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THE ACCURACY ANALYSIS OF THE MEASUREMENT OF AUTOMOBILE VELOCITY BY DOPPLER LASER METERS

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Abstract − Many instruments for velocity measurement are based on the physical phenomenon, a Doppler effect. The paper deals with accuracy of laser meters used in the measurement of automobile velocity. The measurement results of such devices as well as of any other electronic or mechanical devices inevitably possess errors. For error analysis we will use a probabilistic method assuming that only random errors are present. To analyse systematic errors algebraic methods are usually used expressing systematic error as a function of some variables. For that purpose special measurements are performed when the measurement results are compared with the known values of the variables or those having been measured by the devices of higher accuracy.

The accuracy of measurement by Doppler meters depends mostly on the parameters of two kinds – stabilising and measuring laser emission frequencies and the errors of determining the emission speed in the atmosphere. These are the systematic errors of measurement. Other errors appear as a result of direct and indirect measurement of differential frequency. Since measurement is based on Doppler effect, the main parameter of emission affecting measurement accuracy is the frequency of emission. The influence of the changes in emission speed in the atmosphere due to the instability of atmospheric parameters on measurement accuracy is markedly weaker.

A Doppler meter can operate in one system with a pulse-phase range-finder enabling the determination of the distance to a moving object. The measurement accuracy of such system is higher.

Keywords: measurement, Doppler meter

1. INTRODUCTION

The car speed measurement systems are used in many applications beginning from design and testing of new models, applying it in tracking systems during races and, at least, during traffic control operations performed by the traffic police. In all these cases different control equipment is used among which the instrument based on the Doppler effect have a prevailing significance [1, 2]. The equipment used for this purposes have their technical specifications indicating their many technical characteristics and accuracy indicating as relative value from the range of measurement. Nevertheless, the influence factors, as environment influence, sometimes are neglected and their input to the accuracy of measurement can reach rather significant values.

The accuracy of measurements instruments using the Doppler effect depends mainly on the parameters of two types – of the laser stabilisation and of the accuracy of measurement of the laser light emission frequency and of the errors of determination of light speed in the atmosphere [3, 4]. It depends on the errors of the method of the measurement. Other errors occur as the result of the bias of frequency. As the principles of measurements are based on Doppler effect, the frequency of light emission is the main parameter facilitating the accuracy of measurements. The variations of light emission velocity in the atmosphere due to instability of the atmosphere parameters have significant influence on the accuracy of measurements [5]. The Doppler meter can operate as one system with pulse – phase distance measuring equipment (DME) enabling to determine the distance to the moving object and performing the measurements with higher accuracy.

2. THE MODEL OF THE MEASUREMENT

The Doppler frequency f_d while a transmitter and a receiver move relative to each other is determined by the formula [1]:

$$f_d = f_0\left(1 \pm \frac{v_r}{v} + \frac{v_r^2}{v^2} \pm \ldots\right), \tag{1}$$

where f_0 – the frequency of transmitted emission, f_d – the frequency of received Doppler emission,

v_r – radial velocity or the difference of the velocities of movement between the transmitter and the receiver, v – speed of emission in the atmosphere. The signs in the numerators correspond to the shortening of the distance between the transmitter and the receiver.

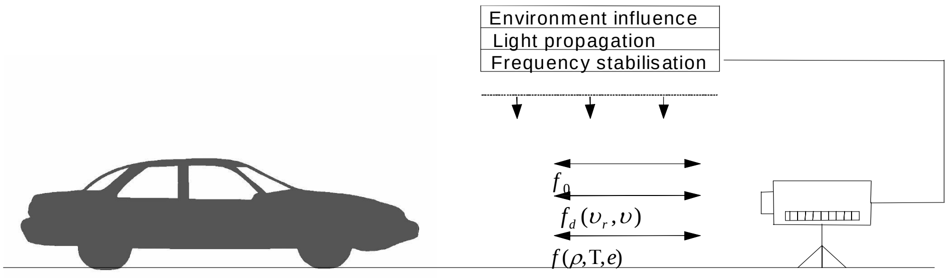

Fig. 1. The diagram of influencing factors in car velocity measurement

The expression of corresponding accuracy parameters will be determined from the series (1) ignoring the members of the second degree and higher as their influence is infinitesimal. As electromagnetic emission reflects from the automobile body and goes back to the receiver, the formula (1) becomes

