XVII IMEKO world Congress
Metrology in the 3rd Millennium
June 22-27, 2003, Dubrovnik, Croatia

VIRTUAL INSTRUMENTATION, OVERCOMING NON-LINEARITIES WHEN ERRORS OF MULTIAXIS MACHINES ARE AMPLIFIED

[1]*José Sánchez,* [2]*Saúl Santillán,* [1]*Sergio Padilla,* [1]*Benjamín Valera,* [1]*Rigoberto Nava,* [1]*Gerardo Ruiz*

[1]Applied Sciences and Technical Research Centre, National University of Mexico, D.F. México
[2]Mechanical Design Centre, Faculty of Engineering, National University of Mexico, D.F. México

Abstract - One purpose of Virtual Instrumentation in Metrology, is to empathise properties that put instrumentation away from idealisations. In actual instrumentation, geometric deviations trend to occur in relations 10^{-5}, 10^{-7} of maximum displacements, requiring their representation in amplified way. If distortion of mechanisms is amplified hundreds times for its representation in CAD, many problems occur: linearity is lost between geometric relations; it is difficult to represent errors simultaneously for many axis and many degrees of freedom; operation of mechanisms doesn't satisfy cinematic of rigid body.

Since linearity is lost between linear and angular displacements, between articulate and operational spaces, and between real and amplified representation, it is necessary to apply functions and mapping procedures for relating them. Those mapping errors and their functions are important goals in the field of calibration, where virtual mechanical instrumentation may illuminate cases of controversy. The manuscript treats with several cases of distortion, their possibilities of amplification, their possibilities of assembling, and diagnostic trough inverse kinematics.

Keywords: virtual instrumentation, distortion

1. CONCEPTS IN VIRTUAL INSTRUMENTATION

The oldest and nearest concepts of virtual machines are in manufacturing, where paths of milling are described with G codes. Earlier developments linked G paths with CAD primitives, facilitating interactivity between CAD and CAM; CAD and CAM however, took machines as with perfect geometry. In the field of calibration, those concepts of virtual machines are useful but not complete, because the main goal of calibration is the representation of properties that put them away from idealisations.

1.1 Virtual machines as calculators of noise.

This concept of machine take source errors for each degree of freedom as a statistical value, which is propagated to operational space trough simple cinematic rules.

Applications of this concept are the calculus of uncertainty in tasks of 3D measures, which may be aided with Montecarlo methods, offering alternatives to ISO-GUM. Since proposal of this kind of virtual machine is the calculus of uncertainties, but not the knowledge and cancellation of their original errors, that concept can not be considered in a wide and complete concept of calibration.

1.2 Models as linear mechanisms with small variations.

In this concept of machines authors consider machines with linear and orthogonal movements, and with small distortions; all errors are considered as independent in between.

Soons [1] developed a general model for this kind of machines, who nest in his equations rotational parameters; a simplified model has been presented by Cresto [2], offering possibilities of calibration with invariability of callipers. Such a models look as universal, but an important characteristic of them is that linearity of equations is valid only for small deviations. These constrain makes it impossible to amplify distortions, since linearity is lost.

1.3 Models for non-linear mechanisms.

Models made with wires, b-splines surfaces, and CAD primitives were propose by Sanchez J. [3], where there are not constrains for distortions of mechanisms; final errors or absolute positions in the operational space can be calculated trough cinematic rules of rigid body. Characteristics of this model are that straightness and tilting are not independent in between, and its algebra can include errors as big as we want, including high tilting, and using equivalencies with CAD primitives.

As a new powerful advantage of this instrument we looked for integrating to it new dynamic characteristics as: a) possibilities of mapping amplified errors in an integrated space, b) possibilities of amplifying linear and angular distortions without significant lost of linearity c) possibilities of coupling *nx* deformed bodies in the articulate space, and c) easy correlation between operational and articulate spaces in order to get reverse diagnose, avoiding heavy process of inverse kinematics.

2. POSIBILITIES OF AMPLIFYING ERRORS

2.1. Scale along one axis.

Due the necessity of representing errors for many axis, we can not draw them in different axis as in ordinary charts of calibration; it is desirable then to draw them along the same axis of movements. That is possible if differences are amplified and added to master values. In order to extract the error from these drawing, it is possible to join master scales with distorted and amplified scales. It is possible too to draw the differences with exponencial factors. See figure 1.

