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APPLICATION OF VECTORIAL DIMENSIONING AND TOLERANCING TO AN UNIQUE 3-D STACKUP ANALYSIS

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Abstract – It is demonstrated that a dependent dimension in assembly, indicated according to the classical dimensioning rules, is not uniquely defined. It is shown that if parts in assembly are defined by the vectorial dimensioning and tolerancing, VDT, then unique specification and verification of the analysed functional dimension in assembly is assured. The stackup of location and orientation vectors on the case of an assembly shaft–sleeve–plate based on experiments carried on coordinate measuring machine, CMM, is analysed.

Keywords: dimensioning, tolerancing, VDT.

1. SPECIFICATION UNCERTAINTY

The capabilities of existing tolerance analysis and synthesis systems are limited in dealing with the majority of real world problems. They are applicable only to 1-dimensional and simple tolerancing assemblies. This limitation is due to the lack of a comprehensive representation scheme for geometrical tolerancing [1].

The practical problems mentioned above are illustrated in Fig. 1a that shows an assembly shaft–sleeve–plate with indicated dependent dimension *X*. With proper dimensioning of each single part (in axial direction using respectively toleranced dimensions N_{-T}, N^{+T}, $N \pm T/2$) the dimensional analysis – calculation of dependent dimension *X* and its limits is a simple task.

The same set with dimension and orientation deviations is given in Fig. 1b. The dimension *X* is not uniquely defined – Fig. 1a does not define how the specified dimension should be verified. On the cross-section, it is shown that measurement results are different for:

- upper edges of sleeve and plate;
- axis of the shaft;
- lower edges of sleeve and plate.

Moreover a measurement in line with the shaft axis cannot be performed without CMM since the indicated points are imaginable theoretical points. Also a measurement on the edges of the sleeve and plate is not practically executable and in some cases it is impossible to determine the examined dimension (for example the dimensions indicated as X5 and X7). Fig. 1b shows also that in each measuring spot, the dimension *X* mentioned above can be evaluated according to three different algorithms:

- in the direction parallel to the shaft axis – as the distance between the common points of shaft axis and selected planes;
- perpendicularly to the front plane of the sleeve – as the distance to the point on the plate plane;
- perpendicularly to the front plane of the plate – as the distance to the point on the sleeve plane.

Each of the mentioned measurements will give different results. These different results are produced due to the lack of datum call out in Fig. 1a. Hence, a more precise method for complete and well-defined specifications has to be used to express assembly functional requirements.

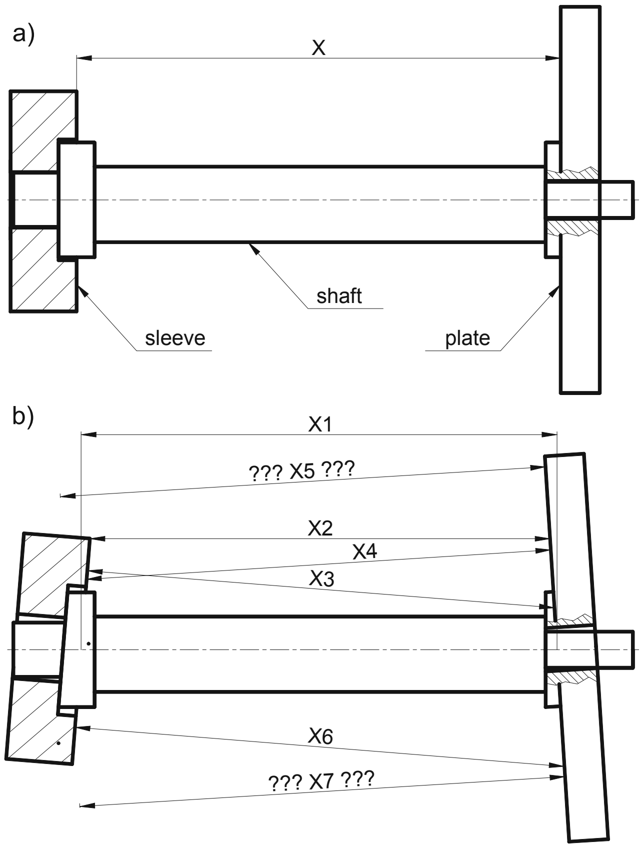

Fig. 1. a) Assembly with dependent dimension X indicated according to classical dimensioning rules b) A few possible interpretations of specified dimension during verification.