$$f_d = f_0\left(1 \pm \frac{2v_r}{v}\right). \qquad (2)$$

The radial velocity v_r is expressed from this equation (in case of shortening of the distance):

$$v_r = \frac{\Delta f}{2 f_0} v, \qquad (3)$$

here $\Delta f = f_d - f_0$.

The equation for the calculation of the standard deviation σ_{v_r} of the radial velocity v_r is derived from the equation (3):

$$\sigma^2_{v_r} = \left(\frac{\partial v_r}{\partial \Delta f}\right)^2_0 \sigma^2_{\Delta f} + \left(\frac{\partial v_r}{\partial f_0}\right)^2_0 \sigma^2_{f_0} + \left(\frac{\partial v_r}{\partial v}\right)^2_0 \sigma^2_v +$$
$$+ 2\left(\frac{\partial v_r}{\partial \Delta f}\right)_0 \left(\frac{\partial v_r}{\partial f_0}\right)_0 \cdot K(\Delta f, f_0)$$

or

$$\sigma^2_{v_r} = \frac{v^2}{4 f_0^2}\sigma^2_{\Delta f} + \frac{v^2 \cdot \Delta f^2}{4 f_0^4}\sigma^2_{f_0} + \frac{\Delta f^2}{4 f_0^2}\sigma^2_v, \qquad (4)$$

here $\sigma_{\Delta f}, \sigma_f, \sigma_v$ - the corresponding standard deviations of differential frequency, transmitted emission frequency and the emission velocity in the atmosphere. The correlation between the values Δf and f_0 is ignored as it is sufficiently small. It is evident from the equation of covariation:

$$K(\Delta f, f_0) = M\{\delta(\Delta f)\cdot \delta f_0\} = M\left\{\tfrac{2v_r}{v}\delta f_0 \cdot \delta f_0\right\} = = \tfrac{2v_r}{v}\sigma^2_{f_0}, \qquad (5)$$

here $K(\Delta f, f_0)$ – covariation between the values Δf and f_0 , M – the symbol of mean value (mathematical expectation); $\delta(\Delta f)$, δf_0 - random errors of relevant values.

In case of v_r =100 km/h the value of covariation is $K(\Delta f, f_0) = 2\cdot 10^{-7}\sigma^2_{f_0}$, so it is negligible.

The analysis of the measurement accuracy under the influence of atmospheric parameters on the velocity of propagation of the electromagnetic radiation v and for the radial velocity v_r can be performed.

The velocity of propagation of the electromagnetic radiation in the atmosphere is

$$v = \frac{c}{n}, \qquad (6)$$

where c=299,792.458 km/s – velocity of propagation of the electromagnetic radiation in vacuum,
n – the coefficient of its diffraction in the atmosphere. For more convenient calculations, the diffraction coefficient is expressed through the diffraction index N:

$$n = 1 + 10^{-6} N \qquad (7)$$

The standard deviation σ_v of the velocity of radiation v has the following expression:

$$\sigma_v = \frac{c}{n^2}\sigma_n = c\cdot 10^{-6}\sigma_N = 0.3\sigma_N \text{ km/s}, \qquad (8)$$

because $n \approx 1.00030$ on average; σ_N – the estimate of the standard deviation of the radiation diffraction in the atmosphere.

