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CONCEPT OF REFERENCE MEASUREMENTS OF CYLINDRICITY PROFILES OF MACHINE PARTS

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Abstract – Products with cylindrical surfaces are manufactured in many (e.g. paper, chemical, steel, heating or shipping) industries. It is required that regular estimations of cylindricity profiles be made during the production process. The reference methods can be used for measurement of cylindricity. The cylindricity deviation includes the roundness deviation in different cross-sections of an object, the relative change of its diameter and the non-centricity of profiles in relation to the nominal axis of the object. The reference measurement systems applied nowadays enable measurement of roundness profiles in relation to fixed points of support in different cross-sections of the measured object. Furthermore, it is possible to evaluate the relative difference of the diameters. However, in traditional reference measurement systems there is no possibility of measurement of the non-concentricity of profiles. That is why appropriate reference systems for measurement of cylindricity have had to be developed..

Keywords: measurement, cylindricity profile, reference method

1. MEASUREMENTS OF CYLIDRICITY PROFILES

The knowledge of the measurement of cylindricity profiles is relatively limited. The works available on the subject concern the application of non-reference methods. Research in this field is being carried out mainly by laboratories collaborating with the leading manufacturers of measuring apparatus or standardization institutions [1,2,3,4]. In Poland, of particular interest are projects realized at Warsaw Research Centre [5,6] or the Kielce University of Technology (for which the authors work) [7]. The non-reference measurements of cylindricity profiles are of a high metrological level. Characterised by high accuracy, they provide complete information of the analysed surface. However, this is true only in the case of instruments applied to the evaluation of the surface of small workpieces under laboratory conditions. In many industries, practice shows that whenever workpieces with cylindrical surfaces are manufactured, their cylindricity profiles need to be controlled throughout the technological process. Proper preparation of cylindrical surfaces ensures high quality of the materials produced, such as paper, metal sheet, etc. It should be noted that some cylinders do not possess perfect cylindrical shapes. Sometimes, they may be deformed under certain operating conditions, e.g. due to a high temperature or great external or internal loads. Consequently, wear and damage of the working surfaces are observed. This is why the industries manufacturing or applying cylinders expect that measurements of cylindricity profiles will be made directly on the machine tool or processing machine. In the reference methods, an appropriate measuring system incorporated into a computer system of signal processing makes it possible to transform a measured profile into a real one. Such system enables full quantitative and qualitative analysis of the measured cylindricity.

2. CONCEPT OF REFERENCE MEASUREMENT OF CYLIDRICITY PROFILES. METHOD PARAMETERS

Methods that can be applied to the measurement of cylindricity directly on the machine tool belong to the above-mentioned reference methods. The cylindricity deviation consists of a roundness deviation in various sections of a workpiece, relative change of its diameter and non-centricity of roundness profiles in relation to the nominal axis of the workpiece. The reference methods applied currently allow the measurement of roundness profiles in relation to fixed points of support in various cross-sections of the measured workpiece [10, 11, 12]. It is also possible to evaluate the relative difference of diameters. In traditional reference measuring systems, on the other hand, we cannot measure the non-centricity of profiles. That is why the authors proposed developing the reference measuring system for the measurement of cylindricity consisting of two connected sets of points of support, in relation to which the measuring sensor would move along precisely constructed shears

The suggested reference system for measuring cylindricity profiles has not been discussed in the literature either at home or abroad. Its application required formulating some theoretical and experimental fundamentals of the reference measurements of cylindricity profiles for a system consisting of two related sets of points of support. The principle of the measurement of roundness profiles with reference methods is given in Fig. 1. In the diagram, the XYZ system was assumed in such a way that the axis Z coincides with the axis of the nominal cylinder, and the plane determined by the X and Y axes coincide with the plane of the left-handed set of fixed points of support. Moreover, the Y axis goes through the point of intersection of tangents to the supports of the measured workpiece and is a bisector of the angle of the position of supports in relation to the inspected element.

Considering the second system of the X'Y'Z' co-ordinates related to the measured workpiece, we can assume that the Z' axis coincides with the real axis of the workpiece, whereas the X' and Y' axes are oriented in such a way that, at zero deviation and zero value of the angle of rotation of the workpiece, the two co-ordinates systems coincide with each other.