2. VDT CONCEPT – AN APPLICATION FOR TOLERANCE STACKUP ANALYSIS

The vectorial dimensioning and tolerancing concept is a new tool for specification of the workpiece geometry that enables the efficient and complete definition and verification of workpiece geometry. In the VDT a workpiece is defined as a set of the substitute features (plane, pair of planes, cylinder, cone, etc.). Each feature is represented by a location vector, an orientation unit vector with toleranced components and a toleranced size (when applicable), [2-5].

Fig. 2 with Tab. 1 and Tab 2 and Tab. 3 specify the geometry of shaft, sleeve and plate according to the VDT rules delineated in [2-5]. The shaft consists of 4 cylindrical surfaces and 6 planes. The sleeve consists of 3 cylindrical surfaces and 4 planes. The plate consists of 1 cylindrical surface and 11 planes. The substitute elements should be numbered and their specified points should be indicated by crossed circles. Therefore at Fig. 2 ten substitute elements should be indicated for the shaft, seven for the sleeve and twelve for the plate. To make the drawing more clear only these features are marked that are relevant to the intended assembly function or used to fix sleeve and plate on the shaft (cylinders No 1, 4, 5 and planes No 2, 3, 6 for shaft; cylinder No 10 and planes No 11, 12, 13, 14 for sleeve; cylinder No 20 and planes No 21, 22, 23 for plate). For each element, the local substitute coordinate system is defined by frames.

For the shaft the substitute coordinate system $Ox_1y_1z_1$ is defined by axis of the substitute cylinder SA and the substitute line SC moved to substitute point SD. The primary substitute datum (xy plane) is defined by the plane containing the axis of the substitute cylinder SA (primary datum feature) and the constructed line – the intersection of the substitute datum plane SB and the substitute plane No3

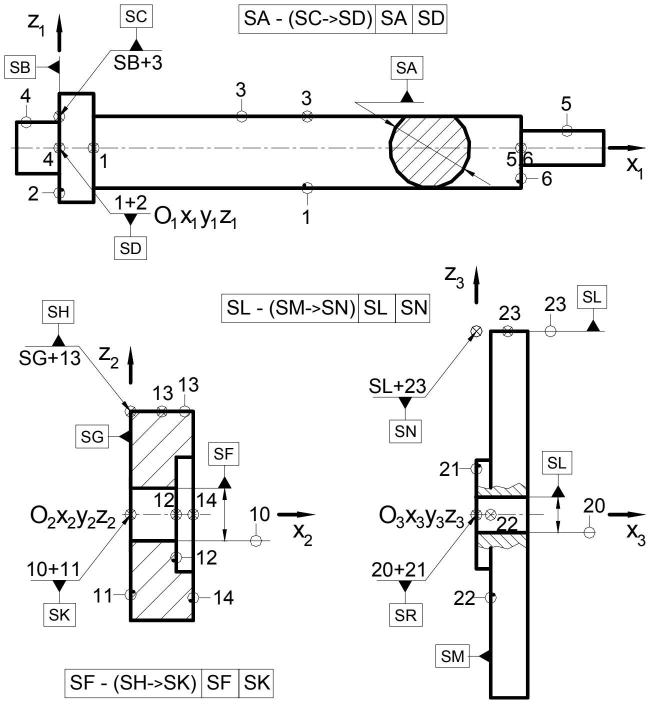

Fig. 2. Shaft, sleeve and plate with selected features indicated according to the VDT rules

TABLE 1. Nominal values of location vector, orientation unit vector and their limit deviations for the shaft.

