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PHASE MEASUREMENT USING A BIAS DERIVATIVE SHIFT TECHNIQUE IN THE PHASE DIFFERENTIATION METHOD

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Abstract − An improvement to phase measurement systems based on differentiation filters is proposed. The new technique involves calculations from measurements conducted using two different phase derivative biases. Advantages of the system are that it can be applied to semi-opaque objects, and that the effects of non-uniformities in the intensity of the light source are negligible. Computer simulations are performed to verify the effectiveness of the technique.

Keywords: phase measurement, differentiation filter, phase derivative

1. INTRODUCTION

Phase differentiation is a method for indirectly visualizing the phase shift of light passing through an object whereby the derivative of the phase shift is converted into optical intensity by some apparatus [1-4]. This technique is applicable to many applications, such as optical testing, material diagnostics and the microscopic analysis of living cells. Phase shift itself can then be calculated by integrating the resulting light intensities, which are measured using a CCD camera. Generally, this technique is only applied to transparent objects, since light being absorbed by the object causes measurement errors. Furthermore, the light source must be uniform and stable, as variations in optical intensity can also produce errors.

In this paper, we propose a new technique to solve these problems, which we will refer to as the shift technique. The theory of the method is presented along with the results of computer simulations of the method.

2. THEORY

The optical arrangement of the new system is the same as is in existing systems, as shown in Fig. 1. A collimated beam of light illuminates the specimen to be analysed. After passing through the object, light is focused onto a differentiation filter by lens L, with the transmission coefficient of the filter proportional to position across it. The intensity of light at the image plane is then given by

$$I_1 = \left[\alpha \left(a_0 + \frac{d\phi}{dx} \right) \right]^2 , \tag{1}$$

where ϕ is the phase shift of light passing through the object, a_0 is a bias and α is a factor describing other influences on the measured light intensity, such as non-uniformities in the light source or absorption of light by the object. If α is constant, the derivative of phase can be calculated easily from this equation. However, if the intensity of the incident light beam is not uniform or the specimen is not transparent, the derivative of the phase shift cannot be calculated.

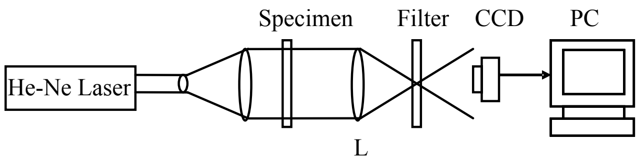

Fig. 1: Optical arrangement.

Now, consider a slight lateral shift in the filter position. This introduces a change in the phase derivative bias a_0. Assuming the change in intensity is smaller than the change in phase, the image intensity then becomes

$$I_2 = \left[\alpha \left(a_0 + \Delta a_0 + \frac{d\phi}{dx} \right) \right]^2 , \tag{2}$$

where Δa_0 is the shift in the bias. Equations (1) and (2) can be combined to give:

$$\frac{d\phi}{dx} = \frac{\sqrt{I_1}\,\Delta a_0}{\sqrt{I_2} - \sqrt{I_1}} - a_0 . \tag{3}$$

Since α is not a factor in this equation, the calculated phase derivative is independent of both absorption within the specimen and non-uniformities in the light source.

3. RESULTS

Computer simulation of the presented technique has been carried out to demonstrate that this technique delivers improved phase measurement.

A specimen in which one surface has a radius of curvature of 1 m over a solid angle of $2\pi\times10^{-4}$ steradians and the other surface is flat is considered. The phase shift of light passing through such a specimen is shown in Fig. 2. Differentiation range of the filter is $0.1\times2\pi/\lambda$ radian·mm^{-1}. Non-uniformity in the intensity of the light source is assumed to be of the order of 20%, according to the profile shown in Fig. 3.

Figure 4 shows simulated signal intensities at the image plane for two filter positions with phase derivative biases of $a_0 = 0.05\times2\pi/\lambda$ radian·mm^{-1} and $a_0 = 0.07\times2\pi/\lambda$ radian·mm^{-1}. Random noise of ±2% in amplitude was introduced into the simulations, with no other errors taken into account. Signal intensity in the ideal case of a uniform light source and perfectly transparent specimen with no noise is shown for comparison.

