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SUBDIVISION OF THE UNIT OF MASS USING WEIGHT SUPPORT PLATES

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Abstract – The physical model for the subdivision of the kilogram into the decade from 1 kg to 100 g was adapted for the measurement system where weight support plates have to be used. That is the case when combinations of weights with different nominal masses are compared. For this purpose the calibration procedure was modified to eliminate the unknown masses of the support plates. The equation was derived to take into account various influences on the measured mass differences. The influence of the plates on the measurement uncertainty budget and on the estimates of unknown masses of weights was studied into the decade. The analysis of results shows that the application of the support plates influences the measurement uncertainty to a small extent and also confirms the adequacy of the used model.

Keywords: mass calibration, subdivision of the kilogram.

1. INTRODUCTION

The Mass Laboratory of Metrology Institute of the Republic of Slovenia (MIRS) is a holder of the Slovenian national standard of mass. At present it is represented by two weight sets of E1 accuracy class [1] in the range from 1 mg to 10 kg. Their traceability is assured by Physikalisch-Technische Bundesanstalt (PTB) [2, 3].

Present work of the laboratory is focused on an autonomous realisation of the Slovenian mass scale in the E1 accuracy level by subdivision and multiplication of the kilogram [4]. The subdivision and multiplication procedure, as described in [5], has to be adapted to the standards and measurement equipment which are at the laboratory's disposal. The design of the comparator balance and weights requires the usage of a set of weight support plates during measurements when combinations of weights with different nominal masses are compared. The influence of the support plates on the measurement process has to be evaluated, especially their influence on the measurement results and their contribution to the measurement uncertainty budget.

The final goal is to realize the national mass standard as a single 1 kg stainless steel weight (or as a group of 1 kg standards) and to carry out its subdivision and multiplication in the range 1 mg to 10 kg with measurement uncertainties equal or better than that of E1 accuracy class.

2. MEASUREMENT SYSTEM

The measurement system consisted of a comparator balance, a set of weight support plates, standard weights and a device for the measurement of ambient conditions. The measurement system is shown in Fig. 1.

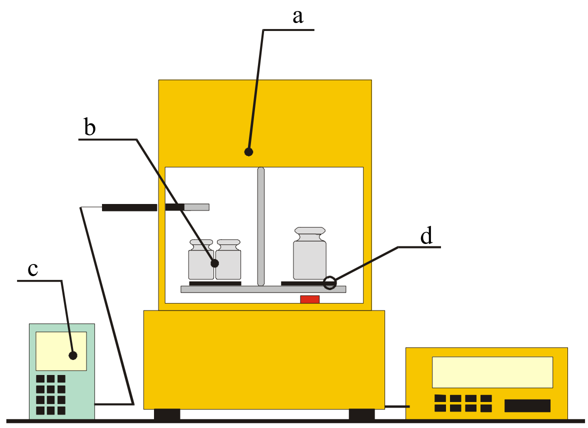

Fig. 1. A measurement system consisted of a) comparator balance, b) standard weight, c) measuring device for ambient conditions, d) weight support plate

Measurements were performed on the Sartorius C1000 comparator with a scale division of 1 µg and a pooled standard deviation from 5 µg to 15 µg for nominal masses from 100 g to 1 kg, respectively.

TESTO 454 data logger with an absolute air pressure sensor and a combined temperature/relative humidity sensor was used to measure ambient conditions. The expanded measurement uncertainties of temperature, relative humidity and pressure sensors were 0,2 K, 2 % and 1 hPa, respectively. Air temperature and relative humidity were measured inside the weighing chamber of the comparator, as shown in Fig. 1.

Four pairs of standard weights with nominal masses 1 kg, 500 g, 200 g and 100 g were selected from two E1 accuracy class weight sets produced by Mettler Toledo and Sartorius. The weights are OIML shaped, their volumes are known from the calibration certificates [2, 3]. One of the 1 kg weights was used as a reference standard with presumed expanded (quoted at k=2 in the whole text) measurement uncertainty of 0,05 mg. Its mass value was taken from the calibration certificate [3]. Their volumes V and standard uncertainties $u(V)$ are presented in Table I. The weights were indexed from $j = 1,\ldots, 8$.

TABLE I. The volumes and the standard uncertainties of standard weights used [2, 3]

Nominal mass and mark	j	V cm^3	$u(V)$ cm^3
1 kg	1	124,271	0,020
1 kg *	2	125,597	0,020
500 g	3	63,652	0,010
500 g *	4	62,754	0,010
200 g	5	24,853	0,006
200 g *	6	24,852	0,006
100 g	7	12,4788	0,0035
100 g *	8	12,5601	0,0035

Two weight support plates, made of aluminium, type YWP02C, producer Sartorius, are shown in Fig. 2. Their volumes V_{P1} and V_{P2} were estimated to 3,2 cm^3 and 3,3 cm^3, respectively, with expanded measurement uncertainty 0,1 cm^3. The support plates were used to place more than one weight simultaneously on the load receptor of the comparator.

