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THE NON-LINEARITY OF PIEZOELECTRIC FORCE TRANSDUCERS AND THEIR ANALYTICAL MODELLING

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Abstract – This contribution deals with possible reasons for the non-linear behaviour of piezoelectric force transducers, previously unknown.

Possible reasons are interactions between isotropic elements such as force introduction and the anisotropic piezoelectric sensor element, geometrical non-linearities of the force transducer and non-linearities of the piezoelectric material.

This paper discusses the influence of geometrical non-linearities. A theory for analytical modelling of piezoelectric force transducers is developed and presented.

Keywords: piezoelectric force transducer, force measurement, non-linearity

1. INTRODUCTION

Piezoelectric force transducers are well-known for their small dimensions and high stiffness. The high resonance frequency compared with strain gauge force transducers is an advantage for measuring dynamic forces. The systems' inherent drift of piezoelectric force measuring devices suggest that static measurements of high accuracy are not possible.

Admittedly, with modern charge amplifiers, a small linear drift allows measurements of static forces for a period of hours or even days. A new calibration method using fast loading and unloading of discrete load steps, developed at PTB, permits the characterisation of piezoelectric force transducers with drift effects taken into account [1,2].

Previous measurements prove that it is not the drift but the non-linearity which is the criterion crucial for the classification of piezoelectric force transducers according to standards commonly applied to strain gauge force transducers [2,3,4].

A measure of the non-linear behaviour of a force transducer is the interpolation error. Fig.1 represents the relative interpolation error for the partial load range up to 2kN and the nominal load range up to 20kN of an examined commercial force transducer.

The investigations show a relative interpolation error of less than 0,02% in the partial load range up to 2kN, which corresponds to an assignment to class 00 according to [4]. In the nominal load range the relative interpolation error is greater than 0,4% and therefore very large. A classification according to the standards usually applied to strain gauge force transducers is not possible.

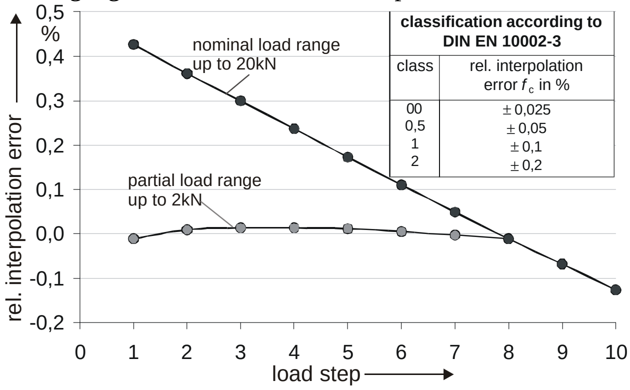

Fig. 1. Relative interpolation error of a force transducer in two different load ranges

These results are ratified by the examination of other commercial piezoelectric force transducers with different load ranges of 5kN and 20kN as shown in Fig. 2.

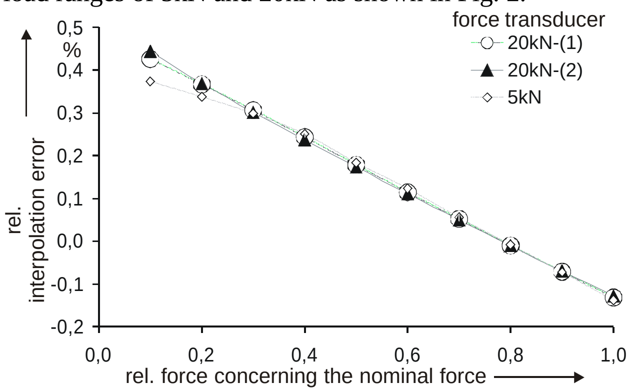

Fig. 2. Relative interpolation error of different force transducers with nominal forces of 5kN and 20kN

All investigated force transducers show about the same interpolation error and consequently point to a systematic non-linear effect. But the reasons for this non-linear behaviour of piezoelectric force transducers were unknown until recently.

