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SETTING THE PROCESS AIM: THE EFFECT OF MEASUREMENT UNCERTAINTY

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Abstract − In this paper, a task-specific measuring capability criterion is described, applicable to select or validate measurement systems that provide data to set the process aim, when the techniques known as “sequence of values” or “difference chart” are used. The criterion is based on the estimation of the “uncertainty of the process mean”, which characterizes the dispersion of the values that could reasonably be attributed to the process mean after the setting procedure. The proposed criterion is compared with the discrimination ratio and with the uncertainty per tolerance ratio, showing that the last one fails to predict the measuring capability for aim-setting operations.

Keywords: setting the process aim, measurement uncertainty, measuring capability

1. INTRODUCTION

The just-in-time manufacturing strategy is one of the most successful answers to the current market conditions: it improves adaptability to product and market changes while reduces costs by eliminating stocks. To be reliable, just-in-time production needs a supply chain integrated by predictable processes, operating on target with minimum variance. In this environment, setting the process aim becomes a critical task, particularly when short production runs are the rule.

Real processes are never on target. Due to the statistical properties of the methods used to set the process aim, the value of the true deviation from target remains unknown, so leading to the concept of *uncertainty of process mean*. This uncertainty depends on three major factors: the process itself, the procedure used to set the process aim and the uncertainty of measurement (Fig. 1).

A process has to be in statistical control before setting the process aim. No efforts should be wasted to regulate processes which mean and/or standard deviation vary in an unpredictable manner. After achieving the state of control by the use of control charts, several statistical techniques can be applied to identify weather or not the process mean is close enough to the target. Some of these techniques use also control charts to test the hypotheses that the process is on target. The technique of the *sequence of values* uses an individual value control chart with the central line set to the target value. The process is considered ready for production when a given number of successive measurements, usually ten, fail to indicate any out of control signal [1]. The technique of the *difference chart* is a variant of the former, to be used when the same process produces several part models of different nominal sizes [1]. Each point in the chart represents the difference between a measured value and the corresponding target value. Individual and difference charts can be also used for process monitoring, simply changing the operating mode once the process is on target. This makes possible using a single tool for the complete requirements of process operation.

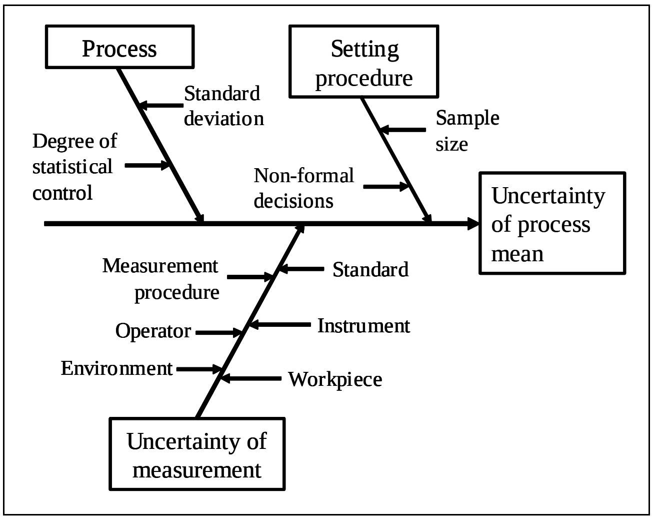

Fig. 1. Factors affecting the uncertainty of process mean

Another possibility is to use *pre-control* charts to set the process aim and, afterwards, to supervise production [2]. Nevertheless, pre-control charts limits are based on product specification and not on process, common-cause, variation. Because of this, pre-control is seriously handicapped for the assessment of statistical control, particularly when process capability is rather high [3]. Under such limitation, one may question about the ability of pre-control charts to set the process aim in an effective manner.

Other techniques to set the process aim can be found in the statistical process control literature: it is not interesting to present them here in detail. A common characteristic of all the techniques is that the uncertainty of process mean is affected by the procedure itself and by the number of measurements used to estimate the process mean and the process standard deviation. More accurate settings can be obtained increasing the number of sample units, but this results in higher operational costs and delays the production launch. Non-formal decisions, i.e. decisions not triggered by the statistical procedure, also affect the uncertainty of process mean, usually enlarging it.

