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A DYNAMIC MODELLING OF MEASURING UNCERTAINTY OF INDUSTRIAL AND ENVIRONMENTAL MEDIA USING A SIGNAL PROCESSING TECHNIQUE

France Pavlovčič

Ministry of environment, spatial planning and energy
Environmental Agency of the Republic of Slovenia
Ljubljana, Slovenia

Abstract - There are many industrial and environmental media (furnaces, climatic chambers, industrial bathes, the weather etc.) where the quantities of the ambient, such as the temperature of fluids and the relative humidity of mixtures of gases, are measured for the purpose of control, when speaking of industrial media, and observations and analyses in many other cases. The measurements are carried out by sensors and instruments calibrated under stationary conditions. But, in industrial and environmental measuring, these sensors meet dynamic conditions of the measured quantity caused by both transients and instability of the media. The measuring uncertainty contributed by an instability of the medium contains two components: a temporal and a spatial measuring uncertainty corresponding to respective kinds of the instability.

The paper deals with a method of an establishment and a propagation of the temporal measuring uncertainty of measured quantity values due to an unstable medium. This method takes into account dynamic characteristics of the employed sensor, its transfer function and time constant. Further on the time function of the quantity of the medium is processed in respect to the measured time-depending values of this quantity. According to the processed time function of the quantity, the measuring uncertainty is determined.

Keywords: uncertainty, industry, environment.

1. INTRODUCTION

The temporal uncertainty of a controlled or observed space of the medium is calculated from a time processed sensor signal, which is the best approximation of the time-varied medium quantity. While the sensor signal is being read out during a defined time interval, its values are processed in real time immediately after each read out by a time signal processing technique, which takes into account the response time constant of the employed sensor and its measuring uncertainties.

Further on the established measuring uncertainty of the medium is considered by technical standards [1,2] to fit the normal probability distribution. Therefore the signal processing technique must deal effectively also with an unknown probability distribution of momentary values of the medium quantity. The measuring uncertainty of the time processed signal has to be processed by this technique in a way that the established uncertainty would attribute to the Gaussian probability distribution, and so a time function describing the instability of the quantity of the medium has to be a Gaussian random signal within limits of a confidence interval. This is necessary condition for later propagation of the temporal uncertainty of the medium.

The measuring uncertainty of the medium, processed by this technique, includes the whole contribution of the temporal instability of the medium. Furthermore, a transformation of its unknown probability distribution to the Gaussian distribution enables to obtain approximately 95 % of a coverage probability by an expanded uncertainty. Due to its normal distribution it is transferable [2], so that it can be directly used as a component of another uncertainty added by quadratic summation.

2. PROCESSING OF THE SENSOR SIGNAL

It is very difficult to obtain the transfer functions of the sensors employed in the measurements. In this paper, the dynamic characteristics of the sensors are limited only to the time constants of the sensors. The time processing algorithm is defined by the first-order differential equation:

$$Y(t) = X(t) + \tau \cdot \frac{dX(t)}{dt} \quad , \tag{1}$$

where the $X(t)$ is the measured sensor signal and the $Y(t)$ is the processed signal, both depended on time.

Thus the establishment of the uncertainty due the instability of the medium is carried out on the basis of so processed sensor signal - Fig. 1, which represents the real time dynamic function of the quantity of the medium. The distinction between the processed sensor signal, representing the time function of the medium quantity, and the measured sensor signal could be also seen by comparison of the relevant diagrams in Fig. 1.

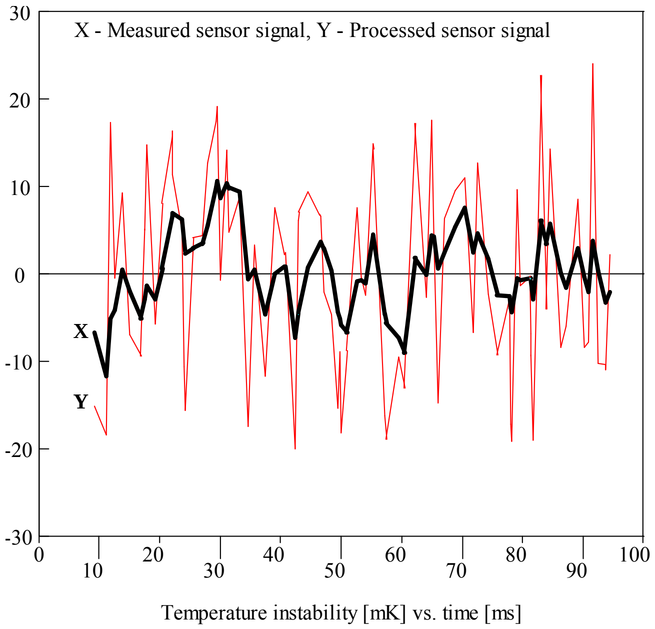

Fig. 1: The measured and the processed sensor signals of the medium quantity

The standard uncertainty of the processed sensor signal due to the temporal instability of the medium is:

$$\tilde{u}_{mt} = \sqrt{\frac{1}{T} \cdot \int_0^T (Y(t) - \overline{Y})^2 \cdot dt} \quad . \tag{2}$$