2. PIEZOELECTRIC FORCE TRANSDUCERS

Possible reasons discussed in this paper is modification of the sensitivity of the transducers caused by geometrical deformations by forces acting on the transducer.

To investigate this geometrical non-linear effects, the mechanical structure of piezoelectric force transducers must be well known. Fig. 3 shows the schematic assembly of the investigated force transducers.

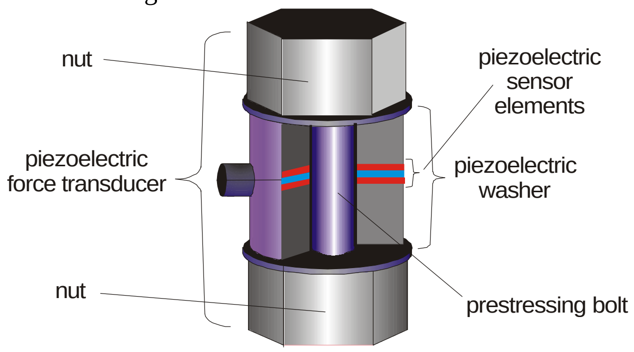

Fig 3. Schematically assembly of the investigated piezoelectric force transducers

The sensors comprise a piezoelectric washer which is prestressed by means of two prestressing nuts and a prestressing bolt. The piezoelectric washer contains one or more piezoelectric sensor elements, wherein a change in the force ΔF induces a charge Q. Besides an optimum force introduction into the piezoelectric washer and sensor elements, the prestressing of the piezoelectric washer enables tensile forces to be measured with the sensor.

3. ANALYTICAL MODEL OF THE SENSITIVITY OF PIEZOELECTRIC FORCE TRANSDUCERS

Because of the prestressing bolt piezoelectric force transducers have a construction-conditioned force shunt which depends on the transducers' geometry and leads to a reduction of the sensitivity. External forces F to be measured acting on the transducer effect deformation of the geometry and therefore a change of the force shunt, and consequently a change in sensitivity.

To investigate the influence of these so-called geometric non-linearities, it is necessary to develop an analytical model which describes the influence of the force shunt on the sensitivity.

Besides the external force F, a preload force F_V, a reset force F_B of the prestressing bolt and a printing force F_U act on the force transducer. The correlation between these forces and their dependence on the elongation Δl_B of the prestressing bolt and Δl_U of the piezoelectric washer is shown in Fig. 4.

An increasing elongation Δl_B of the prestressing bolt results an increasing reset force F_B. Accordingly an increasing printing pressure F_U results in an increasing unsetting deformation $-\Delta l_U$ of the piezoelectric washer.

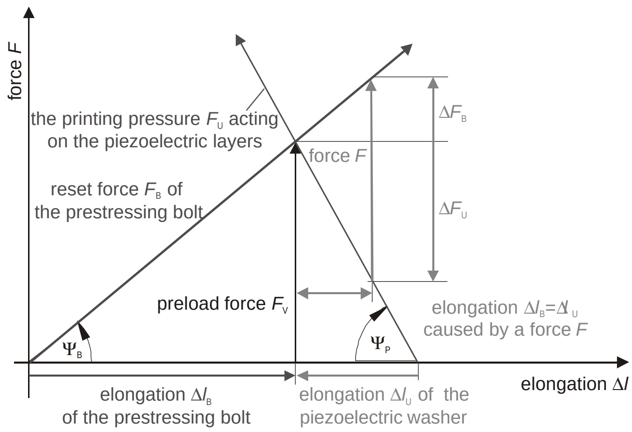

Fig. 4. Correlation between the preload force, reset force and the printing pressure depending on the elongation of the prestressing bolt and the piezoelectric washer

