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ABOUT THE MEANING OF THE HAGEN – RUBENS RELATION TO RADIATION THERMOMETRY

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Abstract – Pyrometry is the only way to obtain temperature measurements when performing fast (µs duration) experiments on metals up to and above their melting points. Converting the pyrometer signals to true temperatures requires some knowledge about the normal spectral emissivity of the target at the operating wavelength of the pyrometer. Because dynamic emissivity measurements are rather difficult, one often has to resort to assumptions about the temperature and wavelength dependence of emissivity instead of a direct measurement. The HAGEN–RUBENS relation between the normal spectral emissivity of a metal and its electrical resistivity can be helpful in making such assumptions. In this work, a method for obtaining good estimates of the normal spectral emissivity of metals and alloys above their melting points as a function of temperature, based on the HAGEN-RUBENS relation, is presented. Its usefulness is examined by comparing results derived from it to actual emissivity data on several liquid metals and alloys that were measured using a pulse-heating technique.

Keywords: HAGEN–RUBENS relation, emissivity, pyrometry, polarimetry.

1. INTRODUCTION

Temperature is a very important quantity in thermophysics experiments, as well as in shock-compression experiments that are crucial in obtaining the equations of state of materials at high temperatures and pressures. Most of these experiments are very fast (ns to µs duration), leaving pyrometry as the only method to measure temperature.

However, a pyrometer typically measures radiance temperature. Some knowledge about the normal spectral emissivity of the sample as a function of temperature is required to convert radiance temperature to true temperature.

Because dynamic emissivity measurements are usually more difficult to perform than pyrometry, and in some cases even impossible, often some assumption about the temperature dependence of the normal spectral emissivity has to be made. One such assumption that is often made in thermophysical property measurements on liquid metals is that the normal spectral emissivity in the liquid state is constant and equal to that at the melting point [1].

In the past few years laser polarimetry has successfully been used for thermophysical property measurements at the Institute of Experimental Physics of the University of Technology in Graz, Austria [2]. The behavior of the normal spectral emissivity in the liquid state of the following metals and alloys has been investigated: tungsten, nickel, niobium, tantalum and inconel 718, which is a nickel based super alloy for high temperature applications.

The results indicate a strong correlation between the normal spectral emissivity and the electrical resistivity (without correction for volume expansion) in the liquid phase. For nickel, both the electrical resistivity and the normal spectral emissivity in the liquid state increase with temperature. For tantalum, both the electrical resistivity and the normal spectral emissivity in the liquid state decrease with temperature (Fig. 1). In the case of niobium, whose electrical resistivity is constant in the liquid state throughout the measurement range, the normal spectral emissivity is constant, as well. This observation led us to a closer examination of the HAGEN-RUBENS relation, which relates these two properties, emissivity and resistivity, for metals in the infrared spectral range.

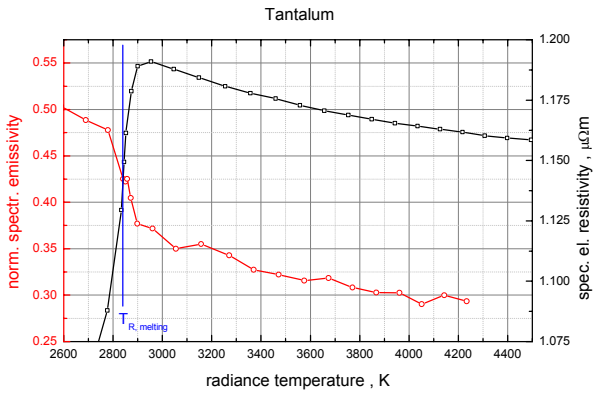

Fig. 1: The normal spectral emissivity at 685nm as well as the specific electrical resistivity (without correction of volume expansion) versus radiance temperature at 650nm.

1.1. The HAGEN-RUBENS relation

In 1903 Hagen and Rubens published a paper dealing with the relation between optical properties and the electrical resistivity of metals [3]. A brief discussion of the derivation of the HAGEN-RUBENS relation (Eq. 1) can be found in Ref. 4.

$$\varepsilon_{\perp,\lambda} = 2\sqrt{2\omega\varepsilon_0\rho} \qquad (1)$$

with: $\varepsilon_{\perp,\lambda}$, normal spectral emissivity; ω, angular frequency; ε_0, dielectric constant of vacuum; ρ, electrical resistivity.

