation in Neper per meter and L is the length of the line. The expressions  $(1)$  and  $(2)$ were derived under the assumption that the victim and the disturber line are perfectly terminated, but in practice this is seldom the case.

In this paper an attempt will be undertaken to include the mismatch effect at the interfaces line extremity - load/ generator of the victim and disturber line. These mismatches allow that waves reflect at the line extremities and next propagate again on the lines causing again crosstalk on the victim line. Also, the induced waves in the victim line can reflect if a mismatch is present. The simple formulae (1) and (2) will strongly increase in complexity when these reflection are taking into account. It will be demonstrated that they exists both out of 4 contributions. An real-world example is given to show the effect of mismatch and to find out the dominant contributions.

## 2. GENERAL CONSTRUCTION AND ASSUMPTIONS

One considers two pairs of metallic wires where each extremity of the pair is terminated with an impedance. A generator E with internal impedance  $Z_{dn}$  drives the disturber line at the near-end (see Figure 1). Note that the first index refers to the disturber or victim line and the second index refers to the near-end or far-end of the same line.



Fig. 1 2 coupled lines where the disturber line is driven by a source at the near-end. A crosstalk voltage can be observed at both extremities of the victim line due to the electro-magnetic coupling between both lines.

In practice, one is only interested in the crosstalk between pairs in the same cable bundle. All wire pairs have the same physical dimensions, except from the twist rate and small locally varying wire positions. In order to keep the analysis surveyable, the following assumptions are used:

**Assumption 1** The victim and disturber lines are of the same type meaning that the propagation function and the characteristic impedance are equal

**Assumption 2** The propagation function and the characteristic impedance are independent of z.

**Assumption 3** The victim and disturber lines are uniform, have equal length and are electromagnetic coupled with each other over the total length.

## 3. COUPLING MECHANISM AND UNBALANCE

Suppose that there exists a point coupling over a length  $dz$  at distance  $z$ . We consider 2 coupling mechanism: inductive and capacitive. The point coupling will introduce waves in the victim line. They originate at distance  $z$  and propagate in opposite directions. These waves will arrive at the victim line extremities were a part of the wave will be absorbed by the termination impedance and the other part will be reflected back into the victim line.



Fig. 2 Point coupling at distance z between 2 balanced Fig. 2<br>circuits

The inductive coupling is caused by the mutual inductances between the conductors of the disturber and victim line (see Fig. 2). Campbell [1] showed that the inductive unbalance  $M_u$  is mainly caused by differences between the mutual inductances in the following way:

$$
M_u = (M_{13} + M_{24}) - (M_{23} + M_{14})
$$
 (3)

One can describe the inductive coupling effect by using a voltage source placed in series with the wires of the victim line (see figure 3). It represents the induced e.m.f.  $dV_v(z)$ into the wires over the length  $dz$ . The nett effect of the induced voltage is practically given in the s-domain by:

$$
dV_{v}(z) \approx M_{u}(z) I_{d}(z) s dz \tag{4}
$$

with  $M_u(z)$  the mutual inductance unbalance at distance z and  $I_d(z)$  the current in the disturber line at distance z.

The capacitive coupling is caused by the mutual capacitance between the conductors of the disturber and victim line (see figure 2). These capacitances form a bridge structure.

This coupling can be described by placing a current source between the 2 wires of the victim line (see figure 3). The current of that source is approximately given by:

$$
dI_{\nu}(z) \approx \frac{C_u(z)}{4} V_d(z) s dz
$$
 (5)

with  $C_u(z)$  the capacitive unbalance at distance z and  $V_d(z)$ the voltage on the disturber line at distance  $z$ .

$$
C_u = (C_{13} + C_{24}) - (C_{23} + C_{14})
$$
 (6)