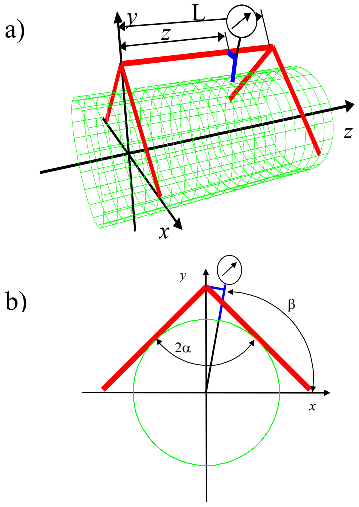

Fig. 1. Principle of measurement of cylidricity profiles with reference methods, where α, β, L, z - are method parameters: a) measured workpiece in the XYZ system b) section of a measured workpiece in the plane of fixed points of support

Due to the occurrence of profile deviations at points of contact of the cylinder and the supports, the real Z' axis of the cylinder is slightly deflected, dependent on the deviation, from the Z axis related to the fixed points of support. As a result, the current indication of the sensor has an influence on both the current position of the workpiece axis and the profile deviation at the point of contact of the workpiece and the sensor. Another effect of deflection of the workpiece axis is a slight additional longitudinal and lateral displacement of the measured surface in relation to the sensor. It is easy to estimate that if γ denotes the deflection angle of the Z' axis in relation to the Z axis, then the additional displacement of the workpiece surface at the point of contact with the sensor is of the order $\Delta l \propto R_0 \sin\gamma$. Also, the angle γ is proportional to the profile deviation, ΔR and inversely proportional to the length of the cylinder, L. Thus, we can write

$$\Delta l \propto \frac{R_0 \Delta R}{L} \qquad (1)$$

We can observe that the quantity of the extra displacements of the workpiece surface in relation to the sensor resulting from the deflection of the axis is of the same order as the value of the profile deviation and may not be taken into account. It should be assumed, thus, that the co-ordinate z' of the point of contact of the sensor and the workpiece is equal to the value of the displacement z of the sensor in relation to the surface of support, while the angular co-ordinate of the point of contact is proportional to the value of the angle of rotation of the workpiece. In reference measurements of cylindricity profiles, the following method parameters were assumed:

2α - angle of arrangement of fixed points of support in relation to the measured workpiece,
β- angle between the direction of the displacement of the contact tip of the measuring instrument (the so-called direction of measurement) and the axis of the X co-ordinate,
L – distance between two sets of fixed points of support,

The equation of a profile deviation in a cross-section of the cylinder can be best written into the polar co-ordinates system with a centre on the workpiece axis. Let us assume that, if the axis of the nominal cylinder coincides with the axis of the real cylinder, the distance between any point of a profile and the surface of the nominal cylinder is equal to:

$$R(\varphi, z), \qquad (2)$$

where φ and z are co-ordinates of the profile point. If the deviation, R, is zero, then the indications of the sensor do not depend on the co-ordinates φ and z. Thus, the indications of the measuring sensor are proportional to the distance of a given point of a profile from the surface of the nominal cylinder..

Due to the fact that profile deviations are observed at the points of contact of the workpiece and the supports, the real cylinder axis, Z', will deviate slightly from the Z axis defined by the nominal cylinder axis. We shall denote by $E_x(\varphi,z)$ and $E_y(\varphi,z)$ the Cartesian co-ordinates of the intersection of the axis of the cylinder turned through the angle φ with a plane perpendicular to the Z axis with the co-ordinate z. We can determine the values of

$$E_{x0}(\varphi) = E_x(\varphi,0), \; E_{y0}(\varphi) = E_y(\varphi,0) \qquad (3)$$

and

$$E_{xL}(\varphi) = E_x(\varphi,L), \; E_{yL}(\varphi) = E_y(\varphi,L) \qquad (4)$$

Let us consider the cross-section of the cylinder defined by the plane $z = 0$. The points of contact of the workpiece and one of the supports coincide with the points of contact of the supports and the nominal cylinder, so the distance of the profile from the nominal cylinder at the point of contact of the workpiece and the supports is equal to zero. The real workpiece turned through the angle φ touches one of the supports at the contact point of a profile with angular co-ordinates $\alpha + \varphi$ and $\pi - \alpha + \varphi$. Thus,

$$0 = R(\alpha + \varphi, 0) + E_{x0}(\varphi)\cos\alpha + E_{y0}(\varphi)\sin\alpha, \qquad (5)$$

$$0 = R(\pi - \alpha + \varphi, 0) - E_{x0}(\varphi)\cos\alpha + E_{y0}(\varphi)\sin\alpha \qquad (6)$$