The unloaded condition of piezoelectric force transducers with F = 0N is characterised by

$$|F_B| = |F_U| = |F_V| . \tag{1}$$

An external force F acting on the transducer generates a change ΔF_B and ΔF_U of the reset force and the printing pressure:

$$F = |\Delta F_U| + |\Delta F_B| . \tag{2}$$

Moreover, the elongation of the prestressing bolt and the piezoelectric washer caused by F is given by

$$|\Delta l_B| = |\Delta l_U| . \tag{3}$$

Taking the Young's modulus E_A, E_B and E_P of the force introduction, the prestressing bolt and the piezoelectric sensor element into account, the sensitivity of the force transducer is specified by

$$S_{ft} = 2d/(f+1) , \tag{4}$$

with a piezoelectric constant d, and a factor

$$f = z \cdot \left(c \cdot \frac{E_B}{E_P} + \frac{E_B}{E_A}(1-c) \right) \tag{5}$$

to describe the force shunt.

The parameters

$$z = A_B / A_U \tag{6}$$

and

$$c = l_P / l_P \tag{7}$$

characterise the geometry of the piezoelectric force transducer. A_B and A_U as well as l_B and l_P are the force introducing surfaces respectively length of the prestressing bolt and the piezoelectric layers. Fig. 5 shows the geometry and Young's modulus of a piezoelectric washer used for the analytical modelling of the piezoelectric force transducer.

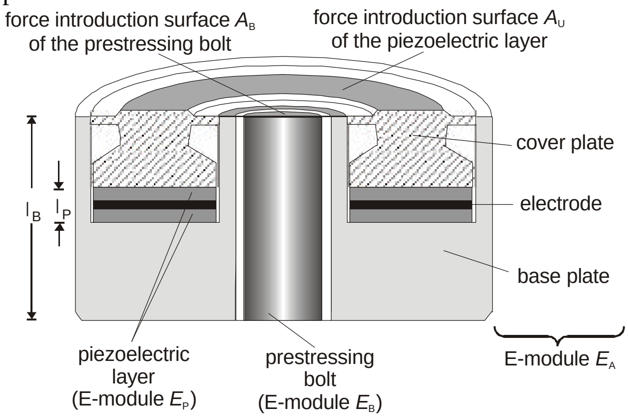

Fig. 5. Geometry and Young's modulus of a piezoelectric washer for analytically modelling piezoelectric force transducer

Because f has a positive value, eq. (4) makes clear that the force shunt caused by the prestressing bolt leads to a reduction of the sensitivity.

To determine the sensitivity according to eq. (4) and compare with experimental results, besides the geometry the modulus of elasticity E_A, E_B and E_P must be known. Admittedly neither the geometry of the force transducer investigated, nor the material properties, are given accurately. So the only possibility to estimate the sensitivity analytically is given by a rough estimate of these geometry factors and material properties.

3.1. Estimation of the geometrial factors z and c

To estimate the geometrial factors the following assumptions are necessary:

$$F_B^{max} = F_V + f \cdot F/(f+1) \cdot \quad (8)$$

is the maximum allowable tension force acting on the prestressing bolt. Accordingly the maximum allowable compression force amounts

$$F_U^{max} = F_V + F/(1+f) \quad (9)$$

The measurement range of the force transducer investigated is the same in compression and tensile force. This is estimated by

$$F_V = F_U^{max}/2 = F_B^{max}/2 . \quad (10)$$

Furthermore A_B and A_U are given by

$$A_B \geq F_B^{max}/R_M . \quad (11)$$

and

$$A_U \geq F_U^{max}/s_P , \quad (12)$$

whereas R_M is the tensile strengh of the prestressing bolt and s_P is the critical value for the mechanical stress of piezoelectric sensor elements.

It follows that z is estimated by

$$z = (s_P/R_M) \cdot f . \quad (13)$$

A prestressing bolt made of tempered steel has a tensile strengh R_M between 1,0GN/m^2 and 2,0GN/m^2. The mechanical stress of quartz used in piezoelectric force transducers is usually given by s_P=150·10^{6}N/m^2 [5]. Through a factor $f \approx 0{,}15$ of the investigated force transducers the geometrical parameter is $z \approx 0{,}02$.