Fig. 3 The crosstalk sources caused by a point coupling at distance *z.*

## 4. THE CROSSTALK CONTRIBUTION CAUSED BY A POINT COUPLING

The voltage and the current on a transmission line at a distance z from the source E with internal impedance  $Z_{dn}$ depend on the values of the source  $e.m.f. E$ , the impedances  $Z_{dn}$  and  $Z_{df}$  and the line properties given by  $\gamma$  and  $Z_c$ . If the value of the termination impedance at the load side differs from the characteristic impedance of the line then standing waves will appear across the line. The voltage and current at distance z are given by the following well known equations [9]:

$$
V_d(z) = \frac{(1 - \Gamma_{dn})}{2} E \frac{(1 + \Gamma_{df} e^{-2\gamma(L - z)})}{1 - \Gamma_{dn} \Gamma_{df} e^{-2\gamma L}} e^{-\gamma z}
$$
 (7)

$$
I_d(z) = \frac{(1 - \Gamma_{dn})}{2 Z_c} E \frac{(1 - \Gamma_{df} e^{-2\gamma(L - z)})}{1 - \Gamma_{dn} \Gamma_{df} e^{-2\gamma L}} e^{-\gamma z}
$$
(8)

with  $\Gamma_{ij}$  the reflection coefficient at the line extremity  $i \in [d, v]$  where d stands for the disturber line and v for the victim line and  $j \in [n, f]$  with n the near-end side and f the far-end side. All the reflection factors are expressed in base  $Z_c$  which means that:  $\Gamma_{ij} = (Z_{ij} - Z_c)/(Z_{ij} + Z_c)$ .

Each coupling mechanism will generate 2 voltage waves which will propagate in opposite directions from the point coupling position z. These waves will propagate on the victim line and cause standing waves if the line termination differs from its characteristic impedance. Again, the voltage seen at any load on the victim line can be calculated using a transmission line model that includes mismatches. Moreover, superposition will be used too to take both sources into account as shown in figure 4. To calculate the near-end voltage (figure 4b), the current source and the impedance will be replaced with its Thevenin source representation. The impedance  $Z_{in2}$  is the input impedance seen when one looks into the victim line, at distance  $z$ , in the far-end direction (figure 4). To calculate the far-end voltage (figure 4c), the current source and the impedance will be replaced with its Thevenin source representation.



Fig. 4 The crosstalk sources (a) caused by a point coupling at distance  $z$  in their Thevenin representation (b and  $\tilde{c}$ ) The impedance  $Z_{in1}$  is the input impedance seen when one

looks into the victim line, at distance  $z$ , in the near-end direction (figure 4).

After elaboration one can show that the contribution to the near-end voltage caused by an infinitesimal coupling at distance  $z$  on the victim line is given by:

$$
dV_{\nu}(0) = \frac{1}{4} \cdot \frac{(1 + \Gamma_{\nu n})(1 - \Gamma_{dn})}{(1 - \Gamma_{\nu n} \Gamma_{\nu f} e^{-2\gamma L})(1 - \Gamma_{dn} \Gamma_{df} e^{-2\gamma L})} \cdot Esdz
$$
  

$$
(C_n(z)e^{-2\gamma z} + C_f(z)\Gamma_{df}e^{-2\gamma L} + C_f(z)\Gamma_{\nu f}e^{-2\gamma L}
$$
  

$$
+ C_n(z)\Gamma_{df} \Gamma_{\nu f}e^{-4\gamma L}e^{2\gamma z})
$$
(9)

Also at the far-end, one can find a similar expression:

$$
dV_{\nu}(L) = \frac{1}{4} \cdot \frac{(1 + \Gamma_{\nu f})(1 - \Gamma_{dn}) e^{-\gamma L}}{(1 - \Gamma_{\nu n} \Gamma_{\nu f} e^{-2\gamma L})(1 - \Gamma_{dn} \Gamma_{df} e^{-2\gamma L})} \cdot Esdz
$$
  

$$
(C_f(z) + C_n(z) \Gamma_{\nu n} e^{-2\gamma z} + C_n(z) \Gamma_{df} e^{-2\gamma (L - z)}
$$
  

$$
+ C_f(z) \Gamma_{df} \Gamma_{\nu n} e^{-2\gamma L})
$$
(10)

with the following definitions for the unbalance functions:

$$
C_n(z) = \frac{C_u(z)}{4} Z_c + \frac{M_u(z)}{2Z_c}
$$
 (11)

$$
C_f(z) = \frac{C_u(z)}{4} Z_c - \frac{M_u(z)}{2Z_c}
$$
 (12)

One can assume that unbalance functions have the following statistical properties:

**Assumption 4** the unbalance functions  $C_n(z)$  and  $C_f(z)$ are normally distributed in amplitude, with zero mean and the covariance  $E[C_f(z)C_f(y)] = k_f \delta(z-y)$  with  $k_f$  a random variable, varying from pair combination to pair combination.

