XVII IMEKO World Congress
Metrology in the 3^{rd} Millenium
June 22-27, 2003, Dubrovnik, Croatia

SPEED OF SOUND MEASUREMENTS IN HUMID AIR USING AN ULTRASONIC FLOW METER

Torbjörn Löfqvist, Kęstutis Sokas, Jerker Delsing

Department of Computer Science and Electrical Engineering, EISLAB
Luleå University of Technology, SE-971 87 Luleå, Sweden

Abstract – This paper presents results from experiments using an ultrasonic gas flow meter measuring the speed of sound in air at varying air velocities, humidities and temperatures. The meter utilises the sing-around technique. The transducers in the meter are silicon-based ultrasonic transducers with a centre frequency of 800 kHz. In order to investigate the performance of the flow meter it was tested in a novel gas flow facility connected to a calibration facility for flow meters used for liquids. The Reynolds' numbers for the investigated flow velocities ranged from 0 to $3{,}2 \cdot 10^4$, the relative humidity varied from 40% to 80% RH and the temperature varied from 20°C to 46°C. It was found that the experimentally measured speed of sound corresponded well with the speed of sound obtained from theory. It is also concluded that the flow meter could potentially be used in determining the relative humidity in flowing air at atmospheric pressures using speed of sound and temperature measurements.

Keywords: ultrasonic, flow meter, air.

1. INTRODUCTION

An inherent property of an ultrasonic, transit-time, flow meter is that it allows both the speed of sound as well as the flow velocity of the flowing medium to be determined. Since the speed of sound depends on the thermo-physical properties of the flowing media, it opens the possibility to use the speed of sound in determining other parameters of interest. Experiments to determine the sound speed in atmospheric air have been performed earlier both on experimental and theoretical levels. Harris [1] investigated experimentally the variation of the speed of sound in humid air at 20°C. Morfey and Howell [2] gave theoretical sound speed predictions, which were in good agreement with the measurements obtained by Harris [1]. Cramer [3] theoretically studied the sound speed in humid air at atmospheric conditions at varying temperature, pressure, carbon dioxide and water vapor mole fractions.

The idea behind the experiments reported here is to explore the possibilities in using a regular ultrasonic flow meter to measure speed of sound in humid air at varying flow conditions and using that information in determining the humidity content of the flowing air. This study contains an experimental part were the sound speed is determined for different flow conditions. The results are compared with values obtained from theory.

2. EXPERIMENTAL METHOD AND THEORY

The flow meter used in the experiments is a transit-time gas flow meter used in sing-around mode. The flow meter is equipped with silicon-based ultrasound transducers, which have a centre frequency of 800 kHz. The basic operating principle of an ultrasonic, sing-around, flow meter is well known and is described in e.g. Lynnworth [4]. From theory, one can derive an equation for the speed of sound in the air flowing through the meter based on the up- and down-stream propagation times for the ultrasonic pulses, i.e.

$$c = \frac{L}{2}\left(\frac{1}{t_1} + \frac{1}{t_2}\right), \tag{1}$$

where L is the distance between the transducers and t_1 and t_2 are the up and down-stream propagation times.

The flow meter's performance is investigated in varying conditions: relative humidities ranging from 40% to 80% RH, flow velocities ranging from 0 m/s to 30 m/s and temperature ranging from +20 °C to +46 °C. The pressure was close to the ambient pressure and varied only slightly in the experiments.

The airflow through the gas-flow meter is created by a pressure difference between two interconnected tanks; one containing air having the desired temperature and humidity and the other containing water. The water tank is also connected to a flow meter calibration facility and its measuring equipment, see figure 1.

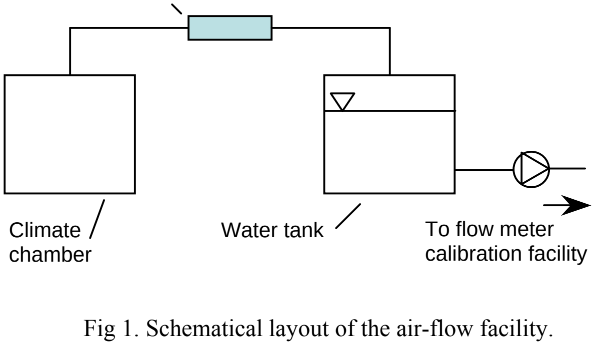

Fig 1. Schematical layout of the air-flow facility.

The air is set in the climate chamber to the desired temperature and humidity. The water tank serves as a reservoir from which water is pumped to the calibration facility and its weighing tanks and measuring systems. The gas flow meter is placed in the pipe connecting the climate chamber and the water tank. Airflow between the climate chamber and the water tank is created by pumping water from the water tank, thus lowering the air pressure in the water tank which forces the air to flow from the climate chamber to the water tank. The volume flow rate of air is controlled by the volume flow rate of water. The volume flow rate of air is kept constant through the time-period required to perform the flow meter measurements for the desired flow rate. To avoid effects on the flow meter measurements from transients in pressure or flow velocity when starting the system, each measurement is controlled not to start until stationary flow conditions are met. By utilizing the weighing tanks and measuring systems in the flow meter calibration facility, the flow rate of air is determined from the flow rate of water.