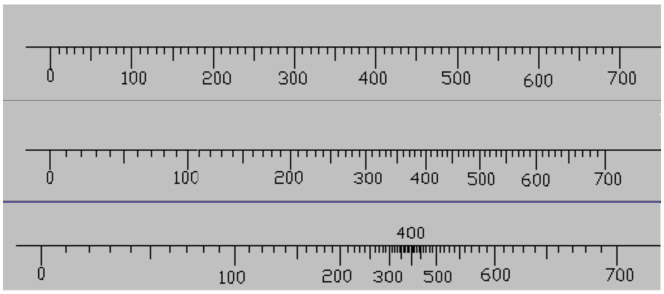

Fig. 1. Master scale, sum of Master scale plus 60x error, and sum of Master scale plus exp. error.

In order to avoid overlapping in drawings, and increasing limits of amplifying, it is useful the employment of polynomials, getting minimisation of local discontinuities. A powerful model useful for drawing amplified errors are the b-splines:

$$x(u) = U(V_x) \quad (1)$$

$$x(u) = U(V_x + \Delta V_x) \quad (2)$$

$$x(u) = U(V_x + k\Delta V_x) \quad (3)$$

Where (1) is the b-spline matrix equation that correlate normalised nominal values *u* in *U,* with real values of scales *x(u)*, through control vertex *Vx* distributed along the axis. The theoretical scale will have the distribution *Vx*; the real scale will have the distribution $Vx+\Delta Vx$ in (2); and the scale with amplified errors will have the distribution $Vx+k\Delta Vx$ in (3). A fast extraction of error may be done by drawing in the same axis the amplified distorted scale and the Master scale with different colours.

2.2. Scale and straightness along one non-straight axis.

In this case we have two distortions: one due to distribution of scale and one due to straightness, the path of scale and its distribution are defined in plane trough next two equations:

$$\begin{aligned} x(u) &= U(V_x + k\Delta V_x) \\ y(u) &= U(V_y + k\Delta V_y) \end{aligned} \quad (4)$$

Apparently the path of *x*-scales in *y* direction is not important in most of cases, but it is important when the reader of scale (a car) is supported on the *x*-guide, and they follow their own non-straight paths.

2.3. Scale and straightness of an arm mounted in a car, which slide along one non-straight axis.

In this case we have three effects: one due to distribution of scale, one due to straightness and one due to tilting. The last known as Abbe effect, will affect in two directions: transversal and parallel to the axis; it depends on the configuration (R_x, R_y) of the supported body. These distortions may be represented with next splines:

$$\begin{aligned} x(u) &= U(V_x + k\Delta V_x + kR_y\theta) \\ y(u) &= U(V_y + k\Delta V_y + kR_x\theta) \end{aligned} \quad (5)$$

Where υ is the tilting of the axis, and may be known through derivatives:

$$\theta = \frac{y^u(u)}{x^u(u)} \quad (6)$$

For more accurate calculus, θ in (5) and (6) may be substituted by *tan(θ)*.

Figure 2 represents a part of distortion in *x*, the *y* part was omitted in order to avoid heaviness of drawings. More complete drawings that include all affects will be showed in next drawings.

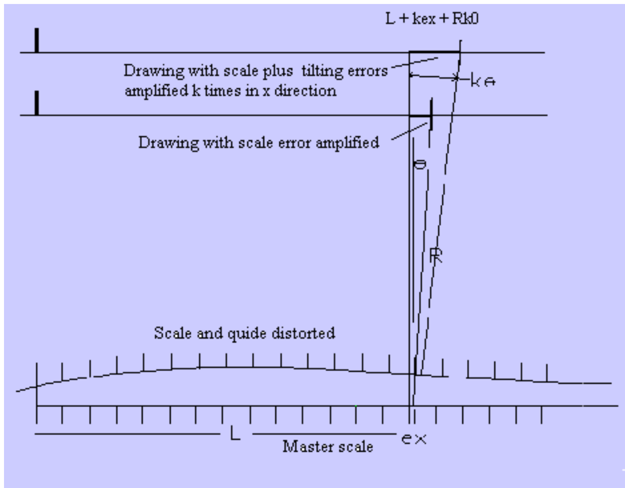

Figure 2. This drawing shows the classical effect of Abbe for x direction.