Because of the simplifying assumptions in its derivation, this equation is only valid for long wavelengths. For copper at a wavelength of 12μm the difference between the experimental value and the computed value is less than 15 % [3]. However, figure 1 suggests a correlation between the two quantities even for wavelengths in the visible region. This correlation has been observed for all metals and alloys that have been investigated so far (i.e., in the liquid phase materials with an increasing electrical resistivity also exhibit an increasing normal spectral emissivity and vice versa).

1.2. The Experiment

All measurements were performed using the microsecond-resolution pulse-heating system of the thermophysics group at the Institute of Experimental Physics of the University of Technology in Graz, Austria. In these experiments, a wire-shaped sample (nominally 0.5 mm in diameter and 50 mm in length) of the material of interest is resistively self-heated by a current pulse of up to 10 kA in amplitude and a duration of about 50 μs. Within this time the sample is heated to the end of the stable liquid phase. Gravitational forces do not destroy the cylindrical shape of the sample that fast, allowing data to be obtained in the liquid state. The measured quantities include the voltage drop across the central part of the sample, the current through the sample, and the sample's radiance temperature. The latter is measured by means of an optical pyrometer that operates at a wavelength of 650 nm with a spectral bandwidth (FWHM) of 37 nm. These data are further reduced to compute the specific enthalpy, specific heat capacity at constant pressure, electrical resistivity, heat of fusion, thermal conductivity, and thermal diffusivity of the sample as a function of temperature. More details about the experimental setup and data reduction can be found in Ref. 6.

In addition, a microsecond-Division-of-Amplitude-Photopolarimeter (μ-DOAP) operating at 685 nm is used to dynamically obtain the index of refraction and extinction coefficient of the sample by measuring the change in the state of polarization of a laser beam upon reflection by the sample surface. From these two optical parameters the normal spectral emissivity of the sample at 685 nm is computed using equation 2. This equation is based on the FRESNEL equations [4], which are valid only for ideally smooth interfaces between two media. Because of this limitation, only data in the liquid state have been analyzed in this work, assuming that upon melting the sample surface rapidly becomes smooth due to surface tension [6]. The μ-DOAP is described in more detail in Ref. 7.

2. RESULTS

The present investigation includes nickel, niobium, tantalum, tungsten and inconel 718.

2.1. Data at room temperature

Figure 2 shows the normal spectral emissivity of tantalum as a function of wavelength computed with the HAGEN-RUBENS relation, using the room temperature value for the electrical resistivity from Ref. 8, and from the index of refraction n and extinction coefficient k, as reported in Ref. 5, using equation 2.

$$\varepsilon_{\perp,\lambda} = \frac{4n}{(n+1)^2 + k^2} \qquad (2)$$

with: $\varepsilon_{\perp,\lambda}$, normal spectral emissivity; n, index of refraction; k, extinction coefficient. This equation is valid for vacuum (air)/metal interfaces only.

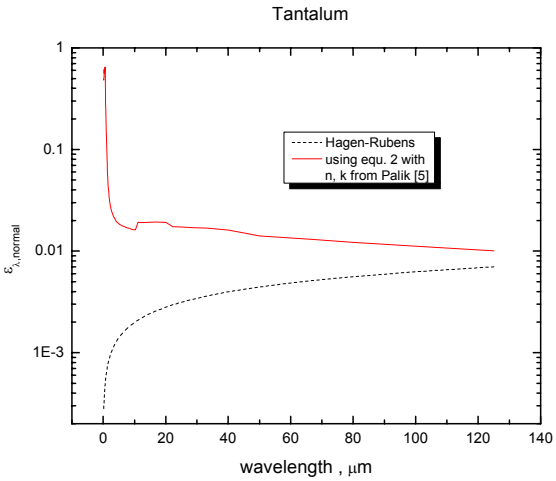

Fig. 2: Normal spectral emissivity calculated using the HAGEN RUBENS relation (dashed line) for room temperature, as well as calculated using the optical parameters n and k from [5] versus wavelength.