**Assumption 5** The reflections on the disturber and victim line will introduce the following covariance

 $E[C_f(z)C_n(y)] = k_{nf}\delta(z-y)$  with  $k_{nf}$  a random variable varying from pair combination to pair combination.

**Assumption 6** The unbalance functions  $C_n(z)$  and  $C_f(z)$ are real valued.

The assumptions are reasonable if the wavelength of the highest frequency component of the signal source is much larger than the twist rate. In that way we can treat the unbalance functions as random variables of which the value randomly varies from grid point to grid point on the length axis.

### 5. CROSSTALK CAUSED BY 2 COUPLED LINES

Next, one can calculate the crosstalk at the near-end or farend of the victim line when there is a distributed coupling over the whole line length  $L$  by using integration of and (10). Because the unbalance functions are random variables, one can only use the Power Spectral Density computation of the crosstalk signals at the line extremities.

$$
NEXT_{PSD} = E[V_v(0) \cdot V_v(0)^*]
$$
 (13)

$$
FEXT_{PSD} = E[V_v(L) \cdot V_v(L)^*]
$$
 (14)

or

$$
NEXT_{PSD} = E\left[\int_{0}^{L} dV_{\nu}(0) \cdot \int_{0}^{L} dV_{\nu}(0)^{*}\right]
$$
 (15)

$$
NEXT_{PSD} = E\left[\int_{0}^{L} dV_{\nu}(L) \cdot \int_{0}^{L} dV_{\nu}(L)^{*}\right]
$$
 (16)

where ' $*$ ' stands for the complex conjugate and  $E[$  ] denotes the mathematical expectation operator.

Inspection of and (10) shows that the equations can be described by the product of 2 functions: one function  $\Phi$  in the variables s and z and the other function  $\varphi$  only dependent of  $s$ . So, and  $(10)$  can be written as:

$$
dV_{v}(0) = \varphi_{n}(s) \cdot \Phi_{n}(s, z) \cdot dz \qquad (17)
$$

$$
dV_{v}(L) = \varphi_{f}(s) \cdot \Phi_{f}(s, z) \cdot dz \qquad (18)
$$

with

$$
\varphi_n(s) = \frac{1}{4} \frac{(1 + \Gamma_{vn})(1 - \Gamma_{dn})}{(1 - \Gamma_{vn} \Gamma_{vf} e^{-2\gamma L})(1 - \Gamma_{dn} \Gamma_{df} e^{-2\gamma L})} \cdot E \cdot s
$$
 (19)  

$$
\Phi_n(s, z) = C_n(z) e^{-2\gamma z} + C_f(z) \Gamma_{df} e^{-2\gamma L} + C_f(z) \Gamma_{df} \Gamma_{vf} e^{-4\gamma L} e^{2\gamma z}
$$
 (20)

and

$$
\varphi_f(s) = \frac{1}{4} \frac{(1 + \Gamma_{vf})(1 - \Gamma_{dn}) e^{-\gamma L}}{(1 - \Gamma_{vn} \Gamma_{vf} e^{-2\gamma L})(1 - \Gamma_{dn} \Gamma_{df} e^{-2\gamma L})} \cdot E \cdot s \quad (21)
$$

$$
\Phi_f(s, z) = C_f(z) + C_n(z)\Gamma_{vn}e^{-2\gamma z} + C_n(z)\Gamma_{df}e^{-2\gamma(L-z)} + C_f(z)\Gamma_{df}\Gamma_{vn}e^{-2\gamma L}
$$
\n(22)

Now, using (17) and (18) the PSD equations (15) and (16) can be generically written as

$$
iEXT_{PSD} = |\varphi_i(s)|^2 \iint_{0}^{LL} E[\Phi_i(s, z)\Phi_i(s, y)^*]dzdy \qquad (23)
$$

where  $i \in [n, f]$  with *n* the near-end side and f the far-end side. Further elaboration of (23) gives

$$
iEXT_{PSD} = |\varphi_i(s)|^2 \iint_{0}^{LL} E\left[\sum_{k=1}^{4} X_{ik}(z) \sum_{k=1}^{4} X_{ik}(y)^* \right] dz dy
$$
 (24)

