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CORRECTIONS OF AERODYNAMIC LOADINGS MEASUREMENT ON VBRATING AIRFOILS

V. Tsymbalyuk, and J. Linhart

(New Technology Research Centre, University of West Bohemia), Pilsen, Czech Republic

Abstract – This paper deals with the specification of method accuracy enhancement for unsteady aerodynamic forces and moments in an airfoil cascade. These forces are induced by forward and angular vibrations of airfoils. The improvement is achieved by taking airfoil and other elements deformations in consideration in the calibration and measurement process.

Keywords: unsteady aerodynamic load, airfoil vibration, electrodynamic vibrator.

1. INTRODUCTION

To analyse the turbomachine blade dynamic stability in a stream, unsteady aerodynamic loads induced by blade oscillations must be measured. Usually, these loads are measured on airfoil cascades.

The cascade of airfoils of an axial compressor is shown in Fig. 1. The aerodynamic load at arbitrary airfoil vibration can be described by both forces L, K and moment M. The influence of airfoil oscillations along the axis X, and also the force K [1] can be neglected. Therefore, it is enough to measure force L and moment M at forward y and angular α vibration displacements of airfoils.

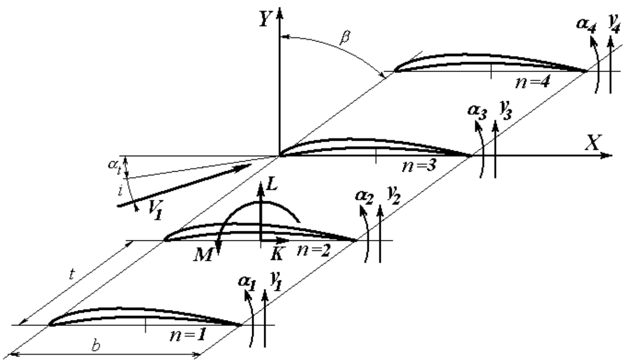


Fig. 1 Diagram of a compressor cascade of airfoils
b - chord of an airfoil, t - step of a cascade, β - stage angle, V_1 – inlet velocity.

The airfoils must be longer in order to reduce the influence of wind tunnel walls on airfoil load measurement due to which the airfoil rigidity and results are reduced and the measured load is changed if additional deformations occur. With thin airfoils in gas-turbine engines being used, the above mentioned contradiction is aggravated nevertheless such airfoils demand careful research of their dynamic stability.

Main principle of loading measurement consists in registration of some measuring system response parameters on this loading and in establishing of these parameters coupling with loading.

The currents in moving coils of electrodynamic vibrators, exciting specific oscillations of an airfoil [2, 3], can be registered as parameters which describe an oscillatory system response to aerodynamic loading. Non-stationary aerodynamic loading is determined from current changes during airfoils oscillations in a flow and in absence of it.

According to one of the method [2], aerodynamic forces are measured by strain gauge dynamometer but the aerodynamic moments are measured by the current in the electrodynamic vibrator voice coil. This vibrator excites airfoil angular vibrations. That method has been further developed in [3], so that angular and forward airfoil vibrations are excited by means of a pair of electrodynamic vibrators. Aerodynamic forces and moments are determined by the current in the voice coils.

However, deformations of an airfoil and elements of vibrator coils fastening are not taken into account in [2] and [3]. The purpose of the present work consists in enhancing precision for the method [3] by respecting deformations influencing the measuring system calibration and measurements of aerodynamic load.

2. PRECISION ENHANCEMENT OF THE METHOD

2.1. Design of the measuring system

The airfoils are located in a working part of a wind tunnel on individual measuring systems (Fig. 2).

Elastic elements of different width form oscillatory system in the shape of elastic parallelogram which makes the airfoil fixation move in a forward and angular way. With the first own frequency of an airfoil being much higher than the airfoil excitation frequency, its displacement is constant along the length. The master-generator of signals and feedback controller are used to control vibration.

The airfoil fixing parts displacements are measured by means of eddy-current contactless sensors. The signal master-generator and controller in a feed-back circuit are used for the control oscillations.