For this uncertainty a degree of freedom is also determined to enable the later calculation of the expanded measuring uncertainty. Presuming that a variance of the standard uncertainty of the processed sensor signal is the square of a B type standard uncertainty of the sensor (u_{Bsens}), the degree of freedom is equal to [2]:

$$\nu_{mt} = \frac{1}{2} \cdot \left(\frac{\tilde{u}_{mt}}{u_{Bsens}} \right)^2 \quad . \tag{3}$$

It could happened that the uncertainty of the processed sensor signal seems to be smaller than it is probable due to the B type measuring uncertainty of the employed sensor. In the latter case an estimation of the standard uncertainty of the processed sensor signal is made, so that the degree of freedom equals one to fulfil minimal terms of statistical confidence.

3. PROCESSING OF THE UNCERTAINTY

The aim of the processing of the expanded uncertainty is to established the confidence interval of the medium quantity with the coverage probability of 95 % or better. With the unknown probability distribution, the confidence interval with 95 % is not obtained by multiplying the standard uncertainty (2) by the coverage factor of two or larger depending on the degree of freedom. The coverage factor (3) as it appears in the t-distribution is a measure of the uncertainty of the expanded uncertainty due to the low degree of freedom [2]. Therefore it cannot overcome determination of the confidence interval of the unknown probability distribution, just because it is unknown. The cover probability of the confidence interval of any distribution is defined by Chebyshev's inequality [3]:

$$P\left(\left|Y - \overline{Y}\right| \le 2 \cdot a \cdot \tilde{u}_{mt}\right) \ge 1 - \frac{1}{4 \cdot a^2} \quad . \tag{4}$$

Presuming the coverage factor equals 2, there is found an additional factor *a* in (4), which deals with the confidence interval of the unknown probability distribution. This factor is rather large for it must be:

$$a = \sqrt{5} \tag{5}$$

to obtain the 95 % confidence level or better.

The probability distribution of the processed sensor signal values has finite bounds of its definition interval for the processed sensor signal is a continuous time function without any singularity. These bounds are known, and they are positive and negative amplitudes of the processed sensor signal. Introducing a form factor of the processed sensor signal, which is commonly used in electrical engineering [4], enables this signal to be statistically evaluated. This form factor is calculated according to the following equation:

$$\tilde{k} = \frac{\sqrt{T \cdot \int_0^T (Y(t) - \overline{Y})^2 \cdot dt}}{\int_0^T \left|Y(t) - \overline{Y}\right| \cdot dt} \quad . \tag{6}$$

Statistically, the form factor is the quotient between standard uncertainty and mean deviation of the values of the processed sensor signal:

$$\tilde{k} = \frac{\sqrt{E\left\{\left(Y(t) - \overline{Y}\right)^2\right\}}}{E\left\{\left|Y(t) - \overline{Y}\right|\right\}} \quad . \tag{7}$$

The processed standard uncertainty, which would give the confidence level 95 % or better when multiplied by coverage factor, is determined by the following equation:

$$u_{mt} = \tilde{k} \cdot \tilde{u}_{mt} \quad . \tag{8}$$

This equation was validated statistically for the time functions of the processed sensor signals having no singularity, which means that the respective probability distributions have finite bounds.

The usage of the form factor of the processed sensor signal is limited by its interval:

$$1 \le \tilde{k} \le \sqrt{5} \quad , \tag{9}$$

otherwise the (4) and (5) are valid for any probability distribution.

The expanded uncertainty of the processed sensor signal with the confidence level of 95 % or better is:

$$U_{mt} = K(\nu) \cdot u_{mt} \quad , \tag{10}$$

where $K(\nu)$ is the coverage factor of the t-distribution depended on the degree of freedom (3). Moreover it can be considered that the established uncertainty attributes to the Gaussian probability distribution.

4. DETERMINING THE SENSOR RESPONSE

In the previous section the measuring uncertainty was determined, which fits the normal distribution. Further on, it was searched for a time function, which corresponds to the normal distribution. This time function is the Gaussian random signal within limits of the confidence interval describing the instability of the medium quantity [5,6]. So far, the confidence interval of $\pm U_{mt}$ is the amplitude of the Gaussian random signal, which is the input signal of the sensor. Generally, if the input signal into any system is the Gaussian random signal the output signal is also the same kind of signal [6]. The sensor was determined as the first-order system, and knowing the input signal the output signal was calculated. The probability distributions of the input and the output signal were also compared to see that the both of them are normal distributions - Fig. 2.