Uncertainty of measurement is expected to affect adversely the uncertainty of process mean. There are no quantitative studies to help defining whether or not a measurement system can be used to set the aim of a given manufacturing process. Because of this, general-scope capability criteria are used, like the uncertainty per tolerance ratio, the gage R&R% [4], the discrimination ratio [5] and others, accepting the hypothesis that a measurement system that satisfies these criteria will be accurate enough for any quality control activity.

This paper proposes a task-specific measuring capability criterion, to be applied when measurements are used to set the process aim. The studies have been focused on the application of the technique of the sequence of values, but the results apply also to the *difference chart* technique. Several indices to the aim-setting performance have been studied: the number of parts (measures) needed to set the process aim satisfactorily, the number of process adjustments needed and the standard deviation of the possible process means that can be obtained after concluding the aim-setting procedure.

It has been shown that only the standard deviation of the possible means is sensitive to the presence of measurement errors. This standard deviation has been used to quantify the uncertainty of process mean, that can be viewed as an index to the capability of measurement systems for process aim-setting tasks.

2. THE AIM-SETTING TECHNIQUE

This research has been centered on the application of the technique known as *sequence of values* (details about this technique can be found in ref. [1]). The technique considers two possible cases. The first one is when the process standard deviation is known, in such a way that control limits are already available when the first unit is produced. The second case, selected for this simulation, is when the standard deviation is not known and has to be estimated from the process outcomes. To do this according to the Shewhart rules for control charts, the standard deviation shall be computed using the mean of a dispersion statistic. In this case, the average of the moving ranges of order two is used:

$$mR_i = y_i - y_{i-1} \tag{1}$$

$$\overline{mR} = \frac{1}{n-1} \cdot \sum_{i=1}^{n-1} mR_i \tag{2}$$

where y are the measured values composing the sample and n is the sample size (n=10, as recommended by the procedure). Then, the estimated standard deviation is:

$$\hat{\sigma}_y = \frac{\overline{mR}}{d_2} \tag{3}$$

being $d_2 = 1.13$ for moving ranges of order two.

The limits of the individuals chart are symmetrically positioned around the process target T and computed by the following equations:

$$LCL_y = T - A_2 \cdot \overline{mR} \tag{4}$$

$$UCL_y = T + A_2 \cdot \overline{mR} \tag{5}$$

being $A_2 = 2.66$ for moving ranges of order two.

Once the control limits are available, the first ten values can be analyzed retrospectively. To interpret the chart, the four decision rules known as *Western Electric Rules* are applied (Fig. 2).

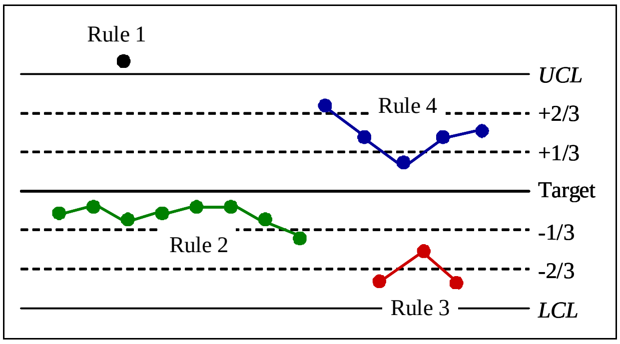

Fig. 2. *Western Electric* decision rules for out of control signals

If any out of control signal is detected, the available information is used to estimate the process mean and compute the value of the correction. After the adjustment, the process is operated and fresh sample units are obtained and measured, looking for out of control signals. If any, a new adjustment is done and more units measured. The procedure ends when ten successive units fail to show an out of control signal.

Like any other statistical tool, the *sequence of values* technique produces outcomes subjected to uncertainty. Measured values and control limits are affected by sampling variation, producing different process means when setting the aim of the same process under repeatability conditions. The number of sample units used to set the process aim and the number of process adjustments are also subjected to heavy sampling variation.

From this brief explanation, some interesting facts can be highlighted regarding the influence of measurement errors on process set up:

- A constant systematic error, affecting to the same extent all measured values, does not modify the control limits. In such situation, the interpretation of the chart leads to inaccurate adjustments, resulting finally in a process that operates deviated form target. The expected value of such deviation is the value of the systematic error.