or

$$
iEXT_{PSD} = |\varphi_i(s)|^2 \iint_{0}^{LL} E\left[\sum_{k=1}^{4} \sum_{l=1}^{4} X_{ik}(z)X_{il}(y)^* \right] dz dy \tag{25}
$$

with the variables  $X_{ij}$  for NEXT ( $i = n$ ) given by:

$$
X_{n1}(z) = C_n(z)e^{-2\gamma z}
$$
  
\n
$$
X_{n2}(z) = C_f(z)\Gamma_{df}e^{-2\gamma L}
$$
  
\n
$$
X_{n3}(z) = C_f(z)\Gamma_{vf}e^{-2\gamma L}
$$
  
\n
$$
X_{n4}(z) = C_n(z)\Gamma_{df}\Gamma_{vf}e^{-4\gamma L}e^{2\gamma z}
$$
\n(26)

and for FEXT T  $(i = f)$ 

$$
X_{f1}(z) = C_f(z)
$$
  
\n
$$
X_{f2}(z) = C_n(z)\Gamma_{vn}e^{-2\gamma z}
$$
  
\n
$$
X_{f3}(z) = C_n(z)\Gamma_{df}e^{-2\gamma L}e^{2\gamma z}
$$
  
\n
$$
X_{f4}(z) = C_f(z)\Gamma_{df}\Gamma_{vn}e^{-2\gamma L}
$$
\n(27)

Using assumptions 4, 5 and 6 and the following two lemma's, one can calculate the crosstalk PSD given by (25).

**Lemma 1** If  $a$  and  $b$  are complex random variables, then  $E[ab^* + ba^*] = 2E[|a||b|\cos(\angle a - \angle b)]$ . The proof can be easily found using polar coordinates for  $a$  and  $b$  and using the relation  $\cos(x) = (e^{jx} + e^{-jx})/2$ .

**Lemma 2** 

$$
\int e^{ax} \cos(bx+c) dx = e^{ax} \frac{a \cos(bx+c) + b \sin(bx+c)}{a^2 + b^2}
$$

The proof is found using  $cos(x) = (e^{jx} + e^{-jx})/2$ .

After elaboration, one obtains 4 crosstalk terms for NEXT:

$$
NEXT_{PSD, 1} = |\varphi_n(s)|^2 k_n \frac{(1 - e^{-4\alpha L})}{4\alpha}
$$
(28)  
\n
$$
NEXT_{PSD, 2} = |\varphi_n(s)|^2 k_n e^{-4\alpha L} |\Gamma_{df}\Gamma_{vf}|
$$
  
\n
$$
\left[\Gamma_{df}\Gamma_{vf} \mid e^{-4\alpha L} \frac{(e^{4\alpha L} - 1)}{4\alpha} + \frac{1}{\alpha \beta} \right]
$$
  
\n
$$
2 \frac{\sin(\angle \Gamma_{vf} + \angle \Gamma_{df}) - \sin(-4\beta L + \angle \Gamma_{vf} + \angle \Gamma_{df})}{4\beta}
$$

$$
NEXT_{PSD, 3} = |\varphi_n(s)|^2 k_f e^{-4\alpha L} L
$$
  

$$
\{|\Gamma_{df}|^2 + |\Gamma_{vf}|^2 + 2|\Gamma_{df}\Gamma_{vf}|\cos(\angle\Gamma_{df} - \angle\Gamma_{vf})\}
$$
(30)

$$
NEXT_{PSD, 4} = |\varphi_n(s)|^2 2 k_{nf} e^{-2\alpha L}.
$$
  
\n
$$
\left\{ |\Gamma_{df}| \theta_1(\Gamma_{df}) + |\Gamma_{vf}| \theta_1(\Gamma_{vf}) + |\Gamma_{vf}| \theta_2(\Gamma_{df})) \right\}
$$
  
\n
$$
|\Gamma_{df}| |\Gamma_{vf}| e^{-4\alpha L} \cdot (|\Gamma_{df}| \theta_2(\Gamma_{vf}) + |\Gamma_{vf}| \theta_2(\Gamma_{df}))
$$
  
\n
$$
\theta_1(X) = \frac{\left(\frac{2\alpha \cos(2\beta L - \angle X) + 2\beta \sin(2\beta L - \angle X) - \rho_2(\Delta L)\right)}{4\alpha^2 + 4\beta^2}}{4\alpha^2 + 4\beta^2}
$$
  