But this value describes the breaking point of the force transducers. In practice z is between $0{,}075 \leq z \leq 0{,}15$ and estimated by the dimensions of the investigated force transducers.

The geometry parameter c is between 0,2 and 0,3.

3.2. Estimate of the material properties E_A, E_B, E_P

To estimate the Young's modulus E_B of the prestressing bolt, E_A of the force introduction and E_P of the piezoelectric sensor element, the following assumptions are necessary:

The prestressing bolt is made of tempered steel whose Young's modulus is unknown for the investigated force transducers. Therefore a Young's modulus between E_B=180·10^{9}N/m^2 and E_B=220·10^{9}N/m^2 is subsequently assumed.

The E-modulus E_A of the force introduction combines the E-modules of the base and cover plate made of steel, but also the electrode and diverse elements to balance the mechanical stress acting on the sensor elements. Thus it is subsequently assumed that the Young's modulus E_A is smaller than E_B and is valued between $E_B/2$ and E_B.

Only the E-modulus E_P=86,6·10^{9}N/m^2 of the piezoelectric sensor element made of quartz is a well-known material constant needing no estimation [5,6].

3.3. Analytical and experimental results

This subsection discusses the results of analytical and experimental investigations. Fig. 6 shows the sensitivity for an modulus of elasticity E_A=E_B=180·10^{9}N/m^2 as a function of the geometrical parameters z and c. The area of the force transducer investigated is given by $0{,}075 \leq z \leq 0{,}15$ and $0{,}2 \leq c \leq 0{,}3$ and indicated 'experimental area'.

As expected the diagram shows a declining sensitivity with increasing geometrical parameters z and c, which conforms with an increasing force shunt or parameter f.

This points out the well-known fact that a maximum sensitivity is realised if thin and long prestressing bolts are used [5]. In this context long means with respect to the sensor element's thickness.

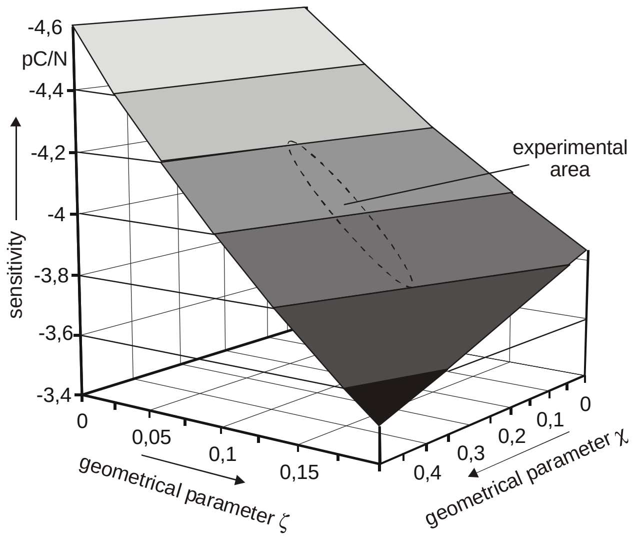

Fig.6. Analytically determined sensitivity of piezoelectric force transducers as a function of the geometrical parameters *z* and *c*.

The determined sensitivity in the ‘experimental area’ estimated for the investigated force transducers is between 3,8pC/N and 4,2pC/N and agrees with the experimental results of sensitivities at about 4pC/N [1,2,3].

As shown in Fig. 7 a significant relationship between the sensitivity and the Young’s modulus E_B of the prestressing bolt is not observed.