- Random (e.g. repeatability) errors inflate the value of the estimated standard deviation, resulting in control limits that are farther from the target. Since the measured values will show also more dispersion, the effect of this kind of errors results dampened.
- Purely linear systematic errors, with value equal to zero at the process target, affect the estimated standard deviation. Errors with positive slope increase the standard deviation of the measured values, inflating the control limits. Errors with negative slope produce the opposite effect. The effect of linear systematic errors seems to be also dampened by the correlated behavior of control limits and measured values.

The model applied in the research considers all these types of errors. It is described in the following section.

3. SIMULATING MEASUREMENT

Measurement uncertainty is defined as "… a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand" [6].

Let us assume that the contributions to measurement uncertainty can be separated into two mutually exclusive groups of physical quantities, one including all random effects and the other, all systematic effects. Representing the uncertainty due to random effects by a normal random variable:

$$E_r \sim \text{normal}\left(0;\sigma_r^2\right) \tag{6}$$

the uncertainty due to systematic effects by a rectangular random variable:

$$E_s \sim \text{rectangular}\left(-E_{\max};E_{\max}\right) \tag{7}$$

and assuming that both variables are statistically independent, the standard measurement uncertainty can be expressed as:

$$u(y)=\sqrt{u^2\left(e_r\right)+u^2\left(e_s\right)} \tag{8}$$

$$u(y)=\sqrt{\frac{E_{\max}^2}{3}+\sigma_r^2} \tag{9}$$

Equation (9) states that, for any value of the measurand within the process dispersion limits, the measurement result will be affected by a random error of standard deviation σ_r and by an unknown systematic error, which value is within the interval $\left[-E_{\max};E_{\max}\right]$.

The mathematical model of the measurement process used in the simulation algorithm is consistent with the uncertainty statement in eq. (9):

$$y = x + e_{rand} + e_{sys} \tag{10}$$

where y is the measured value, x the value of the manufactured characteristic, e_{rand} an event of a random error and e_{sys} the value of the systematic error. In terms of random variables:

$$Y = X + E_{rand} + E_{sys} \tag{11}$$

Normal random variables have been used to model the manufactured characteristic and the random measurement error:

$$X \sim \text{normal}\left(\mu_p;\sigma_p^2\right) \tag{12}$$

$$E_{\text{rand}} \sim \text{normal}\left(0;\sigma_r^2\right) \tag{13}$$

In real measurement processes, systematic errors are a function of the value of the manufactured characteristic. It is widely accepted that, within the limits of process dispersion, most measurement systems present systematic error values that can be interpolated by a straight line. This condition is modeled by the following equation:

$$e_{sys} = \alpha + \beta\cdot(x-T) \tag{14}$$

where α is a constant systematic error and β is a factor determining the value of a linearly dependent error. To fulfill the condition imposed by the uncertainty statement, the values of α and β are chosen at random, in such a way that:

$$-E_{\max} \le e_{sys} \le E_{\max} \quad \forall\ \left(T-4\cdot\sigma_p\right)\le x \le \left(T+4\cdot\sigma_p\right) \tag{15}$$

The values of α and β are maintained constant within each simulation run. This way, all the values of x generated for a complete aim-setting operation are affected by systematic errors obtained from the same pattern. To simulated the effect of the lack of knowledge, the operation is repeated, choosing new values for α and β.

Figure 3 shows three different simulation outcomes of the model described by equations (10) to (15). The graphic in the top shows 100 error values obtained with $\mu_p = 100$, $\sigma_p = 1$ and $E_{\max} << \sigma_r$. The graphic in the middle shows, for the same process, a case with $E_{\max} >> \sigma_r$ and $\alpha \cong 0$ .The graphic in the bottom shows the outcomes when $E_{\max} >> \sigma_r$ and β is small. Note that the two last graphics have been obtained with the same values of $E_{\max}$ and σ_r. Thus, both error characteristics are consistent with the same uncertainty statement.

