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MAGNETIZATION OF MASS STANDARDS AS DETERMINED BY GAUSSMETERS, MAGNETOMETERS AND SUSCEPTOMETERS

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Abstract − Magnetization of a mass standard can lead to weighing errors. This is because most modern balances are sources of non-uniform magnetic induction. Although the problem of unwanted magnetic forces is well known, the characterization of the magnetic properties of mass standards and balances can be problematic. This paper compares the kind of information that can be obtained from three types of instruments: Hall-probe gaussmeter, fluxgate magnetometer and susceptometer.

Keywords: mass standards, magnetic properties.

1. INTRODUCTION

Balances are used in mass metrology to compare the gravitational force on a standard weight and an unknown object. Some well-understood corrections (e.g. air buoyancy) may be applied. A number of parasitic forces (e.g. electrostatic, magnetostatic) defy systematic correction and, therefore, are best reduced to a level where they can be regarded as insignificant. In this regard, the International Organization of Legal Metrology (OIML) has given some guidance regarding magnetic properties of mass standards [1]. The present recommendations are currently under review.

Magnetic forces encountered when weighing objects with volume magnetic susceptibility $|\chi| << 1$ have been reviewed by Davis [2]. Gläser has extended this discussion to larger values of $|\chi|$ and has also obtained essential data on the magnetic environment of several balances used in mass metrology [3].

References [2] and [3] also make clear that magnetization of a weight may not be overlooked as a source of parasitic forces. This report deals with the following aspects: a) it outlines the conceptual problems in determining the magnetization of weights; and b) it shows why common techniques for determining magnetization may give differing results.

2. INDUCED AND PERMANENT MAGNETIZATION

Mass standards used in legal metrology typically have a lifting knob on the top and a recessed base [1], a shape which complicates the analysis of magnetic effects. A useful simplification of this shape is the assumption that such standards are cylinders with an aspect ratio γ (height to diameter) of 1.44 [3]. To introduce the present discussion, however, we will simply assume that each weight is a sphere of volume m_0/ρ, where m_0 is the nominal mass and ρ is the nominal density of the weight. Experimental results will show that this model, though grossly oversimplified, nevertheless is sufficient for a semi-quantitative discussion. Quantitative differences with respect to a more realistic model will be presented together with experimental results.

2.1 Induced Magnetization

In this discussion, we do not assume that determinations of magnetization are carried out in a magnetically shielded room. For practical reasons, measurements must usually be made in the presence of the earth's ambient magnetic field strength, $\vec{H}_E$. If unshielded, the earth's field strength within a laboratory is approximately uniform with a magnitude roughly 50 A/m. The field strength $\vec{H}_E$ is the resultant of vertical and horizontal components which have an approximately dipole form over the surface of the earth. (Field components for a particular location on earth can be estimated conveniently using on-line software that implements the IGRF2000 model [4]).

A weight will, therefore, have an induced magnetization due simply to the earth's magnetic field strength. For the spherical model, the weight acquires an induced magnetization $\vec{M}_i$, given by

$$\vec{M}_i = \frac{\chi}{1 + N\chi}\vec{H}_E \ , \tag{1}$$

with $N = 1/3$ being the well-known demagnetization factor for a sphere. Consequently, the ambient magnetic field strength outside the sphere is $\vec{H}_E + \vec{H}_i$, where $\vec{H}_i$ is the field strength of a spherical dipole of magnetic moment

$$\vec{m}_i = \frac{4\pi R^3}{3}\vec{M}_i \tag{2}$$

and R is the radius of the sphere. The magnetic field strength of a dipole is given by formulas found in all standard textbooks. From (1), we can see that the induced magnetization is proportional to χ for $|\chi| << 1$. However, an asymptotic limit of $3\vec{H}_E$ is approached as the susceptibility of the material increases. For cylinders where $\gamma = 1.44$, N is approximately 0.24 in the range $|\chi| < 1$ [5].