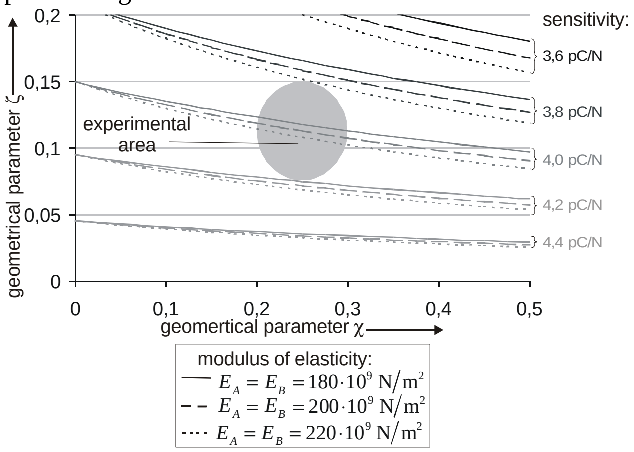

Fig.7. Analytically determined sensitivity of piezoelectric force transducers as a function of the geometrical parameters *z* and *c*.

For all investigated E-moduli E_B the sensitivity in the ‘experimental area’ is more or less between 3,8pC/N and 4,2pC/N and agrees with experimental results.

The relationship between the E-modulus E_A of the force introduction and the sensitivity is pictured in Fig. 8. The diagram shows decreasing sensitivity with a decreasing modulus E_A. But with an modulus E_A=E_B/2 the sensitivity in the ‘experimental area’ is still around 3,7pC/N and 4,0pC/N and proves the experimental results.

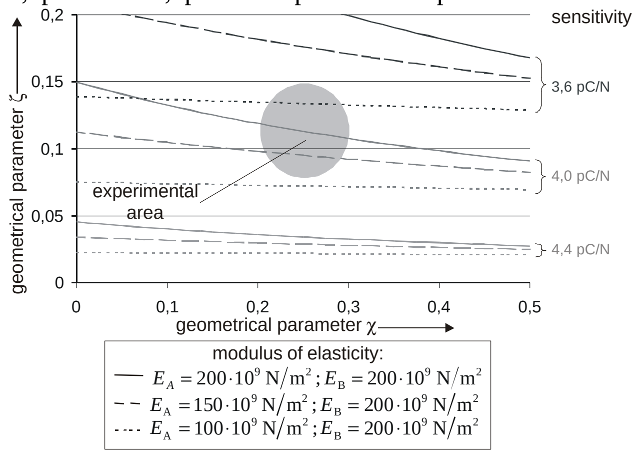

Fig.8. Analytically determined sensitivity of piezoelectric force transducers as a function of the geometrical parameters *z* and *c*.

Furthermore with E_A=E_B/2 the sensitivity of the force transducer is independent of the geometrical factor *c* which describes the aspect ratio between the piezoelectric layers and the prestressing bolt (see eq. 7).

Thus the analytical model offers a satisfactory option to estimate the sensitivity of piezoelectric force transducers. The investigations show that the sensitivity is mainly influenced by the geometry and less by the material parameters.

But by means of this analytical model it is not possible to make statements about the nonlinear behaviour of the force transducers because the parameters used to characterise the force transducers are insufficient.

4. EXPANSION OF THE ANALYTICAL MODEL TO DESCRIBE NON LINEARITIES

Possible reasons for the non-linear behaviour of piezoelectric force transducers are transverse contractions and elongations of the prestressing bolt and the sensor elements, caused by the prestress force F_V and externally acting force F.

It is the aim of the section to develop an expanded analytical model which estimates the influence of these deformations on the sensititvity.

4.1. Expanded analytical model

If geometrical deformation is taken into account the sensitivity $S_{ft}^{def}(F')$ as a function of

$$F' = F + F_V \tag{14}$$

is given by

$$S_{ft}^{def}(F') = S_{ft} + \Delta S_{ft}^{def}(F'), \tag{15}$$

whereas S_{ft} is a sensitivity independent of any deformation (see eq. 4), and $\Delta S_{\mathrm{ft}}^{\mathrm{def}}(F')$ is the change in sensitivity as a result of deformations.