As can be seen, the agreement between the values obtained by means of the HAGEN-RUBENS relation and the values calculated using n and k is very poor in the wavelength range of interest for pyrometry on liquid metals (about 0.65 to 3 μm).

2.2. Data at elevated temperatures

Equation 1 can be written as

$$\varepsilon_{\perp,\lambda} = K_\lambda\sqrt{\rho} \qquad (3)$$

where all temperature independent quantities have been lumped into K_λ. If the normal spectral emissivity is known at one temperature, then $\varepsilon_{\perp,\lambda}$ can be computed for any other temperature, provided that the electrical resistivity is known

as a function of temperature. From Eq. 3 the factor K_λ is obtained easily:

$$K_\lambda = \frac{\varepsilon_{\perp,\lambda}(T)}{\sqrt{\rho(T)}} \tag{4}$$

If the true melting temperature of the sample material is known, its normal spectral emissivity at melting is obtained using:

$$\varepsilon_{\perp,\lambda}(T) = \frac{e^{\frac{c_2}{\lambda T_r}} - 1}{e^{\frac{c_2}{\lambda T_t}} - 1} \tag{5}$$

with: c_2, second radiation constant; T_r, radiance temperature; T_t; true temperature; λ, wavelength.

Then from the measured radiance temperature and the computed normal spectral emissivity (from Eq. 5) at the same wavelength, the true temperature can be obtained:

$$T_t = \frac{c_2}{\lambda}\left\{\ln\left[\varepsilon_{\perp,\lambda}\left(e^{\frac{c_2}{\lambda T_r}} - 1\right) + 1\right]\right\}^{-1} \tag{6}$$

This means that if the electrical resistivity is measured as a function of radiance temperature and the true melting temperature is known, the normal spectral emissivity as a function of radiance temperature and hence true temperature can be calculated using equations 3, 4, 5 and 6. Table 1 shows the factor K_λ, as computed for the materials investigated in the present work[1].

TABLE 1. The factor K_λ for different materials at λ = 685 nm (ρ in $\mu\Omega$.m)

material	K_λ
Ta	0.3392
Nb	0.3498
Ni	0.4163
W	0.3682
Inconel 718	0.3342

Figures 3 and 4 show the normal spectral emissivity as a function of temperature for tungsten and nickel. The solid lines represent the measured values (with the μ-DOAP), whereas the dashed lines represent the values that were computed as described above. In the case of tungsten the agreement is very good over a temperature range of more than 2000 K into the liquid phase. In the case of nickel the deviation is a little more than -5% at 1000 K above melting.

[1] All results reported in this work were computed using el. resistivity data that were not corrected for thermal expansion.

The relative deviation of the computed from the measured values for seven materials as a function of temperature above the melting point is shown in figure 5.

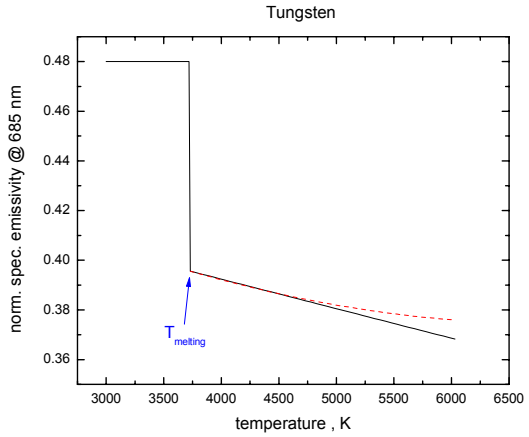

Fig. 3: Normal spectral emissivity as a function of temperature for tungsten. Solid line, measured values; dashed line, computed using the modified HAGEN-RUBENS relation (Eq. 3).

