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ESTIMATION OF UNCERTAINTY FROM UNKNOWN SYSTEMATIC ERRORS IN COORDINATE METROLOGY

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Abstract – In coordinate metrology, a feature (Gaussian associated feature) is calculated from a measured data set by CMM (Coordinate Measuring Machine) using least squares method. This data processing flow is called as "feature-based metrology". In the feature-based metrology, it is a key technique to estimate the uncertainty of measurement in the specific measuring strategy. In this paper, the effects of unknown systematic errors are theoretically analyzed to estimate the uncertainties. The estimation method of uncertainties from two types of unknown systematic errors, such as errors of calibration of ball probe and errors form deviations of measured workpieces, are proposed.

Keywords: CMM (Coordinate Measuring Machine), uncertainty, form deviation

1. FEATURE-BASED METROLOGY

In coordinate metrology, associated features and associated derived features are calculated from measured data sets on real features by CMM (Coordinate Measuring Machine). Then, the associated features are compared with the nominal features indicated on the drawings (see Fig. 1). In this data processing, the features are primal targets to calculate, to evaluate and to process. Consequently, this process is called as "Feature-Based Metrology" [1].

In the feature-based metrology, it is a key technique to estimate the uncertainty of measurement [2] in the specific measuring strategy [3-5]. The estimation method for uncertainties of measured parameters has been already proposed when the only random errors are put in the consideration [6]. The uncertainty of each measured point is defined by error analysis of the CMM and the probing system. From the uncertainty of measured point, the uncertainty of measured feature can be calculated statistically using following equations.

Equation (1) shows an observation equation, a regular equation and a least squares solution, where **A** is Jacobian matrix, **d** is measurements vector, **x** is a parameter vector and **S** is an error matrix. In this paper, we analyze uncertainty measurement of circular features. For the circular features, the uncertainties of parameter s_x, s_y and s_d (X and Y coordinate of the center and the diameter) calculated by Eq. (2).

$$\begin{aligned} \mathbf{d} &= \mathbf{A}\mathbf{x} \\ \mathbf{A}^t\mathbf{S}^{-1}\mathbf{A}\mathbf{x} &= \mathbf{A}^t\mathbf{S}^{-1}\mathbf{d} \\ \mathbf{x} &= (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})\mathbf{A}^t\mathbf{S}^{-1}\mathbf{d} \end{aligned} \tag{1}$$

$$\mathbf{P} = \begin{pmatrix} s_x^2 & s_{xy} & s_{xd} \\ s_{xy} & s_y^2 & s_{yd} \\ s_{xd} & s_{yd} & s_d^2 \end{pmatrix} = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})^{-1} \tag{2}$$

2. UNKNOWN SYSTEMTIC ERRORS

We analyze two types of unknown systematic errors in measurement process of circular workpieces. For simple analysis, all measurement processes are done in two dimensional space. The first factor is uncertainty from the calibration process of the ball probe [6] and the second factor is the effect of the form deviations of measured circular workpieces.

The error matrix **S** is only effected by the systematic errors. When there is no systematic error in the measuring process, the error matrix is the unit matrix multiplied by the random error s_p. For the known systematic errors, we can compensate the measuring values. On the other hand, when the unknown systematic errors influence the measuring results, the error matrix has the factors of covariance by the systematic errors. Using the error matrix, we can estimate the influences from the unknown systematic errors statistically.

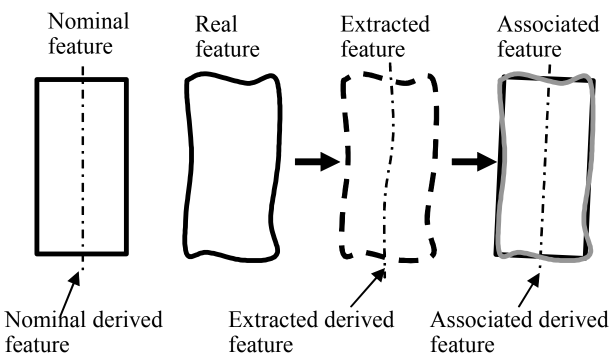


Fig. 1 Data processing flow in feature-based metrology.