Admittedly a complete mathematical solution is not possible. Instead segmentation of the problem in several single solutions supplies a result:

$$\Delta S_{\mathrm{ft}}^{\mathrm{def}}(F') = \underbrace{\Delta S_{\mathrm{B}}^{\Delta l}(F') + \Delta S_{\mathrm{P}}^{\Delta l}(F')}_{\text{elongation}} + \underbrace{\Delta S_{\mathrm{B}}^{\Delta A}(F') + \Delta S_{\mathrm{P}}^{\Delta A}(F')}_{\text{transverse contraction}} + \underbrace{\Delta S_{\mathrm{B,P}}^{\Delta l,\Delta A}(F')}_{\text{negligible}} \tag{16}$$

The terms $\Delta S_{\mathrm{B}}^{\Delta l}(F')$ and $\Delta S_{\mathrm{P}}^{\Delta l}(F')$ quantify the change in the sensitivity due to an elongation of the prestressing bolt and the piezoelectric sensor element:

$$\Delta S_{\mathrm{B}}^{\Delta l}(F') = 2 \cdot d \big/ \left(f_{\mathrm{B}}^{\Delta l}(F') + 1\right) - S_{\mathrm{ft}} \tag{17}$$

with

$$f_{\mathrm{B}}^{\Delta l}(F') = z \cdot \left[c_{\mathrm{B}}^{\Delta l}(F') \cdot \frac{E_{\mathrm{B}}}{E_{\mathrm{P}}} + \frac{E_{\mathrm{B}}}{E_{\mathrm{A}}} \cdot \left(1 - c_{\mathrm{B}}^{\Delta l}(F')\right)\right] \tag{18}$$

and

$$c_{\mathrm{B}}^{\Delta l}(F') = c \cdot \left(1 + \frac{f \cdot F + (f+1) \cdot F_{\mathrm{V}}}{(f+1) \cdot A_{\mathrm{B}} E_{\mathrm{B}}}\right)^{-1} \tag{19}$$

respectively

$$\Delta S_{\mathrm{P}}^{\Delta l}(F') = 2 \cdot d \big/ \left(f_{\mathrm{P}}^{\Delta l}(F') + 1\right) - S_{\mathrm{ft}} \tag{20}$$

with

$$f_{\mathrm{P}}^{\Delta l}(F') = z \cdot \left[c_{\mathrm{P}}^{\Delta l}(F') \cdot \frac{E_{\mathrm{B}}}{E_{\mathrm{P}}} + \frac{E_{\mathrm{B}}}{E_{\mathrm{A}}} \cdot \left(1 - c_{\mathrm{P}}^{\Delta l}(F')\right)\right] \tag{21}$$

and

$$c_{\mathrm{P}}^{\Delta l}(F') = c \cdot \left(1 + \frac{F - (f+1) \cdot F_{\mathrm{V}}}{A_{\mathrm{U}} E_{\mathrm{P}}}\right). \tag{22}$$

The terms $\Delta S_{\mathrm{B}}^{\Delta A}(F')$ and $\Delta S_{\mathrm{P}}^{\Delta A}(F')$ quantify the change in the sensitivity due to transverse contractions of the prestressing bolt and the piezoelectric sensor element:

$$\Delta S_{\mathrm{B}}^{\Delta A}(F') = 2 \cdot d \big/ \left(f_{\mathrm{B}}^{\Delta A}(F') + 1\right) - S_{\mathrm{ft}} \tag{23}$$

with

$$f_{\mathrm{B}}^{\Delta A}(F') = z_{\mathrm{B}}^{\Delta A}(F') \cdot \left[c \cdot \frac{E_{\mathrm{B}}}{E_{\mathrm{P}}} + \frac{E_{\mathrm{B}}}{E_{\mathrm{A}}} \cdot (1 - c)\right] \tag{24}$$

and

$$z_{\mathrm{B}}^{\Delta A}(F') = z \cdot \left(1 - \frac{\left((f+1) \cdot F_{\mathrm{V}} + f \cdot F\right) \cdot n_{\mathrm{B}}}{(f+1) \cdot A_{\mathrm{B}} \cdot E_{\mathrm{B}}}\right)^2 \tag{25}$$