\n
$$
\theta_2(X) = \frac{\left(\frac{-2\alpha \cos(2\beta L - \angle X) + 2\beta \sin(2\beta L - \angle X) - \rho_2(\Delta L)\right)}{4\alpha^2 + 4\beta^2}}{4\alpha^2 + 4\beta^2}
$$
  
\n(33)

and the total near-end crosstalk PSD is obtained with:

$$
NEXT_{PSD} = \sum_{k=1}^{4} NEXT_{PSD,k} \tag{34}
$$

For FEXT similar equations can be found:

*FEXTPSD*, <sup>1</sup>

$$
FEXT_{PSD, 1} = |\varphi_f(s)|^2 k_f L
$$
\n
$$
FEXT_{PSD, 2} = |\varphi_f(s)|^2 k_f e^{-2\alpha L} L |\Gamma_{df}\Gamma_{vn}|
$$
\n
$$
\left\{ \begin{array}{c} |\Gamma_{df}\Gamma_{vn}| e^{-2\alpha L} + \\ 2 \cos(2\beta L - \angle \Gamma_{vn} - \angle \Gamma_{df}) \end{array} \right\}
$$
\n(36)

$$
FEXT_{PSD, 3} = |\varphi_f(s)|^2 k_n
$$
  
\n
$$
\left\{\n\begin{aligned}\n|\Gamma_{vn}|^2 \frac{(1 - e^{-4\alpha L})}{4\alpha} + |\Gamma_{df}|^2 \frac{(1 - e^{-4\alpha L})}{4\alpha} + \n2 |\Gamma_{df}\Gamma_{vn}| e^{-2\alpha L} .\n\end{aligned}\n\right\}
$$
\n
$$
\frac{\sin(2\beta L + \angle \Gamma_{vn} - \angle \Gamma_{df}) - \sin(-2\beta L + \angle \Gamma_{vn} - \angle \Gamma_{df})}{4\beta}
$$
\n(37)

$$
FEXT_{PSD, 4} = |\varphi_f(s)|^2 2 k_{nf}.
$$
\n
$$
\left\{ |\Gamma_{vn}| \Theta_1(\angle \Gamma_{vn}) + |\Gamma_{df}| \Theta_2(\angle \Gamma_{df}) + |\Gamma_{df}| \Theta_1(2\beta L - \angle \Gamma_{df}) + |\Gamma_{df}| |\Gamma_{vn}| e^{-2\alpha L} \cdot \left( |\Gamma_{vn}| \Theta_1(2\beta L - \angle \Gamma_{df}) + |\Gamma_{df}| e^{-2\alpha L} \Theta_2(2\beta L - \angle \Gamma_{vn}) \right) \right\}
$$
\n
$$
\Theta_1(X) = \frac{\left( e^{-2\alpha L} (2\alpha \cos(2\beta L - X) - 2\beta \sin(2\beta L - X)) \right)}{4\alpha^2 + 4\beta^2} (39)
$$
\n
$$
\left( \frac{-2\alpha \cos(2\beta L - X) + 2\beta \sin(2\beta L - X) + 2\beta \cos(2\beta L - X) + 2\beta \sin(2\beta L - X) + 2\beta \cos(2\beta L - X)
$$

$$
\Theta_2(X) = \frac{\left(-e^{2\alpha L}(2\alpha \cos(X) + 2\beta \sin(X))\right)}{4\alpha^2 + 4\beta^2} \tag{40}
$$

and the total far-end crosstalk PSD is obtained with:

$$
FEXT_{PSD} = \sum_{k=1}^{4} FEXT_{PSD,k} \tag{41}
$$

Special case: If the line extremities are perfectly matched meaning that  $\Gamma_{ij} = 0$  for  $i \in [d, v]$  and  $j \in [n, f]$ , then one should obtain the well known crosstalk formulae (1) and (2). Indeed:

(35)

$$
NEXT_{PSD} = \frac{1}{16}|E|^2 \omega^2 k_n \frac{(1 - e^{-4\alpha L})}{4\alpha} \tag{42}
$$

$$
FEXT_{PSD} = \frac{1}{16} |E|^2 \omega^2 e^{-2\gamma L} k_f L \tag{43}
$$

with  $K_n = k_n / 16$  and  $K_f = k_f / 16$ .