3.1 Calibration of ball probe

Fig. 2 shows the two dimensional model for the theoretical analysis of the effect of the calibration process of the ball probe. Firstly, the diameter and the center position of a probe ball are calibrated by measuring a reference circle. The diameter of the reference circle is calibrated with the uncertainty s_c. After the calibration, a measured circle is measured by the ball probe with random measured error s_p. Measured position on the measured circle is indicated by angle t_i shown on Fig. 3.

There are two unknown systematic errors, errors of diameter and center position, in calibration of probe. Two measuring positions (angle) t_1 and t_2 has the measuring errors dr_1 and dr_2 from diameter error on the measured circle (Eq. (3)). Then, variance and the covariance of two measuring data are calculated in Eq. (4).

$$dr_1 = p_1 + \frac{d}{2}, \quad dr_2 = p_2 + \frac{d}{2} \tag{3}$$

$$s_1^2 = s_2^2 = s_p^2 + \frac{c_d^2}{4}, \quad s_{12}^2 = \frac{c_d^2}{4} \tag{4}$$

Two measuring positions (angle) t_1 and t_2 has the measuring errors dr_1 and dr_2 from center position errors on the measured circle (Eq. (5)). The variance and the covariance from center position errors are calculated from the error matrix shown in Eq. (6), where c_x and c_y are center position errors.

$$dr_1 = dx\cos t_1 + dy\sin t_1, \quad dr_2 = dx\cos t_2 + dy\sin t_2 \tag{5}$$

$$\begin{aligned} s_1^2 &= s_2^2 = c_x^2\cos^2 t_1 + c_y^2\sin^2 t_1 = c_x^2 \\ s_{12}^2 &= c_x^2\cos t_1\cos t_2 + c_y^2\sin t_1\sin t_2 = c_x^2\cos(t_1 - t_2) \end{aligned} \tag{6}$$

From two unknown systematic errors, the total variances and covariance are calculated in Eq. (7). Using these Eqs., the uncertainties of measured circle can be defined. If we use two probing system or styli, the total error matrix are defined in Eq. (8).

$$s_i^2 = s_p^2 + c_x^2 + \frac{c_d^2}{4}, \quad s_{ij}^2 = c_x^2\cos(t_i - t_j) + \frac{c_d^2}{4} \tag{7}$$

$$\mathbf{S} = \left(\begin{array}{c|c} \mathbf{S}_1 & \begin{matrix} \frac{s_c^2}{4} & \cdots & \frac{s_c^2}{4} \\ \vdots & \ddots & \vdots \\ \frac{s_c^2}{4} & \cdots & \frac{s_c^2}{4} \end{matrix} \\ \hline \begin{matrix} \frac{s_c^2}{4} & \cdots & \frac{s_c^2}{4} \\ \vdots & \ddots & \vdots \\ \frac{s_c^2}{4} & \cdots & \frac{s_c^2}{4} \end{matrix} & \mathbf{S}_2 \end{array}\right) \tag{8}$$

Fig. 4 shows an example of circle measurements for estimation of uncertainty on 14 measured points by 3 probes. Table 1 indicate the estimated uncertainties by patterns of consideration on center position error of probe, diameter error of probe and certificate error of reference circle, where c_x is 2.25 µm, s_c is 3 µm and s_p is 5 µm. These uncertainties

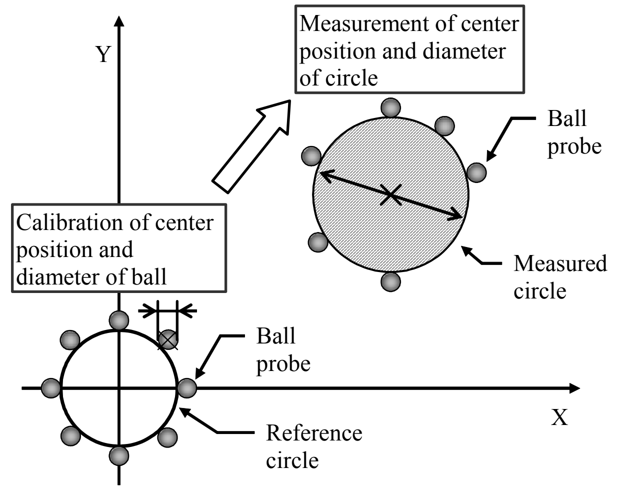


Fig. 2 Model for calibration of ball probe and measurement of circle in 2 dimensional area