respectively

$$\Delta S_{\mathrm{P}}^{\Delta A}(F') = 2 \cdot d \big/ \left(f_{\mathrm{P}}^{\Delta A}(F') + 1\right) - S_{\mathrm{ft}} \tag{26}$$

with

$$f_{\mathrm{P}}^{\Delta A}(F') = z_{\mathrm{P}}^{\Delta A}(F') \cdot \left[c \cdot \frac{E_{\mathrm{B}}}{E_{\mathrm{P}}} + \frac{E_{\mathrm{B}}}{E_{\mathrm{A}}} \cdot (1 - c)\right] \tag{27}$$

and

$$z_{\mathrm{P}}^{\Delta A}(F') = \frac{z}{1 + z \cdot (1 + z) \cdot \left[\left(1 - \frac{\left((f+1) \cdot F_{\mathrm{V}} - F\right) \cdot n_{\mathrm{P}}}{A_{\mathrm{U}} \cdot E_{\mathrm{P}}}\right)^2 - 1\right]} \tag{28}$$

Consequently eq. (16) to (27) enable an estimate of $S_{\mathrm{ft}}^{\mathrm{def}}(F')$, if the specific parameters F, F_{V}, A_{U} and A_{B} of the force transducer are known. In addition the geometrical parameter z and c and the material parameters E_{A}, E_{B}, E_{P}, the Poisson numbers ν_{B} and ν_{P} of the prestressing bolt and the sensor elements have to be taken into account respectively.

4.2. *Worst case estimate*

Besides the estimations of the geometrical factors z and c in subsection 3.1. and the material properties E_{A}, E_{B} and E_{P} in subsection 3.2, a few more assumptions are necessary to determine the worst case of the nonlinear effects caused by geometrical deformations of the force transducer.

If the force introduction surface A_{U} is known the maximum printing pressure $F_{\mathrm{U}}^{\max}$ and the preload force F_{V} is given by eq. (9) and eq. (10). The nominal load of the force transducer is

$$F = (f+1) \cdot F_{\mathrm{U}}^{\max} \big/ 2 . \tag{29}$$

Consequently, a worst case scenario of the non-linear behaviour of piezoelectric force transducers can be estimated.

4.3. *Analytical and experimental results*

Fig. 9 displays the maximum relative change of the absolute sensitivity caused by a nominal compression load F and a Young's modulus of $E_{\mathrm{A}}=E_{\mathrm{B}}=180 \cdot 10^9 \mathrm{N/m}^2$ as a function of the geometrical parameters z and c.

The diagram shows an increasing non-linearity with increasing geometry factors z and c. But in contrast to the experimental results a compression force leads to a declining sensitivity. Within the 'experimental area' the maximum decrease of the sensitivity is smaller 0,01%.

The experimental results of different force transducers show an increasing sensitivity of more than 0,4%.

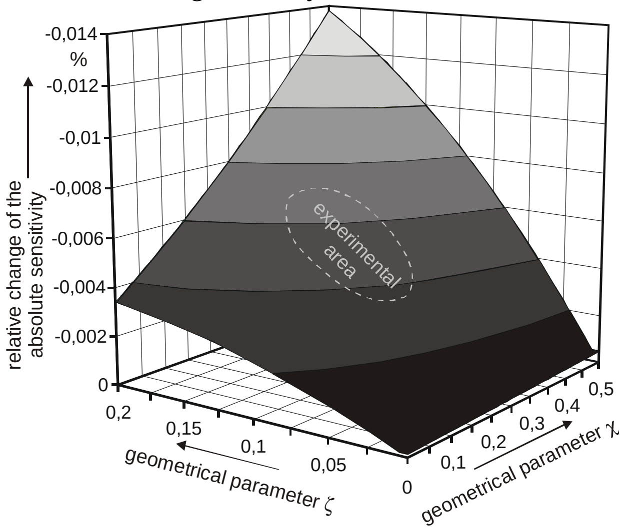

Fig.9. Relative change of the absolute sensitivity caused by a compression load F as a function of the geometrical parameters z and c

Though the relative change of the sensitivity depends on the material parameters E_A and E_B, fundamentally different results are not observed.