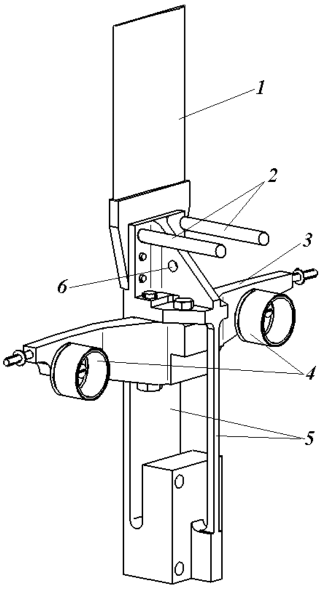


Fig. 2 Measuring system (1 - airfoil, 2 - contactless displacement sensors, 3 – cross-beam, 4 - electrodynamic vibrator moving coil, 5 - basic and auxiliary elastic elements, 6 - place for calibration mass)

2.2. The mathematical model of the measuring system

The aerodynamic loading on an airfoil depends not only on its oscillations but also on the next airfoils oscillations. As an example, oscillations of two cascaded airfoils will be considered.

The centre of masses of the measuring system lies on the axis of torsion. In this case, bending and torsion movements are not mutually mechanically coupled. Therefore as an example, only bending vibration of measuring system will be considered because the airfoil torsion natural frequency is much higher than the flexural one.

The diagram of flexural oscillations of two cascaded airfoils is shown in Fig. 3.

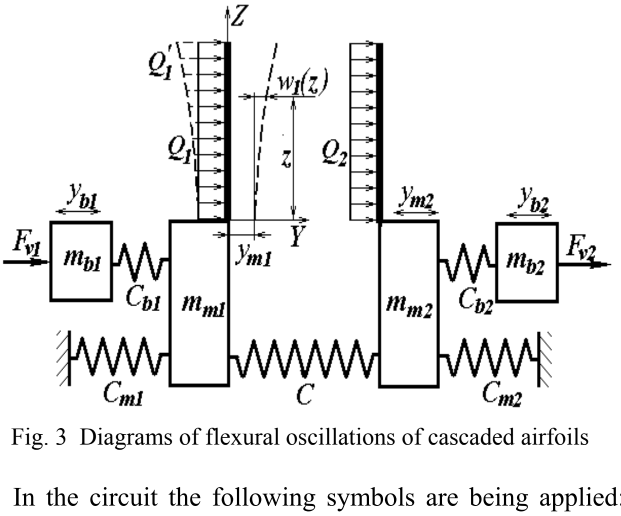


Fig. 3 Diagrams of flexural oscillations of cascaded airfoils

In the circuit the following symbols are being applied: m_{b1}, C_{b1} , m_{b2}, C_{b2} - given masses and bending rigidities of the cross-beam with voice coils, m_{m1}, C_{m1}, m_{m2}, C_{m2} - given masses and bending rigidities of the oscillatory system, y_{b1}, y_{m1}, y_{b2}, y_{m2} – complex vibration amplitudes of cross-beam and oscillatory systems related to the fix co-ordinate system, C –mechanical coupling rigidity of the oscillatory systems mediated by the structure of the rig.

To excite prescribed oscillations of airfoils, hanging on moving coils of electrodynamic vibrators, the forces F_{v1} and F_{v2} must be created. A pair of the oscillatory system vibrators is regarded as one vibrator producing double force

$$F_{v1} = 2\mu_1 i_1 , \qquad (1)$$

where i_1, μ_1 – electrical current and transfer factor of this electrodynamic vibrator.

Let the oscillatory system be supposed to be linear, and with the vibrator being fed with sinusoidal current, the oscillatory system vibration will be harmonic, too.

The distributed uniform loading Q_1 acts on an airfoil the fixed part of which simplifies the harmonic motion with amplitude y_{m1} and frequency ω. This loading has inertial and aerodynamic parts:

$$Q_1 = (m_{p1}\omega^2 y_{m1} + L_1)/h , \qquad (2)$$

where F_p - cross-sectional area of the airfoil, ρ_p - material density of the airfoil, h - airfoil length, L_1 – non-stationary aerodynamic force.

Let also the non-stationary aerodynamic forces be considered to be linearly connected with oscillations [1] at small airfoils oscillations:

$$\begin{aligned} L_1 / h &= q\left(y_{m1} l_{11} + y_{m2} l_{12}\right) \\ L_2 / h &= q\left(y_{m1} l_{21} + y_{m2} l_{22}\right)', \end{aligned} \qquad (3)$$

where q –dynamic pressure, l_{11}, l_{12}, l_{21}, l_{22} - aerodynamic coupling coefficients (ACC) which represent complex factor coefficients of proportionality between the airfoil oscillations and the force, induced by these oscillations.

Then the distributed loading on an airfoil can be written

$$Q_1 = \frac{m_{p1}}{h}\omega^2\left[y_{m1}\left(1+\bar{q}_1 l_{11}\right) + y_{m2}\bar{q}_1 l_{12}\right], \qquad (4)$$

where $\bar{q}_1 = \dfrac{qh}{m_{p1}\omega^2}$ - relative dynamic pressure, m_{p1} – mass of the airfoil.