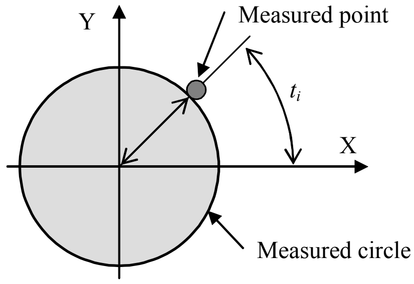


Fig. 3 Measured position indicated by angle t_i

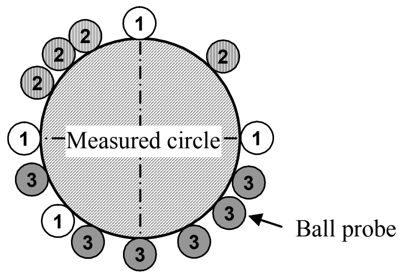


Fig. 4 Example of circle measurements for estimation of uncertainty on 14 measured points by 3 probes

Table 1 Uncertainty of center position and diameter of circle by contributors of calibration errors on Fig. 4 (b)

unit: µm

Center position error of probe c_x, c_y	no	yes	no	yes	yes
Diameter error of probe c_d (s_c not included)	no	no	yes	yes	yes
Certificate error of reference circle s_c	no	no	no	no	yes
X position error of measured circle s_x	1.928	2.441	1.933	2.444	2.444
Y position error of measured circle s_y	1.956	2.473	2.266	2.720	2.720
Diameter error of measured circle s_c	2.733	3.197	3.419	3.805	4.845

indicate diameter error of measured circle varies from 2.733 μm to 4.854 μm relation to the consideration pattern of unknown systematic errors.

3.2 Form deviation of measured workpice

Fig. 5 displays the circular features with random errors and with errors from autocorrelation function in Fig. 6. When form deviation is random function, the error matrix $\mathbf{S}_{\mathrm{ran}}$ is defined by the unit matrix and variance s_f^2 of form deviation in Eq. 9. When form deviation has specified function, the error matrix $\mathbf{S}_{\mathrm{cov}}$ is defined by the autocorrelation matrix $\mathbf{R}_{\mathrm{cov}}$ and the variance s_f^2 of form deviation in Eq. 10.

$$\mathbf{S}_{\mathrm{ran}} = \begin{pmatrix} s_f^2 & & 0 \\ & \ddots & \\ 0 & & s_f^2 \end{pmatrix} = s_f^2 \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = s_f^2 \mathbf{E} \tag{9}$$

$$\begin{aligned} \mathbf{S}_{\mathrm{cov}} &= \begin{pmatrix} s_f^2 & s_{12} & \cdots & s_{1n} \\ s_{12} & \ddots & s_{ij} & \vdots \\ \vdots & s_{ij} & \ddots & s_{n-1n} \\ s_{1n} & \cdots & s_{n-1n} & s_f^2 \end{pmatrix} \\ &= s_f^2 \begin{pmatrix} 1 & r_{12} & \cdots & r_{1n} \\ r_{12} & \ddots & r_{ij} & \vdots \\ \vdots & r_{ij} & \ddots & r_{n-1n} \\ r_{1n} & \cdots & r_{n-1n} & 1 \end{pmatrix} = s_f^2 \mathbf{R}_{\mathrm{cov}} \end{aligned} \tag{10}$$

From two types of error matrix $\mathbf{S}_{\mathrm{ran}}$ and $\mathbf{S}_{\mathrm{cov}}$, there are three types of uncertainties of measured parameters $\mathbf{P}_{\mathrm{ran}}$, $\mathbf{P}_{\mathrm{cov}}$ and $\mathbf{P}_{\mathrm{r+c}}$ are defined in Eqs. 11, 12 and 13.