5. CONCLUSION

This contribution deals with one possible reason of the non-linear behaviour of piezoelectric force transducers. Because of the force acting externally on the transducer the deformation of the force transducers' geometry leads to a change of the force shunt and thus to a change in the sensitivity.

To estimate these non-linear geometric effects a simplified model, which does not take non-linear effects into account, is developed and discussed.

This model assumes that the sensitivity is mainly influenced by the geometry of the transducers. But also material properties like the E-moduli of the force introduction and the prestressing bolt affect the sensitivity.

An expanded model discusses the influence of geometric deformations on the sensitivity. A worst case estimate clarifies that geometric deformations caused by an external force acting on the transducer are not responsible for the systematically non-linear behaviour of piezoelectric force transducers. The worst case estimate results in

a) a much smaller non-linearity than in experimental examinations

b) a decreasing sensitivity with increasing compression force acting on the transducer, which contradicts the experimental examinations.

6. OUTLOOK

Thus multiaxial stress conditions in the contact surface are a remaining reasons for the non-linear behaviour of piezoelectric force transducers. To investigate the influence of multiaxial stress conditions in piezoelectric force transducers under load, simulations using the finite element method (FEM) are necessary (see Fig. 10)

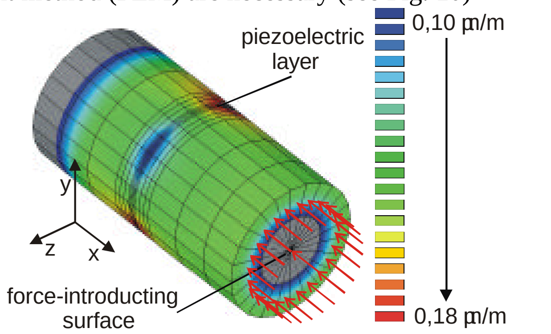

Fig.10. FEM simulation of the direction-dependent deformation in the yz-layer of a piezoelectric force transducer under load

Another possible reason for the non-linear behaviour are effects of non-linear material properties of the piezoelectric sensor elements [6]. To investigate the influence of these non-linear material properties the analytical model of the piezoelectric force transducer developed in this contribution is stringently necessary.

7. REFERENCES

[1] O. Mack, "New procedures to characterise drift and non-linear effects of piezoelectric force sensors", *Proceedings of the IMEKO TC3 Conference*, Istanbul (Turkey), pp. 141-148, 17.-21. Sept. 2001

[2] O. Mack, R. Kumme, "Quasistatic and dynamic investigation methods of piezoelectric force measurement devices", *Proceedings of the IMEKO TC3/APMF*, Taejon, (Republic of Korea), pp.310-320, 14.-18. Sept. 1998

[3] R. Kumme, O. Mack, B. Bill, H. Haab, C. Gossweiler, "Investigation of Piezoelectric Force Measuring Devices in Force Calibration and Force Standard Machines", *Proceedings of the IMEKO TC3 Conference*, Istanbul (Turkey), pp. 92-103, 17.-21. Sept. 2001

[4] EN10002-3, "Metallic materials, tensile test, part 3: Calibration of force providing instruments used for the verification of uniaxial testing machines", 1994

[5] J. Tichý, G. Gautschi, "Piezoelektrische Meßtechnik: Physikalische Grundlagen", *Springer Verlag Berlin, Heidelberg, New York*, 1980

[6] C. K. Hruska, "Last-squares estimates of the nonlinear constants of piezoelectric crystals", Journal of Aplplied Physics, Volume 72, Issue 6, pp.2432-2439, 1992

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