The index N of the radio waves diffraction is the function of atmosphere parameters – temperature T, pressure P and humidity e: $N = N(t, P, e)$. Further it can be written as:

$$\sigma^2_N = \left(\frac{\partial N}{\partial T}\right)^2_0 \sigma^2_T + \left(\frac{\partial N}{\partial P}\right)^2_0 \sigma^2_P + \left(\frac{\partial N}{\partial e}\right)^2_0 \sigma^2_e + \left(\frac{\partial N}{\partial \lambda}\right)^2_0 \sigma^2_\lambda. \qquad (9)$$

The values of partial derivatives of the index of diffraction for medium of meteorological conditions ($T = 288$ K, $P = 745$ mmHg, $e = 12$ mmHg) will be [8]:

$$\left(\frac{\partial N}{\partial T}\right)_0 = -1.4\ N-units/°\text{K},$$

$$\left(\frac{\partial N}{\partial P}\right)_0 = 0.4\; N - units/\text{mmHg},$$

$$\left(\frac{\partial N}{\partial e}\right)_0 = 5.9\; N - units/\text{mmHg}.$$

Air humidity has the strongest influence on the velocity of the propagation of electromagnetic waves. The values of atmospheric parameters are not registered in radiometer devices, and they are not taken into account during measurements. That is why their maximal errors can be considered as the average value of change in the atmosphere parameters:

$$\sigma_T = 20°\text{K},\; \sigma_P = 10\;\text{mmHg},\; \sigma_e = 10\;\text{mmHg}.$$

Using the previously presented values of the partial derivatives and values of errors of atmospheric parameters the calculation gives $\sigma_N = 67$.

Further, from formulas presented here we can obtain the standard deviation of the propagation of electromagnetic radiation in the atmosphere in case when the influence of the atmosphere parameters on the results of measurement is ignored (only the mean values are taken into account): $\sigma_N = 0.3 \cdot 67 = 20$ km/s.

The necessary accuracy parameters are known, so according to formula (4) and after some development, an important index of accuracy, the standard deviation σ_{v_r}, of the velocity measurement of an automobile can be calculated. The value v_r that equals to 100 km/h can be used for this purpose. It can be written:

$$\sigma_{v_r}^2 = 10^{-2} + 10^{-14} + 5 \cdot 10^{-5} \approx 10^{-2}\;\text{km/h}$$

and σ_{v_r} = 0.1 km/h, when v_r =100 km/h.

3. CONCLUSIONS

The accuracy of velocity meters based on the Doppler effect depends on various parameters, the main of which is the accuracy of measurement of the difference of frequencies Δf. The accuracy of velocity measurement using the meters based on the Doppler effect can be determined by standard deviation.

The influence of atmosphere parameters on the determination of velocity of movement using the Doppler effect can be limited by small values.

REFERENCES

[1] M. Bauer, "Vermessung und Ortung mit Satelliten", *Heidelberg: Wichmann*, 1994.

[2] T.J. Bensky, S.E. Frey, "Computer sound card assisted measurements of the acoustic Doppler effect for accelerated and unaccelerated sound sources", *American Journal of Physics*, vol. 69 (12), pp. 1231-1236, Dec 2001.

[3] J. Lozano-Rogado, J.M. Miranda-Pantoja, J.L.Sebastian, "Measurement of velocities in noisy environments with a microwave Doppler effect radar", *European Journal of Physics*, vol. 22(3), pp. 249-255, May 2001.

[4] R.H. Marshall, Y.N. Ning, A.W. Palmer, K.T.V. Grattan, "Accurate displacement measurement using a novel fibre-optic electronically scanned white light interferometer", *Proceedings of the XIV IMEKO World Congress*, vol. II Helsinki, 1997, pp. 144-149, 1997.

[5] J. Skeivalas, "The Accuracy of Determining the Covariation of Random Variables", *Geodesy and Cartography*, vol. XXV, No. 4. Vilnius: Technika, 1999, p. 156–158.

[6] R. Schneider, P. Thurmel, M. Stockmann, "Distance Measurement of Moving Objects by Frequency Modulated Laser Radar", *Optical Engineering*, vol. 40 (1), pp. 33-37, Jan 2001.

[7] B. Lehmann, H. Nobach and C. Tropea, "Measurement of acceleration using the laser Doppler technique", *Measurement Science and Technology*, vol. 13, pp. 1367-1381, Sep. 2002.

[8] J. Skeivalas, V. Giniotis, "The Analysis of Accuracy of Measurement by Doppler Laser Meters", *Optical Engineering"*, (in press).

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