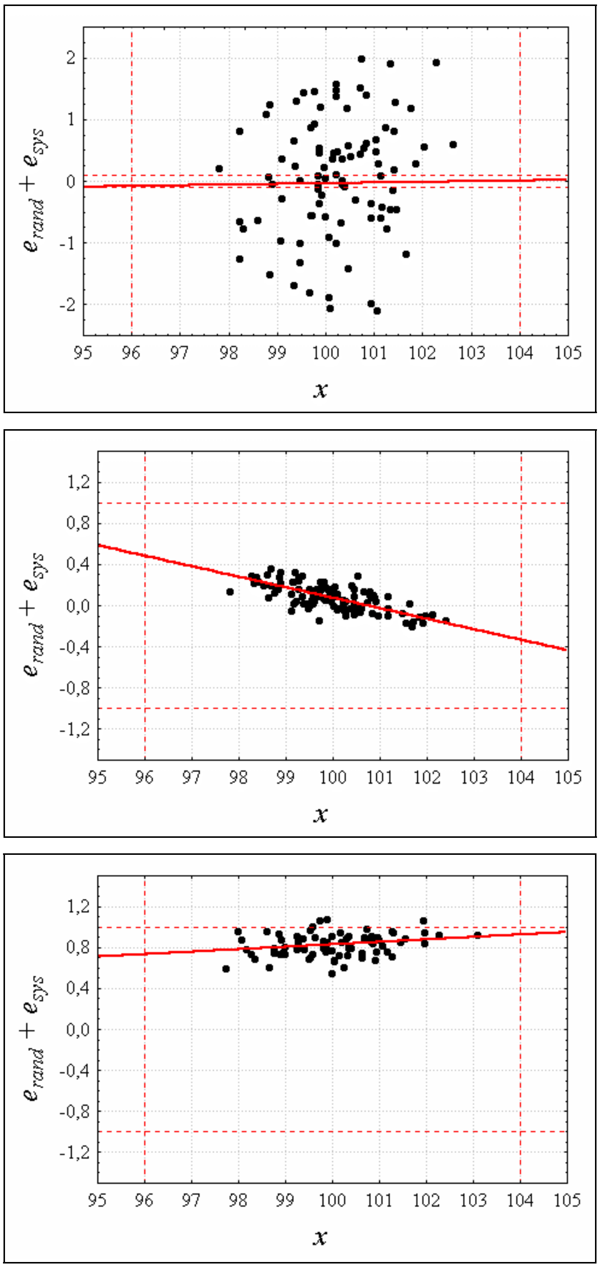

Fig. 3. Three sets of outcomes of the error simulation model

4. RESULTS

The simulation algorithm implementing the aim-setting technique described in section 2 and the error model described in section 3 has been run to study the effect of measurement uncertainty.

To get process-independent results, the manufacturing process mean has been set to $\mu_p = 0$ and all the other parameters have been divided by σ_p. The domain of the uncertainty contributions has been defined by the following inequality:

$$\left(\frac{E_{\max}}{\sigma_p};\frac{\sigma_r}{\sigma_p}\right) \,/\, 0\le\frac{E_{\max}}{\sigma_p}\le 1 \ \wedge\ 0\le\frac{\sigma_r}{\sigma_p}\le 1 \qquad (16)$$

For each point in the uncertainty component domain, a group of 100 cases has been simulated. Each case uses a different set of random error values and a different systematic error function, but fulfills the same measurement uncertainty. This way, it make sense computing an average performance within each group of 100 cases and linking such performance with the effect of measurement uncertainty.

Three performance indices have been evaluated within each group, for all the points in the uncertainty component domain:

- The average number of parts (measures) needed to set the process aim;
- The average number of process adjustments needed to set the process aim;
- The standard deviation of all the true process means, as obtained after the last process adjustment, when the aim-setting procedure has declared the process “on target”:

$$\sigma_{mean}=\sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(\mu_{res\,i}-\bar{\mu}_{res}\right)^2} \qquad (17)$$

where m is the number of cases in the group (m=100), $\mu_{res\,i}$ is the mean of the process for each case, after ending the aim-setting procedure, and $\bar{\mu}_{res}$ the average of the m means within the group.