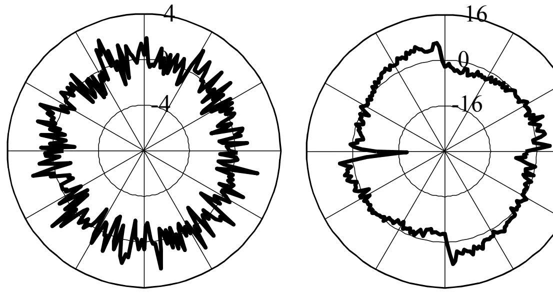
$\mathbf{P}_{\mathrm{ran}}$ is uncertainty matrix of center position and diameter of measured circle when form deviation is ashamed as random function. $\mathbf{P}_{\mathrm{cov}}$ is uncertainty matrix when form deviation has the specified autocorrelation function and calculated using the autocorrelation function. $\mathbf{P}_{\mathrm{r+c}}$ is uncertainty matrix when the form deviation has the specified autocorrelation function and calculated without the autocorrelation function. Usually the calculating program in CMM can not handle the autocorrelation function. Therefore, uncertainties of the normal calculating situation in measuring by CMM are defined by $\mathbf{P}_{\mathrm{r+c}}$.

$$\mathbf{P}_{\mathrm{ran}} = (\mathbf{A}^t \mathbf{S}_{\mathrm{ran}}^{-1} \mathbf{A})^{-1} = s_f^2 (\mathbf{A}^t \mathbf{A})^{-1} \tag{11}$$

$$\mathbf{P}_{\mathrm{cov}} = (\mathbf{A}^t \mathbf{S}_{\mathrm{cov}}^{-1} \mathbf{A})^{-1} = s_f^2 (\mathbf{A}^t \mathbf{R}_{\mathrm{cov}}^{-1} \mathbf{A})^{-1} \tag{12}$$

$$\begin{aligned} \mathbf{P}_{\mathrm{r+c}} &= ((\mathbf{A}^t \mathbf{A})^{-1} \mathbf{A}^t)\, \mathbf{S}_{\mathrm{cov}}\, ((\mathbf{A}^t \mathbf{A})^{-1} \mathbf{A}^t)^t \\ &= s_f^2 ((\mathbf{A}^t \mathbf{A})^{-1} \mathbf{A}^t)\, \mathbf{R}_{\mathrm{cov}}\, ((\mathbf{A}^t \mathbf{A})^{-1} \mathbf{A}^t)^t \end{aligned} \tag{13}$$

When measuring area is all measured circle, $\mathbf{P}_{\mathrm{cov}}$ and $\mathbf{P}_{\mathrm{r+c}}$ are completely same values, because of $\mathbf{S}_{\mathrm{cov}}$ and $\mathbf{S}_{\mathrm{ran}}$ are the same weight function for least squares calculation. Fig. 7 illustrates relationship between number of data n and uncertainty (standard deviation) of diameter s_d and X coordinate of center s_x. On Fig. 7 (a), uncertainty of diameter in 4, 6 and 8 measured data is larger than these in odd number of measured data. This is because autocorrelation function of measured circle (Fig. 6) has large



(a) random errors (b) errors from autocorrelations

Fig. 5 Circular feature with random errors and errors from specified autocorrelations

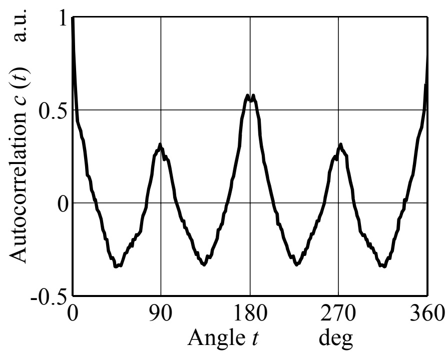
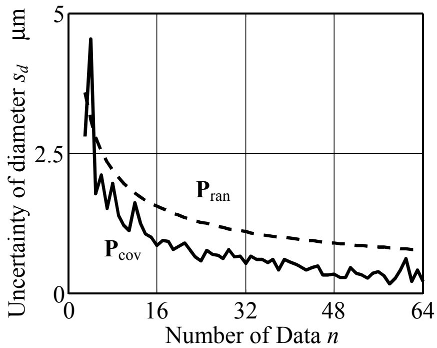
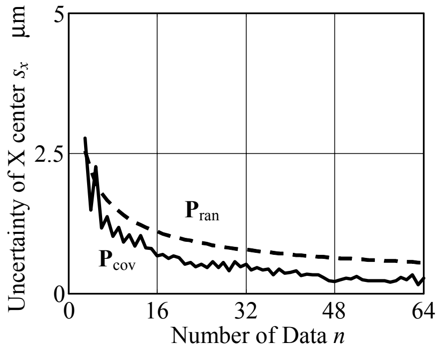


Fig. 6 Autocorrelation function of measured circle (Fig. 5 (b))



(a) Uncertainty of diameter



(b) Uncertainty of X coordinate of center

Fig. 7 Relationship between number of data n and uncertainty (standard deviation) of diameter s_d and X coordinate of center s_x

2 and 4 order frequency values. On the other hand, the uncertainty of center (Fig. 7 (b)) in 3 and 5 measured data are larger than these in even number of measured data.