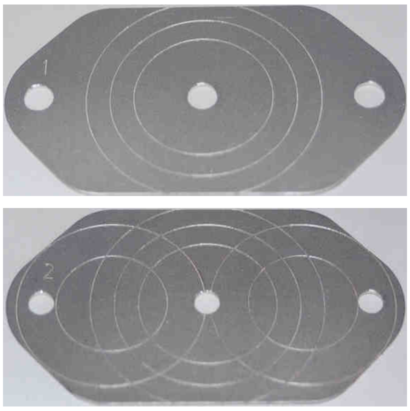

Fig. 2. A set of weight support plates for the C1000 comparator

3. CALIBRATION PROCEDURE

The design shown in Table II was used for the subdivision of the kilogram. The weighing scheme consisted of 10 series of comparison measurements, $i = 1,\ldots,10$, of 8 weights. When a group of weights was compared with a single weight the support plates were used. That was the case during the comparisons $i = 2, 3, 5, 6, 8$ and 9.

TABLE II. Weighing scheme in the range 1 kg – 100 g

	Nominal masses (kg)								plates
i	1	1*	0,5	0,5*	0,2	0,2*	0,1	0,1*	used
1	**1**	**-1**	0	0	0	0	0	0	no
2	**1**	0	**-1**	**-1**	0	0	0	0	yes
3	0	**1**	**-1**	**-1**	0	0	0	0	yes
4	0	0	**1**	**-1**	0	0	0	0	no
5	0	0	**1**	0	**-1**	**-1**	**-1**	0	yes
6	0	0	0	**1**	**-1**	**-1**	0	**-1**	yes
7	0	0	0	0	**1**	**-1**	0	0	no
8	0	0	0	0	**1**	0	**-1**	**-1**	yes
9	0	0	0	0	0	**1**	**-1**	**-1**	yes
10	0	0	0	0	0	0	**1**	**-1**	no

A weighted least square method, as described in [5], was used to estimate unknown masses of the weights. The singularity problem was solved by using the method of Lagrangian multipliers where the mass of the 1 kg weight, denoted as "1" in Table II, was used as a restraint. Unknown masses m_j, $j = 1,\ldots, 8$, were estimated by:

$$m_j = h_j \cdot m_{ref} + \sum_{i=1}^{n} \left(g_{ij} \cdot y_i \right) \tag{1}$$

where h_j denotes a ratio between nominal masses of a respective weight and the reference weight, m_{ref} mass of the reference weight, g_{ij} elements of the design matrix and y_i the mass difference between compared weights in the i-th comparison, $i = 1,\ldots, 10$.

When the support plates are used, the measurement result of a series of comparisons is not reflected only by a difference of apparent masses of compared weights but also includes unknown mass difference between the support plates. Therefore the influence of unknown masses of the support plates on the weighing difference has to be eliminated from the problem. This can be done by a repetition of the measurement series with an exchanged position of the plates while the positions of the weights remained unchanged.

As the illustration of the procedure the simplified weighing equation for the first and second repetition of the measurement series, which considers only the measured weighing difference [6], can be written down. A simplified weighing equation for the first repetition of the measurement series equals:

$$m_T + m_{P1} = m_R + m_{P2} + \Delta_I , \tag{2}$$

where m_{P1} and m_{P2} denote the masses of used support plates, m_T the mass of weights on position T, m_R the mass of weight on position R and Δ_1 the measured mass difference of the first series, while a simplified weighing equation for the second repetition of the measurement series equals:

$$m_T + m_{P2} = m_R + m_{P1} + \Delta_{II} , \tag{3}$$

where Δ_2 denotes the measured mass difference of the second measurement series.

Equations (2) and (3) can be reshaped to expose the difference between the masses of the support plates:

$$m_{P1} - m_{P2} = m_T - m_R + \Delta_I , \tag{4}$$

$$m_{P1} - m_{P2} = m_R - m_T - \Delta_{II} . \tag{5}$$

If the right sides of (4) in (5) are made equal, the result is:

$$m_R - m_T = -\frac{\Delta_I + \Delta_{II}}{2} . \tag{6}$$

The same procedure is used when influencing factors due to weighing under real laboratory conditions are taken into account. The mass difference y_i equals the average measured mass difference corrected for the influences of the difference in levels of the centres of gravity of compared weights, the air buoyancy and the volume thermal expansion of the weights and the support plates. The mass difference was estimated by:

$$y_i = \Bigg[-\frac{\Delta_I + \Delta_{II}}{2} + m_N \frac{\partial g}{\partial z} \frac{z_T - z_R}{g} + \frac{(\rho_{a,I} + \rho_{a,II})}{2}(V_R - V_T) + \frac{(\rho_{a,I} - \rho_{a,II})}{2}(V_{P2} - V_{P1}) + \alpha_{SS} \frac{(\rho_{a,I}(t_I - t_0) + \rho_{a,II}(t_{II} - t_0))}{2}(V_R - V_T) + \alpha_{AL} \frac{(\rho_{a,I}(t_I - t_0) - \rho_{a,II}(t_{II} - t_0))(V_{P2} - V_{P1})}{2} \Bigg]_i \quad (7)$$

where indexes I in II refer to the first and the second repetition of the measurement series, respectively, R and T to the position of weights on the weight exchange mechanism during the measurements, P1 and P2 to the first and the second support plate, SS and AL to stainless steel and aluminium. Δ is the average measured mass difference between compared weights, ρ_a air density, z the level of the centre of gravity of weights, t the air temperature, t_0 the reference temperature, V the volume, α the thermal expansion coefficient, g the value of gravity and m_N the nominal mass of weights. The first summand on the right side of (7) represents the average measured mass difference of the compared weights, the second the correction of the influence due to the difference in levels of the centres of gravity, the third the air buoyancy correction of the weights, the fourth the air buoyancy correction of weights, the fifth the correction due to volume thermal expansion of the weights and the last the correction due to the volume thermal expansion of the support plates. For the measurements where the support plates are not used, (7) reduces to:

$$y_i = \left[m_N \left(1 - \frac{\partial g}{\partial z} \frac{z_T - z_R}{g} \right) - \rho_a (1 + \alpha_{SS}(t - t_0))(V_R - V_T) - \Delta \right]_i \quad (8)$$

4. MEASUREMENT UNCERTAINTY

A combined standard measurement uncertainty u_c of masses m_j, as defined by (1), was estimated by using the equation for independent input quantities [7]:

$$u_c^2(m_j) = \sum_i \left(\frac{\partial m_j}{\partial x_i} \right)^2 u^2(x_i), \quad (9)$$

where x_i are input variables, which are given in (7). The coverage factor k=2 was used to multiply the u_c to calculate expanded measurement uncertainty U.

An exception to (9) was made when the contribution of variability of measurement results of the comparator was estimated. The contribution was introduced as variance of (1). The standard deviation was estimated both on the basis of residuals resulting from the least square method and the pooled standard deviation of the comparator.

5. RESULTS

Based on (7) and (8), estimates of the mass differences are calculated for each series. Whereas the measured mass difference and the air buoyancy correction of weights are not the focus of the article, in Table III corrections due to the thermal expansions of weights, the difference between centres of gravity of weights and the air buoyancy correction of the support plates are presented. The correction of thermal expansions of support plates is not stated, since it is negligible in comparison to the values stated in Table III. Air density ranged between measurements from 1,1481 kg/m^3 to 1, 1615 kg/m^3. The largest difference of air density between the repetitions of series added up to 0,0017 kg/m^3 at i = 6. Temperature between measurements ranged from 21,0 ºC to 21,5 ºC. The largest difference of temperature between repetitions of series added up to 0,3 ºC at i = 9.

TABLE III. Calculated corrections of apparent mass in μg

i	Thermal expansions of weights	Difference between centres of gravity of weights	Air buoyancy correction of support plates
1	-0,080	0,153	0,000
2	-0,130	2,294	-0,002
3	-0,054	2,447	0,011
4	0,075	0,031	0,000
5	0,090	1,407	-0,001
6	0,034	1,377	-0,085
7	0,000	0,000	0,000
8	-0,013	0,226	-0,004
9	-0,011	0,226	-0,065
10	-0,005	0,000	0,000

Values of standard uncertainties resulting from eight different input sources are presented in Table IV for nominal masses from 1 kg to 100 g. The input sources of the standard uncertainties are quoted in the first column of the table.