The analysis of the simulation outcomes made evident the need to filter the results to eliminate statistical extremes. A small number of cases required an atypically high number of units and adjustments to set the process aim. These cases are caused by the underestimation of the process standard deviation during the initial run of 10 units, which itself causes a growth in the false alarm rate. Such condition will be surely avoided by an experienced operator, based on empirical information on how the procedure should function. Then, to prevent distortion of the averages, all cases presenting number of units that can be qualified as extremes or outliers, have been eliminated.

After filtering, it has been shown that the average number of units and the average number of adjustments are not affected by the presence of measurement errors. This makes them unreliable indices to measurement process performance, being so discarded for the purposes of this research.

It has been also shown that, regardless the measurement condition, the distribution of $\mu_{res\,i}$ within each group is close to normal. This brings $\bar{\mu}_{res} \cong T$ in equation (17), in such a way that the sum can be now interpreted as “the sum of the square deviations from target”. The normality permits also expanding the coverage probability by the use of the z factor. This way, a 95% expanded uncertainty has been defined for the process mean:

$$U_{mean} = 1.96 * \sigma_{mean} \quad (18)$$

The behavior of this uncertainty within the domain of measurements uncertainty components can be viewed in the following graphic (Fig. 4):

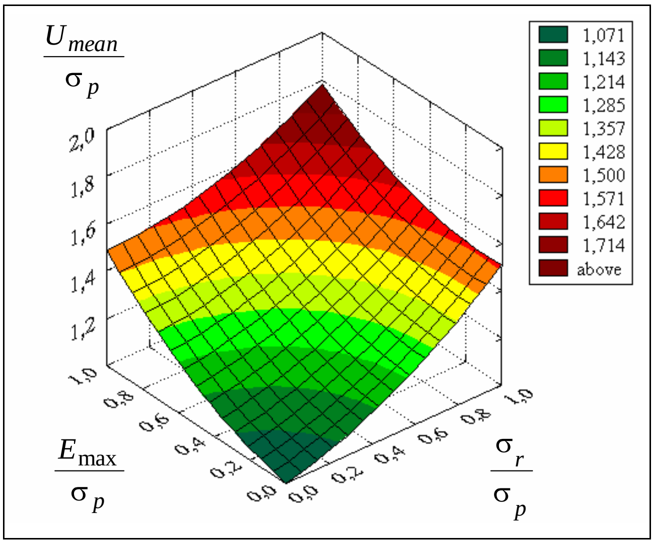

Fig. 4. The behavior of the uncertainty of process mean for different combinations of measurement uncertainty contributions.

It can be noted that, when measurement uncertainty components are brought to zero, 95% of the process means obtained by the application of the sequence of value technique would lie within the interval $T \pm \sigma_p$. As uncertainty contributions grow, the uncertainty of process mean also grows. For the extreme case, defined by $E_{max} = \sigma_p$ and $\sigma_r = \sigma_p$, the uncertainty of process mean grows up to 80%, in such a way that 95% of the true process means would lie within the interval $T \pm 1.8 \cdot \sigma_p$.

The surface depicted in Fig. 4 have been interpolated by polynomial regression, using a complete polynomial of order 2. The equation obtained follows:

$$\begin{aligned} U_{mean} &= \sigma_p + 0.2165 \cdot \sigma_r + 0.1606 \cdot E_{max} + \\ &0.2936 \cdot \sigma_r^2 + 0.3295 \cdot E_{max}^2 - \\ &0.2152 \cdot \sigma_r \cdot E_{max} \end{aligned} \quad (19)$$

This regression model allows explaining 75% of the variance present in the data ($R^2 = 0.75$), so it can be used for most practical purposes.

This way, the uncertainty of process mean becomes an index to the capability of a measurement system for setting the process aim. Unlike other existing capability criteria (e.g. uncertainty per tolerance ratio, *R&R%* [4], *Dr* [5]), it does not seem necessary defining some empirical limit to distinguish capable and non-capable measurement systems. The information given by the index provides insight on the consequences that measurement uncertainty has on the quality of the manufactured characteristics. Knowing the peculiarities of the process at hands and the quality requirements, an engineer or technician could easily decide whether a measurement system can be used for the task or has to be improved.