Hence

$$E_{x0}(\varphi) = -\frac{R(\alpha+\varphi,0) - R(\pi-\alpha+\varphi,0)}{2\cos\alpha}, \qquad (7)$$

$$E_{y0}(\varphi) = -\frac{R(\alpha+\varphi,0) + R(\pi-\alpha+\varphi,0)}{2\sin\alpha}. \qquad (8)$$

Similarly, analysing the cylinder cross-section $z = L$, we obtain

$$E_{xL}(\varphi) = -\frac{R(\alpha+\varphi, L) - R(\pi-\alpha+\varphi, L)}{2\cos\alpha}, \tag{9}$$

$$E_{yL}(\varphi) = -\frac{R(\alpha+\varphi, L) + R(\pi-\alpha+\varphi, L)}{2\sin\alpha} \tag{10}$$

Now we can determine the values of the sensor indications $F(\varphi, z)$. The distance of the profile at the point of contact with the sensor from the nominal cylinder is equal to

$$F(\varphi, z) = R(\varphi+\beta, z) + E_x(\varphi, z)\cos\beta + E_y(\varphi, z)\sin\beta, \tag{11}$$

where

$$E_x(\varphi, z) = \frac{E_{x0}(\varphi)(L-z) + E_{xL}(\varphi)z}{L}, \tag{12}$$

$$E_y(\varphi, z) = \frac{E_{y0}(\varphi)(L-z) + E_{yL}(\varphi)z}{L}. \tag{13}$$

Therefore, the value of the signal measured for given values of the workpiece rotation angle φ and the displacement of the measuring sensor z is defined by Eqs. (11-13).

From the obtained relationships it follows that the value of the deviation can be determined on the basis of the measured signal, if the co-ordinates of the axis displacement, $E_{x0}(\varphi)$, $E_{y0}(\varphi)$ and $E_{xL}(\varphi)$, $E_{yL}(\varphi)$, are known. This, accordingly, requires determining the profiles $R(\varphi,0)$ and $R(\varphi,L)$. We can achieve this by two additional measurements of roundness profiles in the cross-sections $z=0$ and $z=L$. Then, from Eqs. (11), (7) and (8) we have

$$\begin{aligned} F(\varphi,0) &= R(\varphi+\beta,0) + E_{x0}(\varphi)\cos\beta + E_{y0}(\varphi)\sin\beta \\ &= R(\varphi+\beta,0) - \frac{1}{2}R(\varphi+\alpha,0)\left[\frac{\cos\beta}{\cos\alpha} + \frac{\sin\beta}{\sin\alpha}\right] \\ &- \frac{1}{2}R(\varphi+\pi-\alpha,0)\left[-\frac{\cos\beta}{\cos\alpha} + \frac{\sin\beta}{\sin\alpha}\right]. \end{aligned} \tag{14}$$

In the above equation, only the profile $R(\varphi,0)$ is unknown. Thus, the equation should be solved in relation to $R(\varphi,0)$. The easiest way is to do this in the domain of the coefficients is by expansion of the profile in a complex Fourier series.

Let $\hat{F}_{n0}$ and $\hat{R}_{n0}$ be the n-th components of the expansion of the profiles $F(\varphi,0)$ and $R(\varphi,0)$ in a complex Fourier series $n=-\infty,\ldots,-1,0,1,\ldots,\infty$. Then, we get from (14)

$$\hat{F}_{nL} = \hat{R}_{nL}\hat{K}_n \tag{15}$$

where $\hat{K}_n$ is the so-called coefficient of detectability defined by

$$\begin{aligned} \hat{K}_n &= e^{in\beta} - \frac{1}{2}e^{in\alpha}\left[\frac{\cos\beta}{\cos\alpha} + \frac{\sin\beta}{\sin\alpha}\right]. \\ &- \frac{1}{2}(-1)^n e^{-in\alpha}\left[-\frac{\cos\beta}{\cos\alpha} + \frac{\sin\beta}{\sin\alpha}\right] \end{aligned} \tag{16}$$