2.4. Scale and straightness along two guides with cinematic dependence, and therefore with Abbe effects.

Although there are cases without cinematic dependence, the most cases are presented with cinematic dependence. For this last case we have similar equations like (5), but we now need to consider two movements *u, w*, two vectors describing the aspect of supported bodies R_1, R_2, and two tilting of guide-ways $\upsilon 1$, υ_2. The movements of the second supported body is described by equations (7).

$$\begin{aligned} x(u,w) &= U(V_{1x} + k\Delta V_{1x} + kR_{1y}\theta_1) + \\ &\quad + W(V(u)_{2x} + k\Delta V(u)_{2x} + kR_{2y}\theta_2) \\ y(u,w) &= U(V_{1y} + k\Delta V_{1y} + kR_{1x}\theta_1) \\ &\quad + W(V(u)_{2y} + k\Delta V(u)_{2y} + kR_{2x}\theta_2) \end{aligned} \quad (7)$$

Arrays of vertex $V_{2x}+k\Delta V_{2x,y}$ or $V_{2y}+k\Delta V_{2y}$, in (7) describe the distribution of scales and shape of supported guide-ways. Those arrays vary continuously, and may be calculated from (4) and its derivative, by applying transformations of rigid body. With this treatment of straightness, orthogonality is included.

2.5. In 3D with three supported bodies.

The treatment of this case is similar to previous in 2.4 because tilting, squareness and straightness are refereed to planes; therefore an inclination on *x, y* plane will only effect measures on *x, y* plane; and so on *x, z*, and *y, z* planes. The equations (7) then must be adjusted by interchanging sub-indices of supported bodies.

3 OVERCOMING NON LINEARITIES

There are three cases where non linearity are presented: a) in linear arrays (grids) when distortion of length and angle are amplified like a distorted parallelogram; b) non linear arrays or non straight arrays, c) non straight arrays produced by bodies with cinematic dependence (Abbe effects).

3.1. Length and angle for linear 2D arrays

As it was asserted, amplification of length and amplification of angle do not keep linearity, but it is necessary to answer the question: "Is there a limit of amplification, where relative differences are less than expected uncertainty?". To answer this question it is necessary to remember that a result of diagnose, always have a part of uncertainty; then variations of angle are valid if they are less than an expected uncertainty.

Fixing the absolute values of uncertainty as a fifth part of absolute errors, it can fix the limit of amplification. Referring to figure 3., we observe a perfect parallelogram in black, a distorted parallelogram in blue, a diagonal error in the top right corner in blue, a parallelogram and its diagonal amplified five times in green, and a parallelogram amplified five times based on original deviations of angle in red.

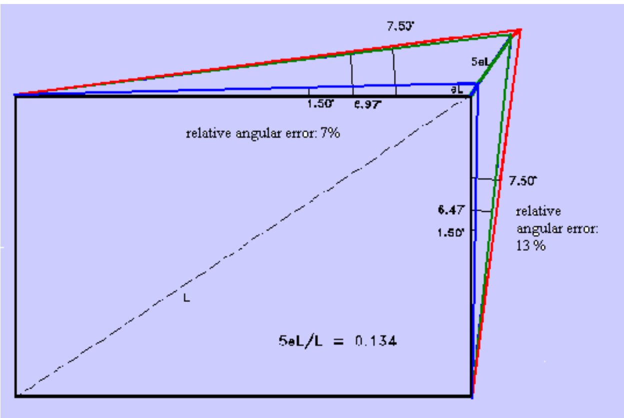

Figure 3. Drawing showing that relative faults of angle are less than a fifth part of angle after amplification.

Taking vertical values of angle, we observe that real distortion is 1.5 degrees; amplifying 5 times we get 7.5 degrees; but the angle obtained from green parallelogram is 6.47 degrees. This case of non-linearity is less than acceptable uncertainty for most of cases. Relation between real diagonal and amplified error of diagonal is near to 7.5; useful quantity for fixing limits of amplification. For most relations of aspect, errors were less than expected uncertainty.