2.2 Permanent magnetization

Induced magnetization is not an intrinsic property of the weight. For instance, it would be negligible in a well shielded laboratory, where the ambient field strength is essentially zero. However, weights may also have a permanent magnetization

$\vec{M}_p$ leading to a magnetic field strength $\vec{H}_p$ outside the sphere. Unlike $\vec{M}_i$, which is uniform in magnitude and direction within spheres (and nearly so within cylinders under usual conditions [5]), $\vec{M}_p$ has no such constraint. In principle, a weight will have a permanent dipole magnetic moment

$$\vec{m}_d = \int \vec{M}_p dV \quad (3)$$

where the integral is taken over the volume of the weight. At distances much greater than the dimensions of the weight, the contribution to the ambient magnetic field strength from this source will have a dipole character. One may, at least in principle, deduce $\vec{m}_d$ from the dipole field strength that it produces. However, this approach is problematic because of the very weak signals involved and the effects of induced magnetization in unshielded laboratories. In addition, knowledge of $\vec{m}_d$ tells us little about $\vec{H}_p$ in regions close to the weight.

3. PRACTICAL METHODS

Although a rigorous characterization of $\vec{M}_p$ is impossible, it is nevertheless well known that permanent magnetization of weights is a real problem for mass metrology, especially for alloys containing iron [3]. Several suggestions for detecting problematic levels of magnetism in weights have been put forward [2,6]. The proposed measuring instruments are Hall-probe gaussmeters [6], fluxgate magnetometers [6] and the so-called BIPM susceptometer [2,6]. None of these methods determines $\vec{M}_p$ (or $\vec{m}_d$) directly, and thus results are comparable in special cases only. It is the purpose of this article to define these special cases and to demonstrate experimentally that it is relatively simple to correct for the effect of the earth's magnetic field strength.

3.1 Hall-Probe Gaussmeter

Hall sensors are solid-state devices designed to measure magnetic induction. They are generally small in volume (a typical form is a thin plate with length and width of about 1 mm), sensitive along a well-defined axis, and may achieve a precision of 100 nT when measuring small changes to ambient fields. The sensor is mounted in a thin wand to form a probe. It is possible to mount three orthogonal sensors in a single wand. A single sensor may be used to measure the component of magnetic induction in the η direction: $B_\eta=\mu_0(H_{E,\eta} + H_{i,\eta} + H_{p,\eta})$, where μ_0 is the magnetic constant ($4\pi\times10^{-7}$ N/A^2). Note that, if the probe is oriented in the East-West direction, $H_{E,\eta}$ is zero. If the axis of a cylindrical weight under test coincides with the axis of sensitivity of the probe, then $H_{i,\eta}$ is also zero.

3.2 Fluxgate Magnetometer [7]

A fluxgate sensor is a special kind of transformer which is sensitive to changes of 1 nT or less in ambient magnetic induction, although a magnetically shielded chamber is required to take full advantage of this sensitivity. As with the gaussmeter, induction is sensed along a well-defined axis with the possibility of placing three orthogonal sensors in a single probe. Typically, the sensor is ring-shaped and senses ambient fields that are in the plane of the ring and directed along the sensitive diameter [7]. A typical ring-core is about 15 mm in diameter.

3.3 Susceptometer

In addition to determining the susceptibility of a sample, a susceptometer may also be used to detect the presence of permanent and induced magnetization [2]. In contrast to the two methods previously mentioned, it measures a change ΔF_Z in vertical force, from which the *gradient* of the ambient magnetic induction may be deduced at a point:

$$\frac{\partial B_Z}{\partial Z} = \frac{\Delta F_Z}{m_s} \quad , \quad (4)$$

where m_s is the dipole magnetic moment of the susceptometer magnet, B_Z is the component of magnetic induction in the vertical direction ($\eta \equiv Z$) and the gradient is evaluated at a distance Z_0 (typically in the range 15 mm to 35 mm). This is the axial distance between the surface of the weight and the centre of the magnet, directly below [2]. Equation (4) is an especially convenient relation when discussing induced magnetization, although a more versatile formalism was developed in [2].