Figure 5 shows the derivative of phase calculated using equation 3, having a signal to noise (S/N) ratio of 20dB, an decrease over the S/N ratio of 34 dB of Fig. 4. Figure 6 shows phase profiles obtained from two different calculations, one using the standard technique of direct calculation from a single measurement, the other using the shift technique. Errors resulting from the shift technique can be seen to be far less than those resulting from the standard technique, with error in the shift technique due entirely to the random noise. When there is no random noise, the curve coincides completely with the ideal curve.

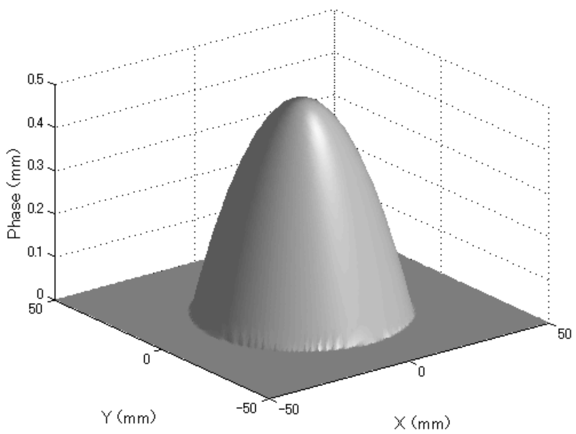

Fig. 2: Phase profile of the specimen used in computer simulations.

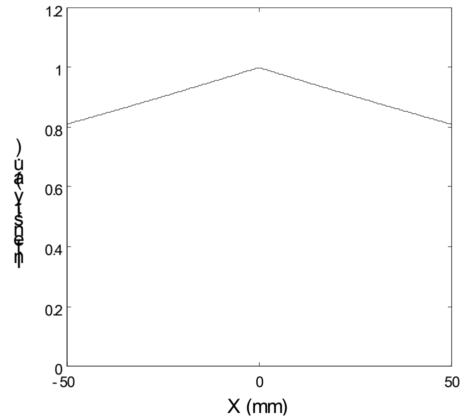

Fig. 3: Intensity profile at y=0 of the light source used in computer simulations.

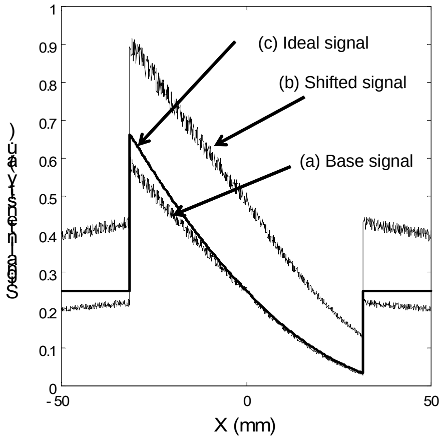

Fig. 4: Simulated signal intensities profiles across the image plane at y=0 for two filter positions: (a) phase derivative bias $a_0 = 0.05\times2\pi/\lambda$ radian·mm^{-1}, (b) $a_0 = 0.07\times2\pi/\lambda$ radian·mm^{-1}, and (c) for the ideal case.

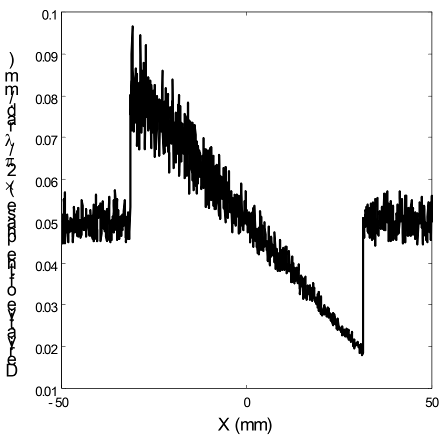

Fig. 5: Phase derivative profile at y=0 calculated using equation 3 from data of Fig. 4.