3.2. Length and angle for non straight 2D arrays

The appearance of equations (5) and (7) show linearity when amplification is applied on distortions; but the appearance or likelihood between drawings before and after amplification do not keep linearity, since straightness is not a linear property. However, a non-straight grid may be represented by an array of linear deformed parallelograms, and the criteria of linear arrays may be applied cell by cell, as seeing in figure 4 and subsequent.

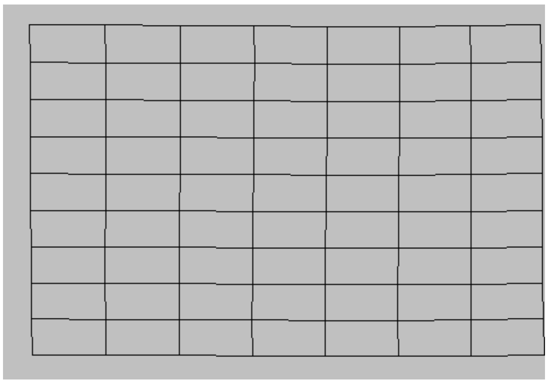

Figure 4. Paths of movements for a 2D machine with cinematic dependence; deviations are equivalent to 0.01 of maximum displacement.

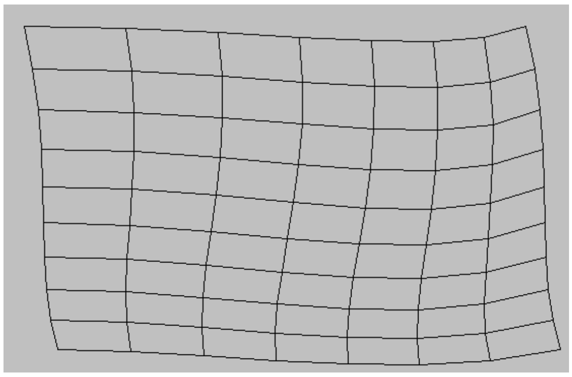

Figure 5. Amplification *10x* of original distortion.

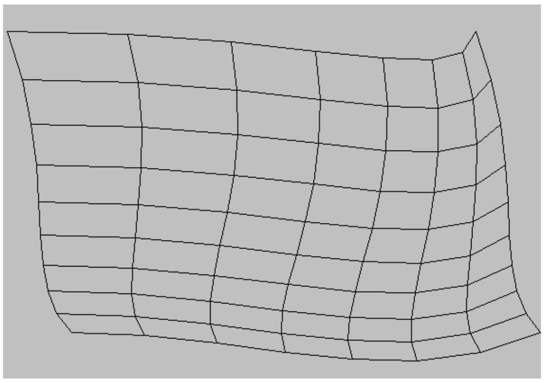

Figure 6. Amplification *20x* of original distortion.

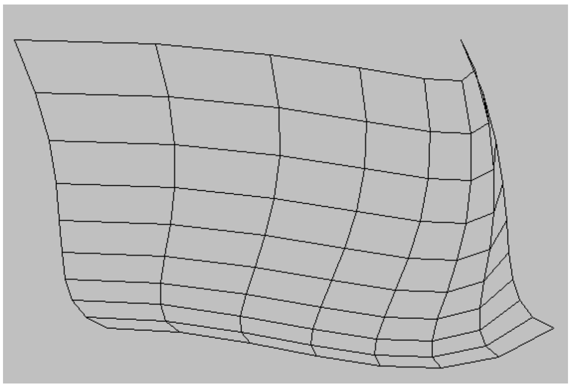

Figure 7. Amplification *30x* of original distortion.

From observation of figures 4 to 6 figures, where amplification is growing, it is easy to suppose that linearity is lost for high amplifications; however a more fine analysis shows that linearity is kept.

For instance, taking the upper right parallelogram for original and 10x, 20x, and 30x drawings, we observe linearity in linear and angular errors, comparing blue (perfect) and black drawings.

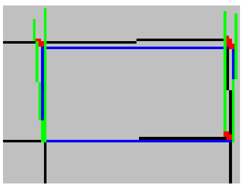

Figure 8.a. Superposition with perfect parallelogram.