Unlike the previous methods, a susceptometer measurement might in principle affect the value of H_p . This will happen if H_p is a function of the small induction field produced by the susceptometer magnet at the base of the sample.

4. EXAMPLES

To illustrate these points, we have made extensive measurements of samples with very different magnetic properties. Three test instruments were used: a Lakeshore model 450 gaussmeter with transverse probe [8]; an Applied Physics Systems model 533 miniature 3-axis probe [8] (with our own electronics added); and a susceptometer based on a Mettler-Toledo UMT5 balance [8], with a cylindrical magnet of moment 0.12 A m^2.

For a spherical sample of radius R and no permanent magnetization, the change in the vertical component of the magnetic induction directly below the sample ($Z_0 \geq 0$) produced by its presence is given by

$$B_{i,Z} = \frac{2}{3}\mu_0 M_{i,Z} \frac{R^3}{(R+Z_0)^3} , \quad (5)$$

where $M_{i,Z}$ is the Z-component of the left-hand side of (1). For a cylinder of height L and radius r, the equation becomes

$$B_{i,Z} = \frac{1}{2}\mu_0 M_{i,Z}\left[\frac{L+Z_0}{r^2+(L+Z_0)^2} - \frac{Z_0}{r^2+Z_0^2}\right] . \quad (6)$$

with (1) modified as described in Section 2.1. Unlike (5), which is a general result, (6) is the asymptotic limit for $\chi \to 0$.

4.1 Brass cylinder

The first sample tested is a brass cylinder with dimensions L = 42 mm and r = 30 mm. Susceptometer measurements showed that $\chi = 0.078(8)$ for this sample. We thus estimate that, in our laboratory, $\mu_0 M_{i,Z} = 3.2(3)$ µT. Using a Hall-probe gaussmeter, $B_Z - B_{E,Z}$ was measured as a function of the distance Z_0 between the bottom of the cylinder and the probe. The resulting data are plotted with square symbols in Fig. 1. The curve is fitted using (6), where $M_{i,Z}$ is the only adjustable parameter. The result is $\mu_0 M_{i,Z} = 4.05(6)$ µT. This is higher than estimated, leaving the possibility of a contribution from $M_{p,Z}$ which is positive in sign and significantly smaller in magnitude than the induced magnetization.

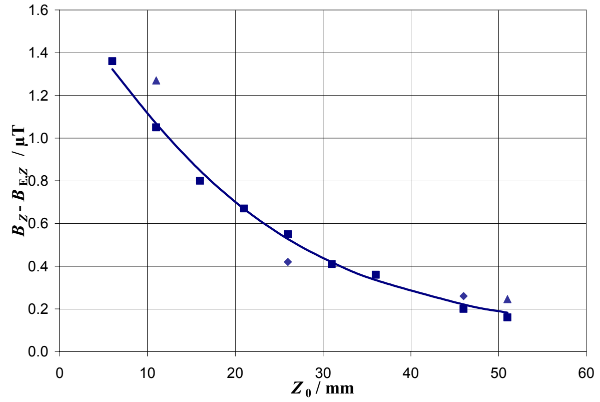

Fig. 1. Data obtained from a brass cylinder.

The same sample was measured using a fluxgate magnetometer having a ring-core sensor. Data were taken at three points, two of which are the triangles shown in Fig. 1. The distance Z_0 is measured from the base of the sample to the centre of the ring, whose diameter is 15 mm. The third point was taken at a spacing of 82 mm, where the measured intensity was found to be about 20% greater than that predicted from the extrapolated curve, so that the inferred value of $\mu_0 M_{i,Z}$ is 5.0 µT. The intensity measured at this point is 80(10) nT, an indication of the great precision of fluxgate sensors.

An additional point was taken with the fluxgate sensor oriented along the E-W direction, where the ambient induction is near zero. Placing the cylinder so that it was coaxial with the sensor axis and with the base a distance Z_0=52 mm resulted in a reading of 32(10) nT, from which we may estimate that $\mu_0 M_{p,Z} = +0.75$ µT, considerably smaller than $\mu_0 M_{E,Z}$.