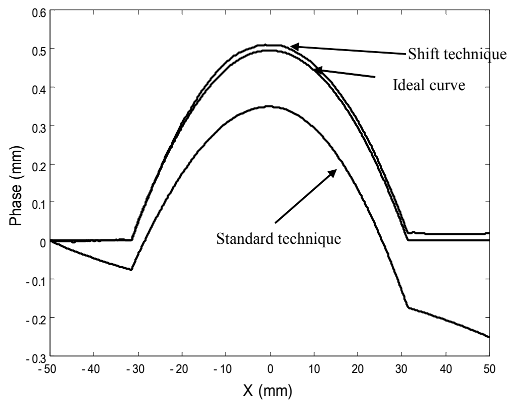

Fig. 6: Phase profiles at y=0 calculated in the simulation, showing the ideal curve, the curve obtained using the standard technique, and the curve obtained using the shift technique. A base bias of $a_0 = 0.05 \times 2\pi / \lambda$ radian·mm^{-1} and bias shift of $0.02 \times 2\pi / \lambda$ radian·mm^{-1} were used in the simulations.

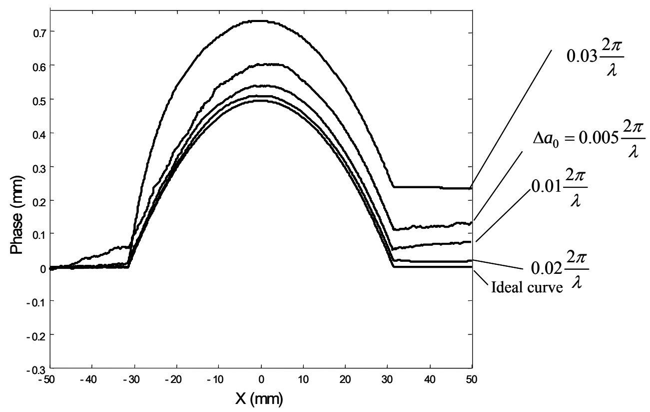

Fig. 7: Phase profiles at y=0 for 4 bias shift values of $\Delta a_0 = 0.005 \times 2\pi / \lambda$, $0.01 \times 2\pi / \lambda$, $0.02 \times 2\pi / \lambda$ and $0.03 \times 2\pi / \lambda$ radian·mm^{-1} and a base bias of $a_0 = 0.05 \times 2\pi / \lambda$ radian·mm^{-1}. The ideal curve is shown for comparison.

Figure 7 shows phase profiles for 4 bias shift values; $\Delta a_0 = 0.005 \times 2\pi / \lambda$, $0.01 \times 2\pi / \lambda$, $0.02 \times 2\pi / \lambda$ and $0.03 \times 2\pi / \lambda$ radian·mm^{-1}. Although noise is averaged by integration of the phase derivative, the influence on the calculated phase is still present. S/N ratios can be seen to decrease with the decrease in bias shift, and the error in calculated phase profile increases. A large bias shift clearly produces better results. However, the amplitude of transmittance of the differentiation filter is limited to values between 0 and 1. This limits the differentiation range, and thus large bias shifts can cause signal saturation. In this case, the filter has a differentiation range of $0.1 \times 2\pi / \lambda$ radian·mm^{-1} and the phase shift profile of the specimen has a maximum derivative of $\pm 0.03 \times 2\pi / \lambda$ radian·mm^{-1}. Therefore a shift of $0.02 \times 2\pi / \lambda$ radian·mm^{-1} is the limit of the bias shift from the base bias of $0.05 \times 2\pi / \lambda$ radian·mm^{-1}.