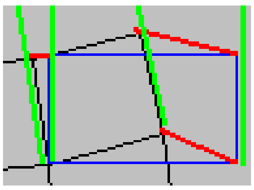

Figure 8.b. Superposition of 10x with perfect parallelogram.

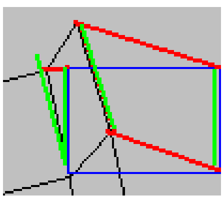

Figure 8.c. Superposition of 20x with perfect parallelogram.

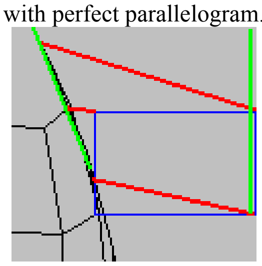

Figure 8.d. Superposition of 30x with perfect parallelogram

The error in red for corners of parallelogram corresponds to their amplification; as well the angular aperture of sides of parallelogram in green correspond to their amplifications.

All previous drawings correspond to real paths of movements, and don't show the shape of guides. The analysis of tilting combined with the aspect of supported bodies, correspond to the main goal of this research: to establish diagnose easily from aspect of drawings.

4. EFFECTS OF TILTING AND REVERSE DIAGNOSE

As we can observe in figure 7, the upper and lower paths are not parallel, not envelop, and if extrapolation were possible, paths will not coincide with guides. Differences mentioned are based on the aspect of supported bodies and their relation Rx/Ry mentioned in (7), as is showed in figure 8.

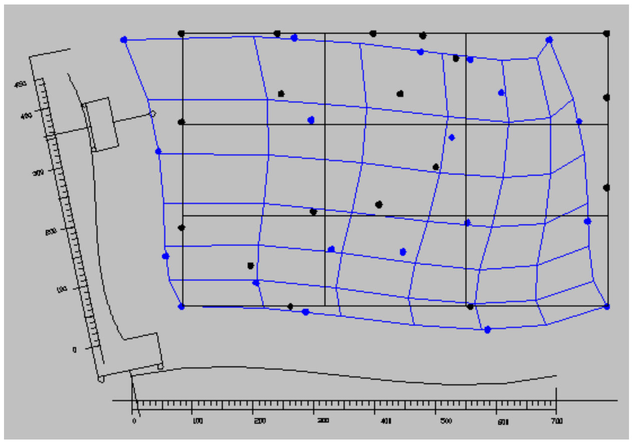

Figure 8. Machine with distortions amplified 20 times. Left and lower paths follow different laws of coincidence with guides, depending on the relation of aspect.

Or principle of correction is based on centre of curvature for all paths including guides, and the size and aspect of bodies that slide on the guides. With this criterion it is possible to correct the aspect of the deformed grid. In general, for configurations like these, the aspect of patches of measures need be compressed vertically.

4.1. Experiments

Using our interfaces of virtual machines, we produced a machine with errors 10 times lower than showed in figure 8. The drawing of measures is the grid showed in figure 9. This grid is deformed 200x and after applying correction of aspect is also showed in figure 9. The results are that all variations of scale, orthogonality, straightness, and curvature of guides , coincide with coincide with original errors, after reducing deviations 200*x*. See figure 9.

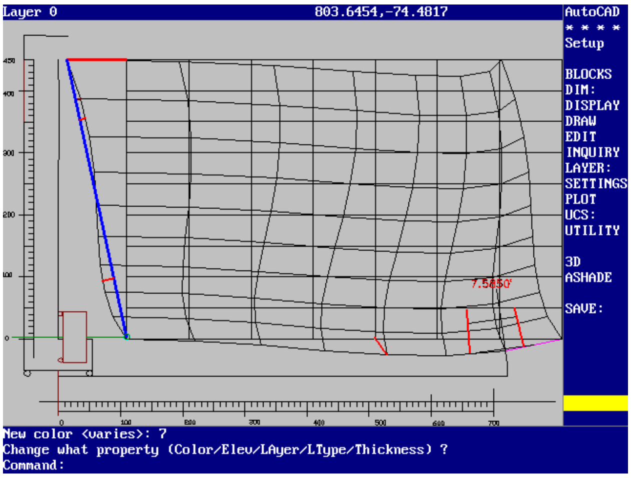

Figure 9. This drawing shows a virtual 2D machine, a mesh of amplified errors, and diagnose in red and blue lines.