Susceptometer measurements were made at the two points indicated by diamonds in Fig. 1. As explained above, the susceptometer is sensitive to field gradients. In order to plot susceptometer data on Fig. 1, it was necessary to make use of (6): first, the gradient of (6) was calculated and combined with (4) to estimate $\mu_0 M_{i,Z}$; then (6) itself was used to calculate $B_{i,Z}$. The susceptometer magnet produces an induction at the base of the sample and this is five times greater in magnitude at the closer of the two settings, amounting to about 1.2 mT.

Normally, only one point would be measured for routine work. That the data of Fig. 1, as well as the point at 82 mm, fall within 25% of the curve is an indication of the practicality of the theoretical model as well as the suitability of the various instruments used to make the measurements. It is possible, but by no means proven, that the field produced by the susceptometer magnet influenced the result at a spacing of 27 mm (the susceptibility measured at the two spacings was the same). Finally, magnetometer measurements in the E-W direction suggest that this sample has a permanent magnetization along its axis, the magnitude of which is much less than $M_{i,Z}$ and the sign of which is positive. This inference agrees semi-quantitatively with the discrepancy between inferred values of the magnetization and the *a priori* estimate of $M_{i,Z}$.

4.2 Ferromagnetic weight

For purposes of illustration, we have also carried out measurements on a ferromagnetic 500 g weight (χ >>1). The weight is very old and is made of a solid piece of metal, possibly nickel. The body is cylindrical with height approximately equal to diameter (42 mm) and there is a narrow lifting knob on the top. Using (1) with N = 0.26 [7], we estimate that $\mu_0 M_{i,Z} \approx 160$ µT.

Measurements were carried out similar to those of the brass cylinder. Data obtained from the Hall probe (squares), fluxgate (triangles) and susceptometer (diamonds) are shown in Fig. 2. A single-parameter fit of the Hall-probe data to (6) is shown as the curve in Fig. (2) with $\mu_0 M_{i,Z}$ = 107(4) µT. We note, however, that the data are not randomly distributed about the curve. In particular, we would not expect that extrapolation of the curve to greater Z_0 would be very reliable. Indeed, a data point taken with the fluxgate sensor at Z_0 = 82 mm is well above the extrapolated curve, leading to an inferred magnetization of about 155 µT. With the sensor oriented along the E-W direction and the cylinder placed so that it was coaxial with the sensor axis and the base a distance Z_0 = 82 mm from the centre of the sensor, a reading of −168(10) nT resulted, from which we may estimate that $\mu_0 M_{p,Z}$ = −21 µT, considerably smaller than the *a priori* estimate of $\mu_0 M_{E,Z}$.

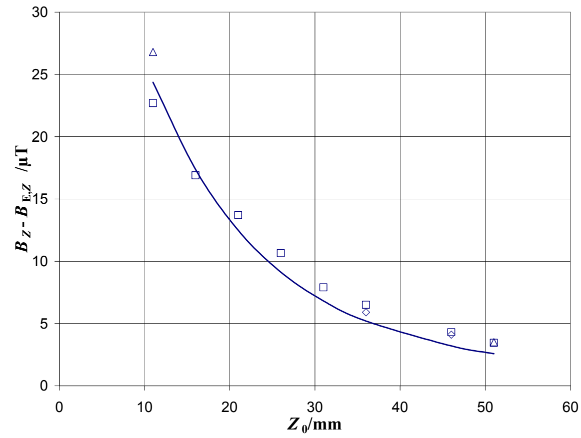

Fig. 2. Data obtained from a 500 g ferromagnetic weight.

4.3 1 kg mass standard in stainless steel

We take as a final example a 1 kg mass standard in stainless steel. Its manufacture predates OIML R111 [1]. Nevertheless, its shape and physical properties approximate a standard of class E2. Susceptometer measurements showed this weight to have a volume magnetic susceptibility $\chi = 0.0108(11)$. We can use (1) to derive the induced magnetization $\mu_0 M_{E,Z}$ = 0.46(5) µT when $B_{E,Z}$ = 42.8 µT. This weight was selected for study because the permanent magnetization along its axis is considerably larger in magnitude than the induced magnetization.