Similarly, for the profile $z=L$, we get

$$\hat{F}_{nL} = \hat{R}_{nL}\hat{K}_n. \tag{17}$$

Finally, rewriting Eq. (14) in the Fourier coefficients domain, we get

$$\hat{F}_n(z) = e^{in\beta}\hat{R}_n(z) - \left(\frac{L-z}{L}\hat{R}_{n0} + \frac{z}{L}\hat{R}_{nL}\right)\hat{M}_n, \tag{18}$$

$$\begin{aligned} \hat{M}_n &= \frac{1}{2}e^{in\alpha}\left[\frac{\cos\beta}{\cos\alpha} + \frac{\sin\beta}{\sin\alpha}\right]. \\ &+ \frac{1}{2}(-1)^n e^{-in\alpha}\left[-\frac{\cos\beta}{\cos\alpha} + \frac{\sin\beta}{\sin\alpha}\right] \end{aligned} \tag{19}$$

Equations (15)-(19) constitute the complete mathematical model for precise calculation of the constant value of a deviation of the nominal cylinder.

3. EXPERIMENTAL VERIFICATION OF THE CONCEPT OF THE REFERENCE MEASUREMENT OF CYLINDRICITY PROFILES

3.1 Model test stand

The model measuring device for reference measurements of cylindricity profiles presented in Fig. 2 was designed and constructed using the assumptions of the newly developed method.

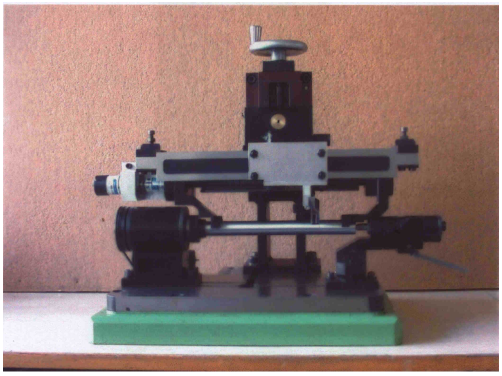

Fig. 2 Model measuring device for reference measurements of cylindricity.

The measuring device consists of a machine tool - a device equipped with lathe centres - in which a measured workpiece is fixed, and a set of two V-blocks. The element connecting the two V-blocks functions also as a slideway, along which the inductive measuring sensor moves. The V-blocks are lightly pressed down to the workpiece by a set of springs. This assures a stable contact of the V-blocks and the workpiece surface during the rotation. The V-blocks are also linked with the machine tool to prevent their moving along the workpiece. Both the angle of rotation and the displacement of the sensor are controlled by the computer. Cylindricity is measured by scanning of the workpiece surface with the sensor along a certain path. The scanning is performed by appropriate control of the angle of rotation and the displacement of the sensor.

3.2 Standardised function of the intercorrelation of cylindricity profiles

The experimental verification of the new method conducted on the model device, which was discussed in section 3.1, involved measuring the cylindricity of an element. The results were then compared with those obtained during a measurement of the same element on the TALYCENTA made by Taylor Hobson applying the standard non-reference method. To make the comparison as reliable as possible, it was essential to develop special software, and this was done using the MATLAB package. The software was responsible for the qualitative and quantitative comparison of cylindricity profiles. In order to determine the quantitative coincidence of profiles, the standardised function of intercorrelation of cylindricity profiles was proposed. It had the form of:

$$r\,(\tau)=\frac{2\int_0^L\int_0^{2\pi}R_p(\varphi,z)R_a(\varphi+\tau,z)d\varphi dz}{\int_0^L\int_0^{2\pi}R_p{}^2(\varphi,z)d\varphi dz+\int_0^L\int_0^{2\pi}R_a{}^2(\varphi,z)d\varphi dz} \qquad (10)$$

where

R_p, R_a - compared cylindricity profiles

τ – phase shift between the compared profiles

It is easy to show that $-1\le r(\tau)\le 1$ and $r(\tau)=1$, only if $R_a(\varphi,z)=R_p(\varphi,z)$.

3.3 Experiment

The experiment consisted in measuring the same cylindrical element with the proposed reference method and also the standard non-reference one. The signals obtained during both measurements were processed, and the results were as follows.

TABLE 1. Parameters of cylindricity for the reference and non-reference measurements

Parameter	Non-reference measurement on the TALYCENTA	Measurement with the proposed reference method
intercorrelation coefficient	r=0.9213	
cylindricity deviation ΔC	20.9 μm	19.9 μm
max. profile peak P more than the least squares cylinder	10 μm	10.5 μm
max. profile valley V less than the least squares cylinder	-10.9 μm	-9.4 μm

It has been reported that the cylindricity profiles obtained with both methods are characterised by high correlation. This testifies to their good coincidence, illustrated also in the diagrams showing two compared profiles. In Fig. 3, the three-dimensional diagrams of the workpiece are determined on the basis of the non-reference measurement and one performed with the proposed reference method.