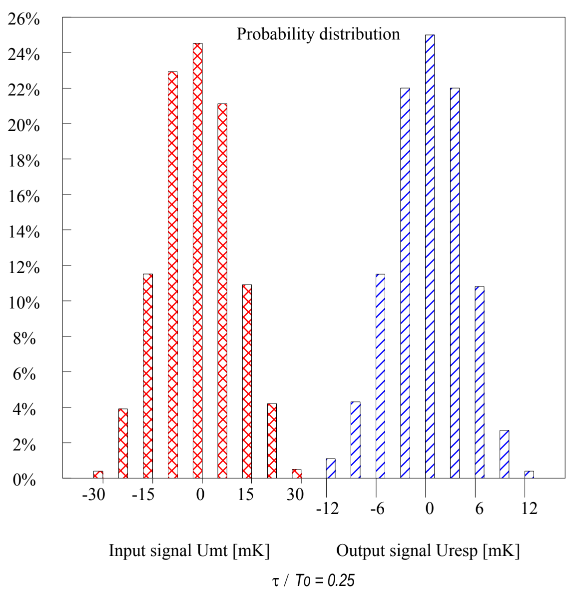

Fig. 2: The comparison of the probability distributions of the input and the output signal

Further on, the response function of the sensor was represented as a ratio between the output signal with the amplitude of $\pm U_{resp}$, and the input signal with the amplitude of $\pm U_{mt}$ - Fig. 3.

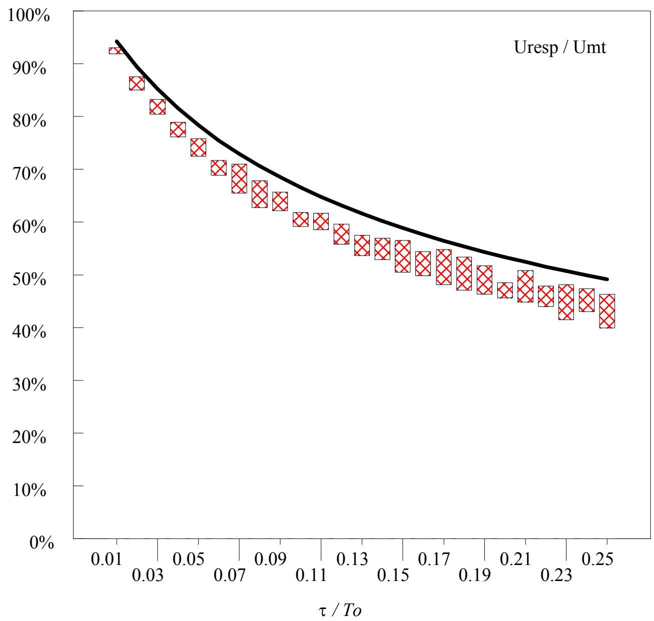

Fig. 3: The sensor response function of the Gaussian random signal of the medium quantity

The ratio between the output and the input signal amplitudes is defined by the following equation:

$$A = \frac{U_{resp}}{U_{mt}} = \frac{1}{\sqrt{1 + 2 \cdot \omega_0 \cdot \tau}} \quad , \tag{11}$$

where the τ is the time constant of the employed sensor, and ω_0 is the cyclic frequency due to the basic time period T_0 of the statistical repeating of the signal values. The ratio is shown by the diagram in Fig. 3 as the range of dispersed data and the maximum response function.

The expanded uncertainty U_{mt} corresponds to the processed sensor signal $Y(t)$, and U_{resp} does to the measured sensor signal $X(t)$. If the measured sensor signal is regarded as Gaussian random signal with the expanded uncertainty U_{resp} the expanded uncertainty of the processed sensor signal is determined by the following equation:

$$U_{mt} = U_{resp} \cdot \sqrt{1 + 2 \cdot \omega_0 \cdot \tau} \quad . \tag{12}$$

So far, if the expanded uncertainty of the measured signal was determined by any algorithm to obtain the 95 % confidence level or better the expanded uncertainty due the instability of the medium is calculated by (12) instead by the time processing procedure.

5. CONCLUSIONS

This algorithm of the dynamic modelling of the temporal uncertainty enables that the temporal expanded uncertainty due to the medium instability, sensed by measuring device, is transferred to the medium quantity according to the response function of any measurement device.

The feature of this method is also, that the obtained expanded uncertainty has the confidence level of 95 % or better.

The expanded uncertainty is assigned to normal probability distribution, it is fully transferable, and so it can be added to the combined uncertainty by quadratic summation in any further calculations.

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Author: Dr. France Pavlovčič

Affiliation: Ministry of environment, spatial planning and energy
Environmental Agency of the Republic of Slovenia
Ljubljana, Slovenia

Mail: Vojkova 1b
SI - 1000 Ljubljana
Slovenia

Tel.: +386 1 478 4098
Fax: +386 1 478 4054
E-mail: france.pavlovcic@gov.si