Results obtained with the three instruments used at different settings of Z_0 are shown in Table 1. The last line in the table is the result when the fluxgate sensor is oriented along the East-West direction and the weight is placed horizontally so that its base is toward the sensor and its axis aligned with the sensor axis. All instruments were used at a spacing of 52 mm, although it is evident that the precision of the Hall probe is much less than that of the other two devices. Type B uncertainties (including inadequacies of the model) are not included in the standard uncertainties given in column 3 of the Table. Column 4 is obtained by subtracting 0.46 µT from column 3, except on the last line where the induced magnetization is negligible. Results are consistent to within about 25%.

Table 1. Data obtained from 1 kg stainless steel weight. HP=Hall Probe; FG=Fluxgate; S=Susceptometer.

Z_0 /mm	B_Z-$B_{E,Z}$ /nT	$\mu_0 M_{Z,tot}$ /µT	$\mu_0 M_{p,Z}$ /µT	instrument
2	1000	2.3(2)	1.9	HP
11	510	1.90(3)	1.5	FG
22	288	1.95(6)	1.5	S
27	274	2.40(7)	2.0	S
32	240	2.56(8)	2.1	S
32	225	2.4(1)	2.0	FG
52	*300*	*7.3(3.0)*	*6.9*	*HP*
52	97	2.4(2)	2.0	FG
52	86	2.15(10)	1.7	S
52	79	2.0(2)	2.0	FG (E/W)

5. CONCLUSIONS

A Hall-probe gaussmeter with a vertically-oriented sensor centred any distance Z_0 below the weight measures $B_{i,Z}$ directly, so that $M_{i,Z}$ may be inferred from (6) and then compared with (1), the *a priori* prediction. A significant discrepancy may be attributed to $\vec{M}_p$. A fluxgate magnetometer may be used in the same way. However, since the magnetic induction may vary over the considerable size of some fluxgate sensors, data obtained can be difficult to interpret. Ideally, Z_0 should be significantly greater than the characteristic dimension of the sensor (the diameter of the ring, in the case of a ring-core design). Nevertheless, we see that the fluxgate results at Z_0 = 11 mm, which is less than the core diameter, give reasonable results and one may wonder why. The influence of sensor size should be more pronounced when measuring small weights but even in these cases (not reported here) we have found that results obtained with the ring-core sensor remain consistent with those from the Hall probe.

We suggest the explanation is that the ring-core geometry may be self-compensating for the sample configurations discussed above. Simple (but possibly naïve) calculations carried out by the author tend to confirm this. These calculations are based on a two-dimensional model described by Clarke [9], but with additional assumptions in order to treat nonuniform fields such as those due to a magnetic point dipole. Further work in this area will be necessary before firm conclusions can be drawn.

In any case, it is interesting to note that results obtained from samples oriented first vertically and then horizontally in the East-West direction may be combined to yield a value of $M_{p,Z}$ as well as a rough value of χ (or N, if $\chi >> 1$). Perhaps this feature might be exploited for routine screening of materials.

A susceptometer measures the *gradient* of $B_{i,Z}$ at some distance Z_0 below the weight, and thus the measured value of $M_{i,Z}$ may be inferred from the gradient of (6), combined with (4). We point out that this algorithm is equivalent to a more general method of computing $M_{i,Z}$ already published [2].

Inferred values of $M_{i,Z}$ will have an unwelcome dependence on Z_0 for all methods unless (6) is a realistic model. In the special case that permanent magnetization in the vertical direction is uniform, the magnetization in the numerator of (6) will contain this additional term and the same arguments apply. Since an induced magnetization $\vec{M}_i$ is unavoidable in most mass laboratories, a requirement that $|\vec{M}_p| << |\vec{M}_i|$ would seem to be unreasonable.

If the permanent magnetization is non-uniform, there is no simple relation such as (6) to connect measurements of magnetic induction in free space with the internal magnetization of the weight.

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