5. DISCUSSION

In this section, comparisons will be made between the uncertainty of process mean criterion and other measuring capability criteria. For the sake of simplicity, only two measuring capability criteria have been included, one based on the comparison with the standard deviation of the process (*Dr*) and other on the comparison with the tolerance (U/Tol).

According to Wheeler [5], the discrimination ratio *Dr* can be computed as:

$$Dr = \sqrt{\frac{2 \cdot \sigma_m^2}{\sigma_e^2} + 1} \quad (20)$$

where σ_m is the standard deviation of part measurements and σ_e is the standard deviation of a repeatability error, evaluated according to [5]. A system leading to $Dr = 4$ (just capable according to Wheeler) will have:

$$\sigma_e = 0.365 \cdot \sigma_p \quad (21)$$

Considering $\sigma_r = \sigma_e$ and assuming that $E_{max} = \sigma_e$, to allow for some systematic residual errors that would not appear in the repeatability study (like calibration residuals, long-term drift in the environmental conditions, etc.), the uncertainty of process mean will result, according to eq. (19), $U_{mean} = 1.19$.

This means that a measurement system considered capable according to the *Dr* criterion would probably produce less than 20% enlargement of the uncertainty of process mean. Nevertheless, care should be taken with the influence of unknown and residual systematic errors. If they are much bigger than the repeatability error, the uncertainty of process mean could grow drastically.

Let suppose now a measurement system presenting an uncertainty per tolerance ratio $U/Tol = 0.2$, highly common in industrial quality control. Suppose also that the uncertainty is composed by random and systematic contributions according to eq. (9), in such a way that $E_{max} = \sigma_r$. If the capability of the manufacturing process were $C_p \cong 1.67$, $E_{max} = \sigma_r = 0.884 \cdot \sigma_p$. Replacing these values in eq. (19), the uncertainty of process mean results $U_{mean} = 1.65$.

In this case, a measurement system that is found capable regarding the uncertainty per tolerance ratio, could affect heavily the performance of the sequence of values technique. A better uncertainty per tolerance ratio would be necessary (e.g. better than $U/Tol = 0.1$), to maintain low the uncertainty of process mean.

The difference between the two cases analyzed above can be explained with regard to the mechanism by which the measurement errors affect the position of the process. As it has been advanced in section 2, measurement errors make the measured values deviate from the corresponding values of the quality characteristic. The control limits, computed from statistics relating those measured values, result also affected. Then, the comparison of the deviated values with distorted control limits produces mistaken decisions and inadequate process adjustments. The intensity of the cause and effect relationships within this chain depends on the relevancy of measurement errors, when compared with the standard deviation of the process. No reference is made to the product tolerance. This is the reason why the uncertainty per tolerance ratio presents a poor correlation with the uncertainty of process mean. A similar behavior should be expected from other indices based on product tolerance, like R&R(%Tol) [4], *Cg* and *Cgk* [7].

6. CONCLUSION

In this paper, a task-specific measuring capability criterion has been described. It can be applied to select or validate measurement systems that provide data to set the process aim, when the techniques known as “sequence of values” or “difference chart” are used.

The criterion is based on the estimation of the “uncertainty of the process mean”, which characterizes the dispersion of the values that could reasonably be attributed to the process mean after the setting procedure. This dispersion includes the sampling variation, characteristic of the statistical technique, and the incremental uncertainty due to measurement. Unlike other criteria, it does not need empirical limit values to judge the measuring capability, because the uncertainty of the process mean is directly related to product and process quality. This way, highly specific information is brought to the engineer or technician, who would be able to judge considering the peculiarities of the process in hands.

It has been also shown that the uncertainty per tolerance ratio can fail when used to assess the measuring capability of systems used to set the process aim. This problem, associated with the use of the tolerance instead of the process standard deviation as reference value, could be also common to other indices like *Cg*, *Cgk* and *R&R(%Tol)* (not studied in this paper).

In the opinion of the authors, more efforts are necessary to relate the metrological characteristics of measurement systems with their effect on product quality and process economy. The result of these efforts should be a complete set of simple tools to decide the adequacy of measurement systems for specific shop-floor applications. This will make metrology more valuable for the production personnel, making it possible justifying the expenditures in better measurement systems where they are truly needed.

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