Additional airfoil displacement distribution due to this loading in compliance with the book [4] is

$$w_1(z) = \frac{Q_1 h}{m_{p1}} \sum_{i=1}^{\infty}\left[\frac{w_{0i}(z)}{\Omega_i^2 - \omega^2} \cdot \frac{\int_0^h w_{0i}(z)dz}{\int_0^h w_{0i}^2(z)dz}\right], \qquad (5)$$

where Ω_i , $w_{0i}(z)$ - natural frequencies and modes of airfoil oscillations, i - number of the mode. The detailed description of $w_{0i}(z)$ is following:

$$w_{0i}(z) = \cosh(\beta_i l \cdot z) - \frac{\cosh(\beta_i l) + \cos(\beta_i l)}{\sinh(\beta_i l) + \sin(\beta_i l)} \times$$
$$\times\left[\sinh(\beta_i l \cdot z) - \sin(\beta_i l \cdot z)\right] - \cos(\beta_i l \cdot z) . \qquad (6)$$

The computation on the first member of the sum can be limited with error less 1% which means i=1, $\beta_1 l$=1,875 for Ω_1/ω >2 in the formula (5). In this case, according to (7), additional displacement of the first airfoil can be received as

$$w_1(z) = \frac{\pi}{4}\frac{w_{01}(z)}{k_{f1}^2 - 1}\left[y_{m1}\left(1+\bar{q}_1 l_{11}\right) + y_{m2}\bar{q}_1 l_{12}\right], \quad (7)$$

where $k_{fl}=\Omega_l/\omega$ - the relation of the lowest natural frequency of the first airfoil to operational frequency ω.

Similarly for airfoil 2

$$w_2(z) = \frac{\pi}{4}\frac{w_{01}(z)}{k_{f2}^2 - 1}\left[y_{m2}\left(1+\bar{q}_2 l_{22}\right) + y_{m1}\bar{q}_2 l_{21}\right]. \quad (8)$$

With displacements of separate cross-sections of an airfoil being known, the new more exactly distributed loading on an airfoil, respecting its deformation, can be found out:

$$Q_1' = \frac{m_{p1}}{h}\omega^2 \left\{\left[y_{m1} + w_1(z)\right]\cdot\left(1+\bar{q}_1 l_{11}\right) + \right. \\ \left. + \left[y_{m2} + w_2(z)\right]\cdot \bar{q}_1 l_{12}\right\}, \quad (9)$$

Then more exact resulting force acting on an airfoil is

$$R_1' = \int_0^h Q_1' dz. \quad (10)$$

After integration, it results

$$R_1' = m_{p1}\omega^2 y_{m1}(1+\theta_1) + qh\left(y_{m1}l_{11}' + y_{m2}l_{12}'\right), \quad (11)$$

$$R_2' = m_{p2}\omega^2 y_{m2}(1+\theta_2) + qh\left(y_{m1}l_{12}' + y_{m2}l_{22}'\right). \quad (12)$$

In the last two formulas following substitutions are adopted:

$$\theta_1 = \frac{0{,}613}{k_{f1}^2 - 1}, \quad \theta_2 = \frac{0{,}613}{k_{f2}^2 - 1} \quad (13)$$

and also

$$\begin{aligned} l_{11}' &= (1+2\theta_1)l_{11} + \theta_1\bar{q}_1 l_{11}^2 + \theta_2\bar{q}_2 l_{12}l_{21} \\ l_{12}' &= l_{12}(1+\theta_1+\theta_2+\theta_1\bar{q}_1 l_{11} + \theta_2\bar{q}_2 l_{22}) \\ l_{21}' &= l_{21}(1+\theta_1+\theta_2+\theta_1\bar{q}_1 l_{11} + \theta_2\bar{q}_2 l_{22}) \\ l_{22}' &= (1+2\theta_2)l_{22} + \theta_2\bar{q}_2 l_{22}^2 + \theta_1\bar{q}_1 l_{12}l_{21} \end{aligned}. \quad (14)$$

Let us record equations of oscillatory system motion with a cross-beam and replace the airfoil by its reaction R_1':

$$\begin{cases} \left(C_{b1} - m_{b1}\omega^2\right)y_{b1} - C_{b1}y_{m1} = 2\mu_1 i_1 \\ \left(C_{b1} + C_{m1} + C - m_{m1}\omega^2\right)y_{m1} - C_{b1}y_{b1} - C\cdot y_{m2} = R_1' \end{cases}, \quad (15)$$

2.3. Coupling determination for the electrical current in a vibrator moving coil with aerodynamic force.

From the first equation of the system (15), the displacement of the cross-beam is found as

$$y_{b1} = \left(\frac{2\mu_1 i_1}{C_{b1}} + y_{m1}\right)\eta_1, \quad (16)$$

where $\eta_1 = \frac{k_{b1}^2}{(k_{b1}^2 - 1)}$, a k_{b1}^2 - 2nd power of the relation of the cross-beam natural frequency to the operational frequency ω.