The same profiles in polar coordinates are presented in Fig. 4.

Figure 5 shows roundness profiles of one of the cross-sections of the element measured with the reference and non-reference methods.

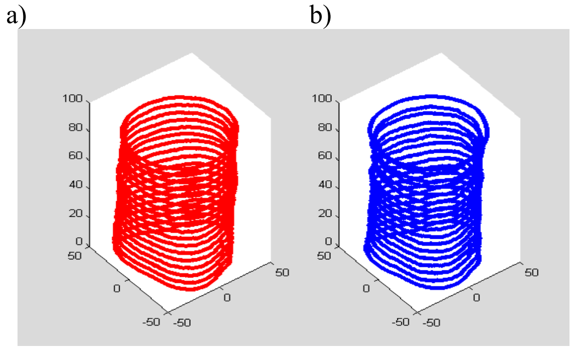

Fig. 3 Three-dimensional representation of the measured element on the basis of the results obtained by applying: a) the standard non-reference method; b) the proposed reference method

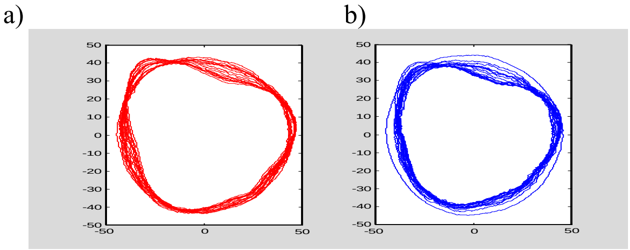

Fig. 4. Polar representation of the compared profiles:

a) profile obtained with the non-reference method

b) profile obtained with the reference method

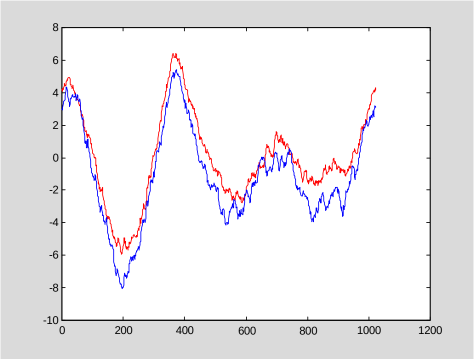

Fig. 5 Roundness profiles of one of the cross-sections

blue – profile obtained by means of the reference measurement

red - profile obtained by means of the non-reference measurement

From Table 5 and diagrams 3,4 and 5, it follows that the compared profiles are characterised by good coincidence. In order to check the reliability of results in the case of cylindricity profiles measured with the proposed reference method, it is necessary to perform a certain number of measurements so that statistical tests can be carried out.

Also, it would be advisable to determine the errors resulting from the application of the reference method for cylindricity measurement and compare them with those discovered during the standard non-reference measurement. The statistical tests are planned for the near future.

4. CONCLUSIONS

The theoretical fundamentals of the reference methods developed for the proposed measuring systems were applied to the evaluation of cylindricity profiles. It was possible to conduct a mathematical transformation of a measured cylindricity profile into a real one and develop a mathematical model of reference methods for the evaluation of cylindricity. The mathematical model was used for computer simulations, and, in consequence, the determination of optimal parameters ensuring a precise measurement of cylindrical profiles. It was necessary to verify the suggested concept of the mathematical transformation by performing some computer simulations, and constructing a prototype of a special computer-aided test stand. The stand was equipped with an original measuring system where a high-quality inductive sensor was responsible for transmitting the information of the registered surface irregularities.

Measurement accuracy has been tested of the proposed reference method. For this purpose a cylindrical workpiece was measured on a model device as well as on a high-quality non-reference instrument (TALYCENTA by Taylor Hobson). The results of both tests were characterised by high coincidence. However, to confirm their reliability in the case of the reference method, we need to conduct some statistical tests on a greater population of samples, which is planned for the near future. Hopefully, the tests will enable the elimination of potential sources of systematic errors that may affect the measuring results, for example, a straightness deviation of the shear axis or shear inclination to the axis of the measured workpiece. In addition, it will be possible to determine the errors resulting from the application of both the reference method and the standard non-reference one.

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