TABLE IV. Standard uncertainty contributions in mg

	1 kg *	500 g	200 g	100 g
Reference weight	$2,5{\cdot}10^{-2}$	$1,3{\cdot}10^{-2}$	$5,0{\cdot}10^{-3}$	$2,5{\cdot}10^{-3}$
Difference between centres of gravity	$7,0{\cdot}10^{-4}$	$3,6{\cdot}10^{-4}$	$1,9{\cdot}10^{-4}$	$1,0{\cdot}10^{-4}$
Volume of support plates	$5,1{\cdot}10^{-6}$	$1,9{\cdot}10^{-5}$	$3,4{\cdot}10^{-5}$	$2,0{\cdot}10^{-5}$
Volume of weights	$1,3{\cdot}10^{-2}$	$7,6{\cdot}10^{-3}$	$4,6{\cdot}10^{-3}$	$2,9{\cdot}10^{-3}$
Air density	$7,8{\cdot}10^{-4}$	$5,0{\cdot}10^{-4}$	$2,4{\cdot}10^{-4}$	$1,4{\cdot}10^{-4}$
Thermal expansion of weights	$8,7{\cdot}10^{-6}$	$5,9{\cdot}10^{-6}$	$2,7{\cdot}10^{-6}$	$1,6{\cdot}10^{-6}$
Thermal expansion of plates	$4,0{\cdot}10^{-7}$	$3,7{\cdot}10^{-7}$	$3,7{\cdot}10^{-7}$	$2,9{\cdot}10^{-7}$
Repeatability of comparator	$1,2{\cdot}10^{-2}$	$7,9{\cdot}10^{-3}$	$5,2{\cdot}10^{-3}$	$3,8{\cdot}10^{-3}$

Relative portions of values of squares of the standard measurement uncertainties for each single nominal mass are shown in Fig. 3.

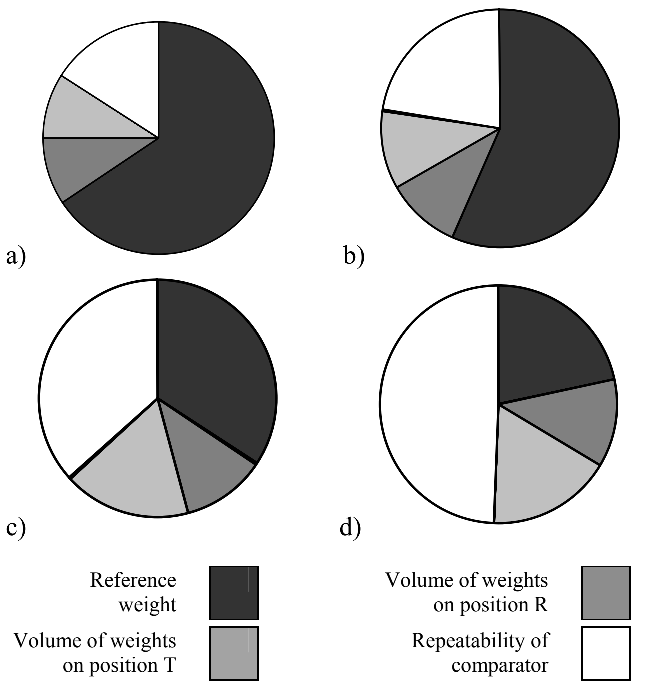

Fig. 3. Relative portions of squares of the standard uncertainty contributions for a) 1 kg *, b) 500 g, c) 200 g and d) 100 g weight

Estimated deviations from nominal masses δ and belonging expanded uncertainties U are presented in Table V. The results of the subdivision procedure are compared with data from the calibration certificates [2, 3]. The differences in values are normalised by using [8]:

$$E_n = \frac{\delta_{subdivison} - \delta_{certficates}}{\sqrt{U^2_{subdivision} + U^2_{certficates}}}. \qquad (10)$$

TABLE V. Comparison of measurement results of subdivision procedure and results form their calibration certificates [2, 3]

nominal mass	subdivision		calibration certificates		E_n
	δ mg	U mg	δ mg	U mg	
1 kg	*-0,810*	*0,050*	-0,81	0,15	/
1 kg *	0,674	0,062	0,65	0,15	0,15
500 g	1,508	0,033	1,521	0,075	0,16
500 g *	0,326	0,033	0,306	0,075	0,24
200 g	-0,176	0,017	-0,163	0,030	0,38
200 g *	-0,196	0,017	-0,175	0,030	0,61
100 g	-0,057	0,011	-0,053	0,015	0,22
100 g *	0,042	0,011	0,043	0,015	0,05

6. CONCLUSIONS

According to (2), the use of the support plates does not significantly influence the calculation of the mass difference under laboratory conditions presented. The influence is reduced with smaller difference in the volume of the support plates and the difference in the air density between the repetitions of the series. The application of the support plates also does not introduce significant uncertainty contributions (see Table IV and Fig. 3). The most significant uncertainty components remain the uncertainty of the reference, the uncertainty of the volumes of standards and the uncertainty due to the repeatability of the comparator. The measurement equipment, which is at the laboratory's disposal, enables the subdivision of 1 kg in the range to 100 g with uncertainties lower than those of the E1 accuracy class. The comparison of results of the subdivision to those from the certificates (see Table III) shows that there is no evidence that the usage of weight support plates systematically affects the measurement results. The adequacy of the model used can therefore be confirmed.

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