Two phenomena must coincide into tolerances: the measures of a machine where original errors are amplified 200*x*, and the drawings of measures deformed 200*x* from the machine with errors amplified 1*x*. Figure 10 shows the two cases.

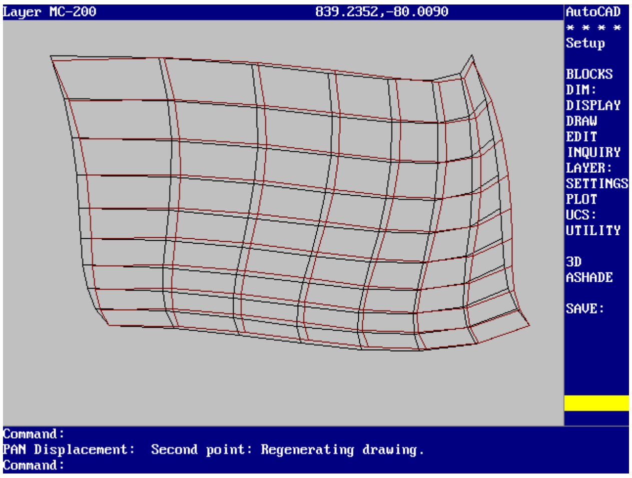

Figure 10. Drawing from machine deformed 200x in black, and drawing deformed 200x from original machine in red.

5. CONCLUSIONS

It is difficulty to resume the power of those virtual tools, their analysis and their experiments, but perhaps two could express our expectations:

a) "It is necessary to fix an universal model and process of calibration, and to ease the interpretation of results; our virtual instrument looks a good candidate "

b) b) "it is necessary to develop universal tools to convince experts and organisations about the results of calibrations; our analysis looks a good candidate".

6. REFERENCES

[1.] J. A. Soons, et al, Modelling the errors of multi-axís machines a general methodology. PRECISION ENGINEERING. January 1992, vol. 14, n 1

[2] Pier Carlo Cresto. Self-calibration with application to CMM's geometry error correction. **Proceedings of the Workshop.** Advanced Mathematical Tools in Metrology, Torino, Italy, October 1993, Elsevier.

[3] Sanchez Jose et al. "Evaluating Masters and Procedures of Calibration with non Linear Computational Mechanisms", XIV IMEKO World Congress, 1 - 6 June, 1998, Tampere, Finland. Proceedings.

First author: Ms Eng. José Sánchez, Metrology Laboratory, Applied Sciences and Technical Research Centre, University of Mexico, 04510, D.F. México, Phone: 52+5556228602 ext 121, Fax: 52+5556228624, email: sanchezj@aleph.cinstrum.unam.mx.

Dr. Saúl Santillan. Mechanical Design Centre CDM, Faculty of Engineering, National University of Mexico, D.F. Phone: 52+5556228051 Fax: 52+5556228051, email:saulsan@servidor.unam.mx..

Eng. Sergio Padilla,, Metrology Laboratory CCADET, University of Mexico, 04510, D.F. México, Phone: 52+5556228602, Fax: 52+5556228624, email: padillas@aleph.cinstrum.unam.mx.

Ms Eng. Benjamín Valera, Metrology Laboratory, CCADET, University of Mexico, 04510, D.F. México, Phone: 52+5556228602, Fax: 52+5556228624, email: valerab@aleph.cinstrum.unam.mx.

Eng. Rigoberto Nava, Metrology Laboratory, CCADET, University of Mexico, 04510, D.F.México, Phone: 52+5556228602, Fax: 52+5556228624, email: navar@aleph.cinstrum.unam.mx.

Eng. Gerardo Ruiz, Metrology Laboratory, CCADET, University of Mexico, 04510, D.F. México, Phone: 52+5556228602, Fax: 52+5556228624, email: ruizb@aleph.cinstrum.unam.mx.

Acknowledgement:
National Council of Science and Technology