Substitute element No:			1	2	3	4	5	6
Location [mm]	Nominal	P_x	12	0	77,5	0	161,96	161,96
		P_y	0	0	0	0	0	0
		P_z	0	0	10,98	0		00
	Limit dev. (±)	T_{Px}	•	•	×	×	×	0,02
		T_{Py}	•	•	×	0,01	0,02	×
		T_{Pz}	•	•	0,02	0,01	0,02	×
Orientation	Nominal	E_x	-1	-1	0	-1	-1	1
		E_y	0	0	0	0	0	0
		E_z	0	0	1	0	0	0
	L. dev. (±)$\times 10^{-3}$	T_{Ex}	•	–	0,07	–	–	–
		T_{Ey}	•	0,13	0,58	0,33	0,34	0,18
		T_{Ez}	•	0,13	–	0,33	0,34	0,20
Size [mm]	Nominal		22,99	n.a.	n.a.	17,99	11,99	n.a.
	L. dev. (±)		0,01	n.a.	n.a.	0,01	0,01	n.a.
Form	Nominal		cylinder	plane	plane	cylinder	cylinder	plane
	L. dev.		0,012	0,01	0,01	0,008	0,008	0,01

TABLE 2. Nominal values of location vector, orientation unit vector and their limit deviations for the plate

Substitute element No:			20	21	22	23
Location [mm]	Nominal	P_x	0	0	0	9
		P_y	0	0	0	0
		P_z	0	0	5	48,98
	Limit dev. (±)	T_{Px}	•	•	0,01	×
		T_{Py}	•	•	×	×
		T_{Pz}	•	•	×	0,02
Orientation	Nominal	E_x	-1	-1	-1	0
		E_y	0	0	0	0
		E_z	0	0	0	1
	L. dev. (±)$\times 10^{-3}$	T_{Ex}	•	–	–	0,67
		T_{Ey}	•	0,13	0,08	0,08
		T_{Ez}	•	0,13	0,10	–
Size [mm]	Nominal		12,38	n.a.	n.a.	n.a.
	L. dev. ±)		0,02	n.a.	n.a.	n.a.
Form	Nominal		cylinder	plane	plane	plane
	L. dev.		0,008	0,01	0,01	0,01

• limit deviation with default value 0 due to the construction of datum system (diversified only in this paper usually specified as 0)
× component not defined due to feature degree of freedom in respective direction (diversified only in this paper usually marked by –)
– negligible component (close to 0 value of minor component, not specified to keep absolute value of orientation unit vector)
n.a. not applicable

that is conceived to be projected to the substitute point SD. The substitute point SD is the intersection of the axis of the substitute cylinder SA and the substitute plane SB. The secondary substitute datum (xz plane) is defined by the plane perpendicular to the primary datum containing axis of the primary datum feature – the substitute cylinder SA – which becomes thereby also the secondary datum feature. The tertiary substitute datum (yz plane) is defined by a plane perpendicular to the primary and secondary datums and containing the substitute intersection point SD.

The substitute coordinate systems $Ox_2y_2z_2$ for the sleeve and $Ox_3y_3z_3$ for the plate are defined similarly.

TABLE 3. Nominal values of location vector, orientation unit vector and their limit deviations for the sleeve.

Substitute element No:			10	11	12	13	14
Location [mm]	Nominal	P_x	0	0	16	11	21,99
		P_y	0	0	0	0	0
		P_z	0	0	0	35,98	0
	Limit dev. (±)	T_{Px}	•	•	×	×	0,01
		T_{Py}	•	•	0,01	×	×
		T_{Pz}	•	•	0,01	0,02	×
Orientation	Nominal	E_x	-1	-1	-1	0	1
		E_y	0	0	0	0	0
		E_z	0	0	0	1	0
	L. dev. (±)×10^{-3}	T_{Ex}	•	–	–	0,45	–
		T_{Ey}	•	0,13	0,25	0,33	0,13
		T_{Ez}	•	0,13	0,25	–	0,132
Size [mm]	Nominal		18,38	n.a.	n.a.	n.a.	n.a.
	L.dev. (±)		0,02	n.a.	n.a.	n.a.	n.a.
Form	Nominal		cylinder	plane	plane	plane	plane
	L.dev.		0,008	0,01	0,01	0,01	0,01