After having substituted (11) and (16) into the second equation of a system (15), we shall receive

$$H_1 y_{m1} - Cy_{m2} = 2\eta_1\mu_1 i_1 + qh\left(y_{m1}l_{11}' + y_{m2}l_{12}'\right), \quad (17)$$

where

$$H_1 = C_{b1}(1-\eta_1) + C_{m1} - m_{m1}\omega^2 - m_{p1}\omega^2\left(1+\theta_1\right). \quad (18)$$

On the other hand, at absence of flow (q=0)

$$H_1 y_{m01} - Cy_{m02} = 2\eta_1\mu_1 i_{01}, \quad (19)$$

where i_{01} and y_{m01} , y_{m02}- coil current and displacement of the both oscillatory systems without a flow that are equal.

With the control system ensuring the same oscillations of both systems in the flow and in calm fluid

$$y_{m1} = y_{m01}, \quad y_{m2} = y_{m02}, \quad (20)$$

the equation (19) can be substituted into the equation (17). We shall receive as a result

$$\left(y_{m1}l_{11}' + y_{m2}l_{12}'\right) = \frac{2\eta_1\mu_1}{qh}\cdot\left(i_{01} - i_1\right), \quad (21)$$

$$\left(y_{m1}l_{21}' + y_{m2}l_{22}'\right) = \frac{2\eta_2\mu_2}{qh}\cdot\left(i_{02} - i_2\right). \quad (22)$$

With similar measurements on another version of linearly independent system oscillations being made, two further equations, similar to (21) and (22), are received. This four-equation system will help to find unknown values of $l_{11}', l_{12}', l_{21}', l_{22}'$. Afterwards, it enables to find more exact value of ACC l_{11}, l_{12}, l_{21}, l_{22} from the equation set (14).

2.4. The measuring system dynamic calibration

We shall find the unknown transfer factor μ_l by dynamic calibration of an electrodynamic vibrator. For this purpose with the flow being absent, the known inertial force to the system will be applied, with attaching an additional mass Δm to the mass m_{m1}.

The second equation of the system (15) will get the form

$$\left[C_{b1} + C_{m1} + C - \left(m_{m1} + \Delta m\right)\omega^2\right]y_{m1} - \\ - C_{b1}y_{b1} - Cy_{m2} = R_1'. \quad (23)$$

After having inserted (11) at q = 0 and (16) into the equation (23), we shall receive

$$H_1\cdot y_{m1\Delta} - Cy_{m2\Delta} = 2\eta_1\mu_1 i_{1\Delta} + \Delta m\omega^2 y_{m1\Delta}, \quad (24)$$

where $i_{1\Delta}$ и $y_{m1\Delta}$ - current and displacement with additional mass.

With the control system supporting the same oscillations of both systems with additional masses and without them

$$y_{m1\Delta} = y_{m01}, \quad y_{m2\Delta} = y_{m02}, \quad (25)$$

the equation (19) can be inserted into the equation (24). Then it follows :

$$2\mu_1\eta_1 = \frac{\Delta m\omega^2 y_{m1\Delta}}{i_{01} - i_{1\Delta}}. \quad (26)$$

We obtain $\mu_2\eta_2$ by similar way. Thus, the specified dynamic calibration automatically takes the influence of a cross-beam deformation into account and enables the products $\mu_1\eta_1$ and $\mu_2\eta_2$ to be found at once in order to be used in the formulas (21) and (22).

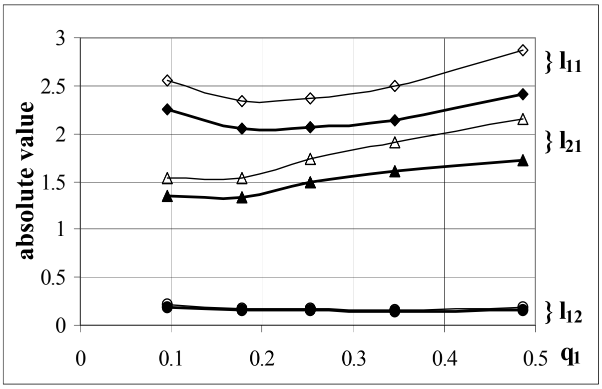
In case the additional masses Δm have been attached on the cross-beam ends, the calibration relations are more complex.