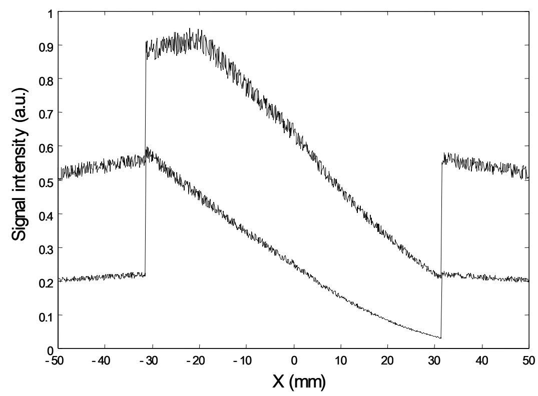

Fig. 8: Signal intensities across the image plane at y=0 for two filter positions for a base bias of $a_0 = 0.05 \times 2\pi / \lambda$ radian·mm^{-1} and bias shift of $0.03 \times 2\pi / \lambda$ radian·mm^{-1}.

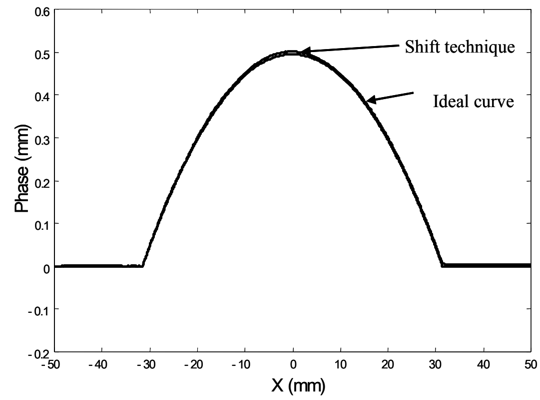

Fig. 9: Phase profiles at y=0 for a base bias of $a_0 = 0.035 \times 2\pi / \lambda$ radian·mm^{-1} and bias shift of $\Delta a_0 = 0.03 \times 2\pi / \lambda$ radian·mm^{-1}. The ideal curve is shown for comparison.

When a bias shift of $0.03 \times 2\pi / \lambda$ radian·mm^{-1} is used, error increases as shown in Fig. 7, because the sum of maximum phase derivative, base bias and bias shift comes to $0.11 \times 2\pi / \lambda$ radian·mm^{-1}. This is beyond the range of the filter, causing saturation of the phase derivative signal as shown in Fig. 8.

For a base bias value of $0.035 \times 2\pi / \lambda$ radian·mm^{-1}, the phase derivative is within the range of the filter. Error in the calculated phase for a bias shift of $0.03 \times 2\pi / \lambda$ radian·mm^{-1} is smaller than in the case of a bias shift of $0.02 \times 2\pi / \lambda$ radian·mm^{-1}. The calculated phase profile is shown in Fig. 9.

For a large bias shift without saturation of the signal, the optimal base bias (phase derivative bias a_0) is 1/3 of the filter range. The range of the filter for the standard technique is given by the following equation [4].

$$\Delta \gamma \approx \frac{2\pi}{\lambda f} W \tag{4}$$

where W is the width of the filter and f is the focal length of the lens. The optimal base bias, therefore, is

$$\frac{2\pi}{3\lambda f}W \text{ (radian·mm}^{-1}\text{)} \tag{5}$$

This correspond to the *W*/3 mm form the edge of the filter. The maximum bias shift such that the signal does not saturate is then

$$\frac{2\pi}{3\lambda f}W \text{ (radian·mm}^{-1}\text{)} \tag{6}$$

which corresponds to a filter shift of $W/3$ mm. The differentiation range for the bias derivative shift technique is thus

$$\pm\frac{2\pi}{3\lambda f}W \text{ (radian·mm}^{-1}\text{)} \tag{7}$$

The S/N ratio decreases 9.6 dB over the S/N ratio of the original signal.

4. CONCLUSION

A new phase derivative measurement system was proposed. The advantage of the new method is that opacity of the specimen being analysed and non-uniformity of the light source do not produce errors in the results. The theory was successfully verified by computer simulation. Optimal conditions for reducing noise in the signal were investigated, giving.

· Focal point is the *W*/3 of the filter.

· The shift of the filter is *W*/3.

Under these conditions, the differentiation range for the bias derivative shift technique is $\pm\frac{2\pi}{3\lambda f}W$ (radian·mm^{-1}).

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