When the measured data are in the measured area a for partial circle measurement in Fig. 8, $\mathbf{S}_{cov}$ and $\mathbf{S}_{ran}$ are not same weight functions. Fig. 9 shows the relationship between the uncertainty of diameter s_d and the number of data n in the partial circle measurement of angle a = 180, 90 and 30 deg, by 3 calculated methods $\mathbf{P}_{r+c}$, $\mathbf{P}_{cov}$ and $\mathbf{P}_{ran}$. For the partial circle measurement, $\mathbf{P}_{ran}$ is under estimation and $\mathbf{P}_{r+c}$ is over estimation for the uncertainty, when the number of measured data is large.

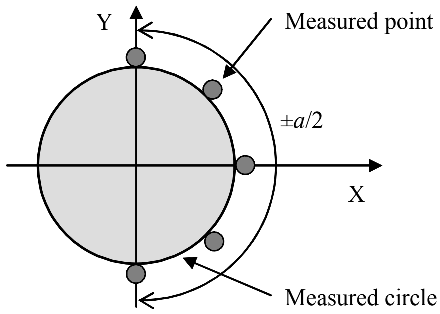
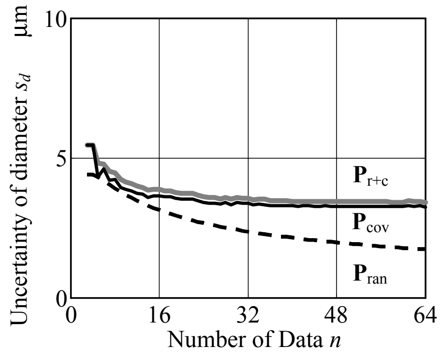
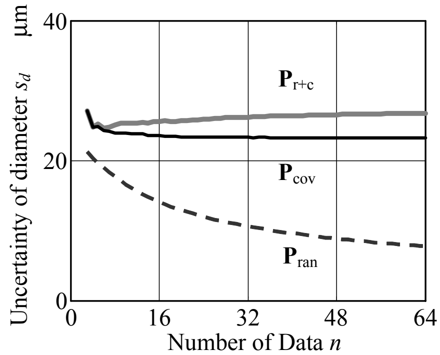


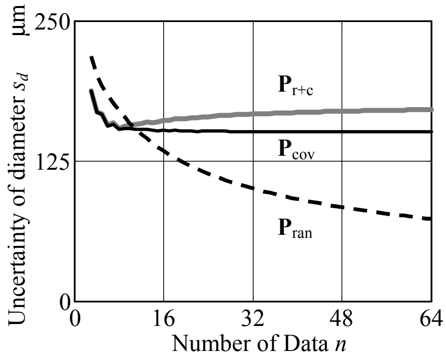
Fig. 8 Measured area a for partial circle measurement, 5 measured points in ±90 deg (a = 180 deg)



(a) a = 180 deg



(b) a = 90 deg



(c) a = 30 deg

Fig. 9 Relationship between uncertainty of diameter s_d and number of data n in partial circle measurement of angle a = 180, 90 and 30 deg, by 3 calculated methods $\mathbf{P}_{r+c}$, $\mathbf{P}_{cov}$ and $\mathbf{P}_{ran}$

4. CONCLUSIONS

In this paper, we theoretically analyzed the effects of the unknown systematic errors in feature-based metrology. The calibrated error of the ball probe and the form deviation of circular feature treated as the unknown systematic errors. These errors propagate as unknown systematic errors to the uncertainties of measured parameters, such as the center position and the diameter of a measured circle. The method to calculate the error matrix **S** was derived when the center position and the diameter of the circle are measured.

Using this method, the uncertainties of the measured parameters can also be calculated in the complex measuring strategy. The series of simulations for this method in statistical way directly implies that the concept and the basic data processing method in this paper are useful to the feature based metrology.

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