As it is shown in Fig. 2 and Tab. 1-3 the VDT applies in geometrical specification quite different notation that engineers are familiar to use on design drawings. On one hand the lack of dimensional and extension lines, hence the explicit absence of dimensional chains considered using 1-D approach, makes difficult the tolerance stackup for design or manufacturing engineer. On the other hand the vectorial specification enables clear 3-D tolerance stackup specification, so the analysis and verification can be computer aided. Fig. 3a shows the assembly shaft–sleeve–plate for which location and orientation between sleeve plane (plane No14) and plane of plate flange (plane No22) are crucial for proper function of the assembly. It should be mentioned that Fig. 3a presents just the concept of assembly and details of fixture are not shown. To keep the parts together a screw was used to tight up sleeve to shaft and nut was used to fix the plate. The parts were designed in such way that clearance on the cylindrical surfaces was assured. Therefore the sleeve and the plate are located and orientated by the planar surfaces. Fig. 3b shows the case of the assembly of the real features. Due to the assumed functional requirements for the assembly the relative location and orientation between substitute planes No14 and No22 are defined in the substitute assembly coordinate system Oxyz. The assembly substitute coordinate system is defined in different way when compared with substitute coordinate systems for particular parts. Primary substitute datum (xy plane) is defined by substitute plane SR. Respectively the secondary substitute datum (plane xz) is defined by a plane perpendicular to primary datum and containing the constructed line – the intersection of the substitute datum plane SR and the substitute plane No13 that is conceived to be projected to the substitute point SU. The substitute point SU is the intersection of the substitute plane SR and the axis of the substitute cylinder No1. The tertiary substitute datum (yz plane) is defined by a plane perpendicular to the primary and secondary datums and containing the substitute intersection point SU.

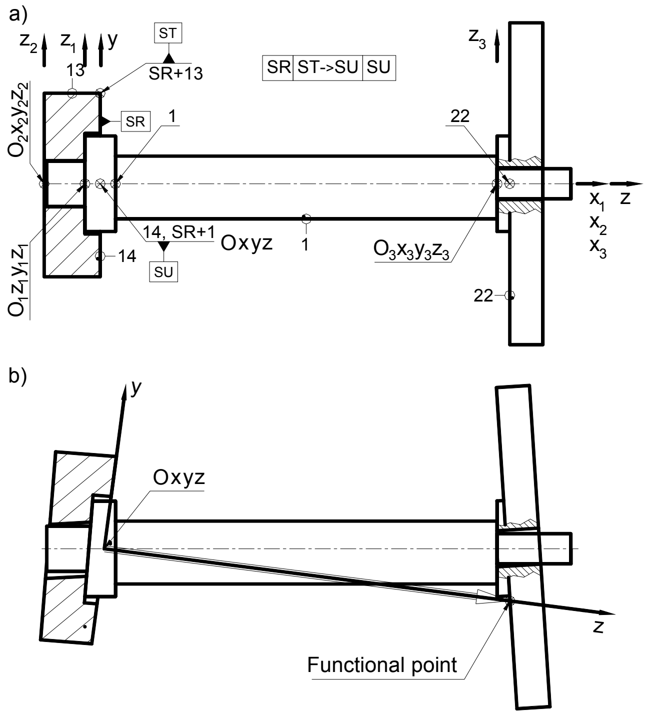

Fig. 3. a) Functional requirements for assembly defined according to VDT; b) Interpretation for real features (exaggerated deviations for the parts).

Tab. 4 specifies components of the required nominal location vector, the nominal orientation unit vector and their limit deviations for plane No 22 as well as for three geometrical features that are used to establish the assembly substitute coordinate system.

Fig. 4 illustrates stackup of the three local coordinate systems $Ox_1y_1z_1$ for the shaft, $Ox_2y_2z_2$ for the sleeve and $Ox_3y_3z_3$ for the plate, that are used in calculations of relative location and orientation between substitute planes No14 and No22 in the assembly coordinate system Oxyz. The location vectors $[P_{xi}, P_{yi}, P_{zi}]$ and the orientation unit vectors $[E_{xi}, E_{yi}, E_{zi}]$ specified in the local coordinate systems for each geometrical feature of the separate part are transformed to the assembly coordinate system (Fig. 5).