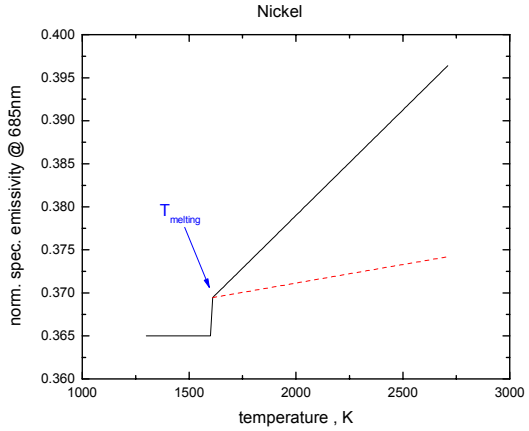

Fig. 4: Normal spectral emissivity as a function of temperature for nickel. Solid line, measuredvalues; dashed line, computed using the modified HAGEN-RUBENS relation (Eq. 3).

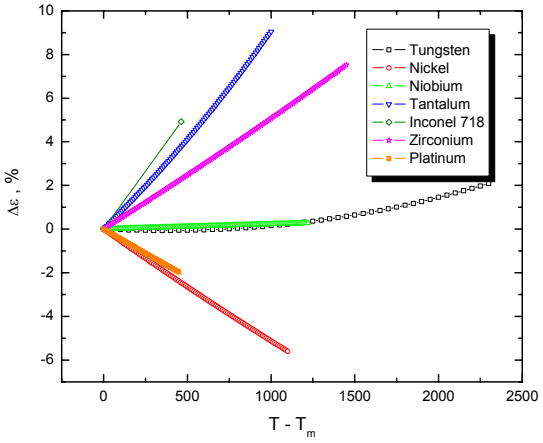

Fig. 5: Relative error of the computed normal spectral emissivity ($100\cdot(\varepsilon_{HAGEN\text{-}RUBENS} - \varepsilon_{experimental}) / \varepsilon_{experimental}$) as a function of temperature difference to the melting temperature for different materials. Data for Zirconium and Platinum from Ref. 9.

3. UNCERTAINTY ANALYSIS

The normal spectral emissivity that was computed as a function of temperature using the modified HAGEN-RUBENS relation (Eq. 3) does not deviate more than 10 % from the measured values for temperatures up to 1000 K above melting for all materials investigated in this work.

Using WIEN's approximation to PLANCK's law (Eq. 7) to calculate the temperature uncertainty due to an uncertainty in emissivity leads to equation 8.

$$\frac{1}{T_t} - \frac{1}{T_r} = \frac{\lambda}{c_2} \ln \varepsilon \tag{7}$$

$$\frac{dT_t}{T_t} = \frac{\frac{\lambda}{c_2}}{\left(\frac{\lambda}{c_2} \ln \varepsilon + \frac{1}{T_r} \right)} \frac{d\varepsilon}{\varepsilon} \tag{8}$$

For a radiance temperature of 2500 K at 650 nm the uncertainty in true temperature is 1.3 %, assuming a value of 0.35 for the normal spectral emissivity and an uncertainty in this value of 10 %.

Whereas if there is no knowledge about the normal spectral emissivity, and therefore the reasonable boundaries of $0.05<\varepsilon_{\perp,\lambda}<1.00$ are assumed the uncertainty in true temperature for the example above is as high as 33.2%.

4. CONCLUSIONS

The results presented in this work indicate that the modified HAGEN-RUBENS relation (Eq. 3) can provide good estimates for the normal spectral emissivity of liquid metals and alloys at temperatures up to 1000 K above their melting points. This equation can be used if the electrical resistivity is known as a function of (radiance) temperature in the liquid phase and the normal spectral emissivity can be obtained for at least one temperature. Since electrical resistivity data are already available for many liquid metals and the MATHIESSEN rule can be used to estimate the electrical resistivity of alloys made of these metals [10], the modified HAGEN-RUBENS relation may prove useful in many applications. However, more research is needed to examine the applicability of this equation to these materials.

This conclusion will, in general, not apply to measurements in the solid state. The reason is that the emissivity of a material does not only depend on its intrinsic optical properties, as expressed by n and k, but also on the surface texture of a sample made of this material [11]. Above melting, surface tension is believed to rapidly smoothen the surface of the liquid sample. In addition, surface oxides that might be present on the surface of a solid sample, often exhibit higher vapor pressures than the pure material, so that they evaporate by the time the sample reaches temperatures that exceed the melting point.

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