3. ESTIMATION OF THE AIRFOIL DEFORMATION INFLUENCE ON THE FORCES MEASUREMENT RESULTS

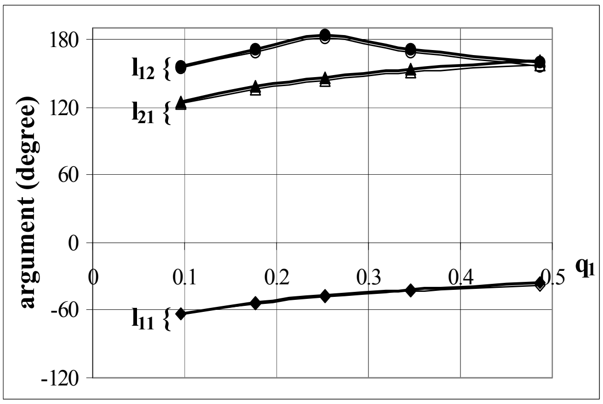
The dynamic vibrators calibration was applied in the article [3] in the above shown way with automatically taking a cross-beam deformation into account. However, the airfoil deformations were considered neither in this technique nor in the one of [2].

With the airfoils being considered absolutely rigid ($\theta_1 = \theta_2 = 0$), it follows from the formulas (14) $l'_{jk} = l_{jk}$. It means that l'_{jk} represents the "old" ACC which can be determined by old technique of aerodynamic loading measurement without taking the airfoils deformation into account. Thus, the formulas (14) enable to refine the "old" ACC.

The "old" ACC l'_{jk} l'_{jk} (thin lines) and the more exact ACC l_{jk} (bold lines) are presented as a function of relative dynamic pressure in Fig. 4. The absolute values of the complex ACC are given in Fig. 4a. Their arguments (phase shifts between aerodynamic force and airfoil oscillations) are shown in Fig. 4b. As in our example, $l'_{11} = l'_{22}$ is valid practically due to which l'_{22} and l_{22} are not shown in the figure.



a)



b)

Fig. 4 Relations of ACC to relative dynamic pressure (bold line – more precise ACC).

The old results were measured on the airfoils that were rather thin (the thickness of an airfoil was 0,033 of its chords). These were made of a composite material on the basis of carbon filaments. The relation of the lowest natural frequency of airfoils to the operational frequency is $k_{f1} = k_{f2} = 3{,}29$.

As seen on Fig. 4, the error of ACC determination increases with relative dynamic pressure augmentation. The error increases with the measured aerodynamic force, too. The above mentioned error of measurements achieves 18 % for module and 2° for argument of l'_{11}. For ACC l'_{12} and l'_{21}, these errors have appeared to be even greater (24 % for absolute value and 4° for argument).

On the one hand, with this airfoil being made of steel, the relative dynamic pressure will decrease approximately 4 times, but its natural frequency becomes smaller ($k_{f1} = k_{f2} = 2{,}28$), too. As a result, neglecting the steel airfoil deformation will cause an error of 35% in ACC determination.

4. CONCLUSIONS

1. The mathematical model of the measuring system for non-stationary aerodynamic loading on vibrating cascade airfoils has been developed. The mathematical model takes the final rigidity of airfoils and other structural parts of the measuring system into account.
2. The way of refinement of non-stationary aerodynamic loading per unit of length, as obtained by the old technique, has been found. The carbon composite airfoils material is shown to increase the loading determination accuracy.
3. The dynamic calibration correctness is proved. As demonstrated, the dynamic calibration will respect the cross-beam deformation automatically if additional masses are located in the place of the airfoil fixation. The calibration is more complex if additional masses are attached to the cross-beam ends.

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Authors: Ing. Volodymyr Tsymbalyuk, Ph.D., tel. (+420) 377634705, fax (+420) 377634702, tsym@ntc.zcu.cz,
Prof. Ing. Jiri Linhart, Ph.D., tel. (+420) 377638132, fax (+420) 377638102, linhart@kke.zcu.cz,
New Technology Research Centre, University of West Bohemia in Pilsen. Univerzitni 22, 306 14 Pilsen, Czech Republic.