## 6. APPROXIMATED NEXT AND FEXT

A first order approximation of the final crosstalk PSD expressions can be realized by assuming that reflections, either on the disturber as the victim line, do not significantly contribute to crosstalk. The mismatch at the line extremities are the main source for the observed deviation between the idealized and true crosstalk expressions. These approximations are:

$$
NEXT_{PSD} = |(1 + \Gamma_{vn})|^2 |(1 - \Gamma_{dn})|^2 \frac{|E|^2}{16} \omega^2 k_n \frac{(1 - e^{-4\alpha L})}{4\alpha}
$$
\n(44)

$$
FEXT_{PSD} = |(1 + \Gamma_{vf})|^2 |(1 - \Gamma_{dn})|^2 \cdot \frac{|E|^2}{16} \omega^2 e^{-2\gamma L} k_f L
$$
\n(45)

#### 7. CROSSTALK IN A SIMULATED ENVIRONMENT

The results depend on the cable behaviour. The VUBmodel [7][8] was used to simulate the propagation function γ and the characteristic impedance  $Z_c$ . The parameters of the VUB-model were estimated from data taken from a 20 pair 0.5mm polyethylene cable (BELGACOM 32 20). The coupling constants were chosen based on a NEXT measurement, as shown in figure 5, from pair 1 to pair 6. A NEXT coupling constant of -62dB@1MHz was found, which results in  $k_n = 2.8 \cdot 10^{-21}$  and  $k_n = k_f = k_{nf}$  was assumed. The disturber impedance =  $135\Omega$ , the victim Impedance =  $100\Omega$ , and the line length amounted to 300m  $k_n = 2.8 \cdot 10^{-21}$  and  $k_n = k_f = k_{nf}$ 135Ω

As can be seen in figures 6-9, the approximated crosstalk model (44) and (45) is very useful above 30kHz. In figures 7 and 9, a comparison is made with the crosstalk model from the standards (ETSI, ITU and ANSI). There is an over estimation of 1,5 dB with the standard crosstalk formulation (1) and (2). For frequencies below 30kHz, the true model should be used, otherwise the approximated model will do the job. It can also be observed that the dominant contribution to the crosstalk PSD is contribution 1, which describes the crosstalk when there are no reflections on the lines.

Remark that the ETSI-models are even further reduced, using the high frequency approximation  $\alpha \approx K \sqrt{\omega}$ , into:

$$
NEXT_{PSD} = |E|^2 \omega^{1.5} K_n' \tag{46}
$$

$$
FEXT_{PSD} = |E|^2 \omega^2 e^{-2\alpha L} K_f L \tag{47}
$$



Fig. 5 The measured NEXT transfer function from pair1 to 6 (matched case).



Fig. 6 The near-end crosstalk contributions.



Fig. 7 The difference between the NEXT formulations.



Fig. 8 The far-end crosstalk contributions



Fig. 9 The difference between the FEXT formulations.

## 8. CONCLUSIONS

Exact expressions to describe the PSD of the near-end and far-end crosstalk signals when the line extremities were terminated with an impedance different than the characteristic impedance were derived. If one neglects the crosstalk caused by the reflections on both the disturber line and the victim line, very simple formulae are obtained which take into account de power loss due to the mismatch. It was shown that the approximated formulae perform very well above 30kHz.

The presented model can also serve as a basis to study the crosstalk behaviour on access networks when for example a bridged taps exist in a loop or when a loop is put together with different line types. Hereto, the loop can be subdivided in segments Each segment can be treated as a transmission system where mutual couplings exist and where the lines may be terminated arbitrary.

The crosstalk expressions gave rise to a new crosstalk constant namely  $k_{nf}$ . It's value is rather difficult to obtain from measurements because a measured crosstalk transfer function is not a straight line any more in the dB-logfrequency plane but it shows a pole-zero behaviour at frequencies typically above 100kHz even in the matched case. Moreover, most telecom operators have conducted crosstalk measurements in order to gather information about  $k_n$  and  $k_f$ , but not  $k_{nf}$ . Therefore further investigation is necessary to derive  $k_{nf}$  from  $k_n$  and  $k_f$ .

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