TABLE 4. Nominal values of location vector, orientation unit vector and their limit deviations for the assembly.

Substitute element No: (in assembly)			1	13	14	22
Location [mm]	Nominal	P_x	0	0	0	0
		P_y	0	36	0	0
		P_z	6	-11	0	160,98
	Limit deviation (±)	T_{Px}	×	×	•	×
		T_{Py}	×	0,01	•	×
		T_{Pz}	0,01	×	•	0,01
Orientation	Nominal	E_x	0	0	1	0
		E_y	0	1	0	0
		E_z	-1	0	0	-1
	Limit Deviation (±) ×0.001	T_{Ex}	0,07	0,33	•	0,69
		T_{Ey}	0,07	–	•	0,67
		T_{Ez}	–	0,45	•	–

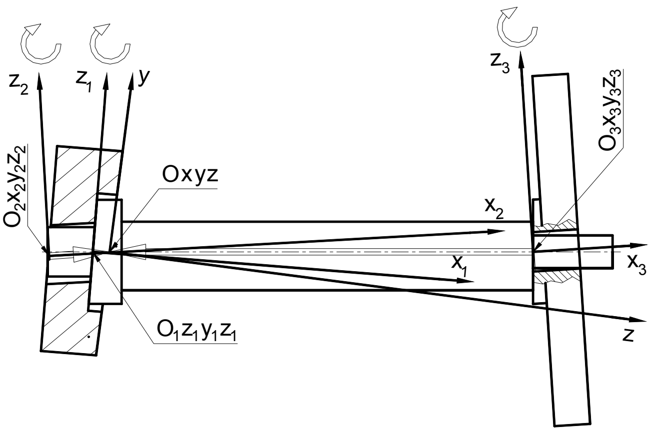

Fig. 4. Assembly of real features – location vectors and orientation unit vectors that determine analysed functional dimension.

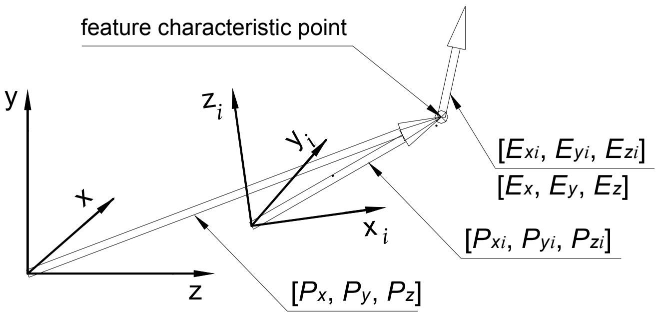

Fig. 5. Specified characteristic point of the geometrical feature with its location vectors and orientation unit vectors expressed in the global and local coordinate systems.

Presented analysis was verified experimentally. In this research as test pieces three shafts, three sleeves and three plates were manufactured and measured separately. Next various combinations of the parts were assembled and measured as the set shaft–sleeve–plate. The performed measurements have confirmed that the location vectors and the orientation unit vectors for the analysed plane No22 were in the range of the values predicted by the stackup of the location vectors and orientation unit vectors for the separate parts. The workpieces and the assemblies were measured with the coordinate measuring machine at University of Kalmar. The standard outputs of the CMM software *JoWin* were adopted to extract data that are necessary for application of vectorial dimensioning and tolerancing to 3-D stackup analysis.

3. CONCLUSIONS

The classical dimensional analysis includes inherent uncertainty due to ambiguities existing in the specification itself. Main disadvantage of the classical GDT is insufficient datum call out in the drawings. The current ISO and European standards do not sufficiently reflect 3-D models that are widely used in CAD, CAM and other CAx systems. Therefore the standards developed for 2-D drawings should be revised, extended or substituted by standards that address volumetric models. The VDT is a simple and unambiguous method of representing the expected nominal geometry of the workpieces and permissible limits of the variations of this geometry. The application of the VDT for 3-dimensional and geometrical tolerance stuckup gives a tool for an unique mathematical specification and verification of intended functional characteristic in an assembly.

Theoretically, mathematically predicted results of 3-D stackup analysis based on the VDT were confirmed during measurements on the CMM.

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