Pressure, temperature and humidity are measured at the inlet of the flow meter. All measurements as well as the whole operation of the air flow facility and the calibration facility were performed automatically and under computer control. The calibration facility, its instrumentation and performance, is thoroughly described in Carlander [5].

Each experimental set in the air flow facility consists of three repetitions of a sequence of predetermined air flow-rates. At each predetermined flow-rate, the ultrasonic flow meter measures continuously during 100 seconds and concurrent measurements are performed on pressure, temperature and humidity. This is repeated six times before changing to the next predetermined air flow-rate. Each data point is then calculated as the arithmetic mean of the sampled values. For each data point, statistical measures of pressure, temperature, humidity, transit times and speed of sound are also calculated. This is done for each repetition of the air flow-rate sequence. During each experimental set the relative humidity is allowed to vary during the course of the experiments.

In order to compare the experimentally obtained speed of sound from (1) with theoretical data, the following approximate equation, derived by Cramer [3], for the zero frequency speed of sound in moist air was used.

$$\begin{aligned} c_0\left(t, p, x_w, x_c\right) &= a_0 + a_1 t + a_2 t^2 + \left(a_3 + a_4 t + a_5 t^2\right) x_w \\ &+ \left(a_6 + a_7 t + a_8 t^2\right) p + \left(a_9 + a_{10} t + a_{11} t^2\right) x_c \\ &+ a_{12} x_w^2 + a_{13} p^2 + a_{14} x_c^2 + a_{15} x_w p x_c \end{aligned} \tag{2}$$

Above c_0 is the zero frequency speed of sound, t is the temperature in Celsius, x_w and x_c are the water vapor and carbon dioxide mole fractions respectively and p is the pressure in Pa (N/m^2). Values on the constants a_i are given by Cramer [3]. The relation between water vapor mole fraction, x_w, and relative humidity, h, could be found in Cramer [3]. The range of validity of (2) is 0 °C - +30 °C in temperature, 75 – 102 kPa in pressure, up to 0.06 water vapor mole fraction and carbon dioxide concentrations up to 1%. The effect of dispersion on the speed of sound could be included, but this is not considered in this study. Temperature, pressure and water vapor mole fractions were registered during the experiments in this study. The carbon dioxide mole fraction, or concentration, was not measured in the experiments but assumed to remain constant and equal to the level in standard air.

3. UNCERTAINTY ANALYSIS

The zero frequency speed of sound, c_0 , (2), and the speed of sound obtained from the flow meter, c, (1), offers each the possibility to determine the uncertainty limit of c_0 and c using the uncertainty data of the sensors as well as of the flow meter. A general error propagation analysis, Coleman and Steele [6], is performed on both (1) and (2). We then obtain the uncertainty limits on both c_0 and c from the following expressions.

$$\delta_{c0} = \left[\sum_{i=1}^{4} \left(\frac{\partial c_0}{\partial \xi_i} \delta \xi_i \right)^2 \right]^{1/2} , \tag{3}$$

$$\delta_c = \left[\sum_{i=1}^{3} \left(\frac{\partial c}{\partial \psi_i} \delta \psi_i \right)^2 \right]^{1/2} , \tag{4}$$

where δ_{c0} and δ_c are the uncertainty limits on the zero frequency speed of sound and speed of sound obtained from the flow meter respectively. ξ_i and ψ_i represents the i:th variable in each equation and $\delta\xi_i$, $\delta\psi_i$ are their respective uncertainty limits.

The uncertainties for the different parameters are shown in table 1.

TABLE 1. Uncertainty values for the zero frequency speed of sound, (2). The uncertainties are related to the different sensors. In the table, fr denotes full range. The carbon dioxide mole fraction uncertainty is estimated as the double difference between its estimated upper and lower bound.

Variable	Uncertainty (% of fr)	Full range
t	1%	-10...+120 °C
p	0.5%	100 - 102 kPa
h	0.5%	12-90 %RH
x_c	300 ppm	350-500 ppm

TABLE 2. Uncertainty values for the ultrasonic gas flow meter.

Variable	Value	Uncertainty $\delta\psi_i$
L	61,4 mm	0,1mm
t_1, t_2	~ 0,178·10^{-3} ms	3,3·10^{-8} sec

The uncertainty limit range for the zero frequency speed of sound is found to be | δ_{c0} | = 0,79 – 0,75 m/s, or ~ 0,23 %.

The uncertainty limit range for the speed of sound obtained from the flow meter is found to be | δ_c | = 0,54 – 0,57 m/s, or ~ 0,16 %. Both uncertainty limits are calculated at a 95% confidence level. The uncertainty limit δ_{c0} could be reduced considerably by using a temperature sensor with better precision. For example, by reducing the uncertainty limit for the temperature sensor to 0,3%, which is the uncertainty of the temperature sensor built into the humidity sensor used in these experiments, the uncertainty limit is reduced to | δ_{c0} | = 0,24 – 0,23 m/s, or ~ 0,07 %.

By studying the uncertainty magnification factors (UMFs, Coleman and Steele [6]) for each variable, we are enabled to discriminate the relative impact of each variables uncertainty to the uncertainty limit. The result for the zero frequency speed of sound is shown in figure 2.

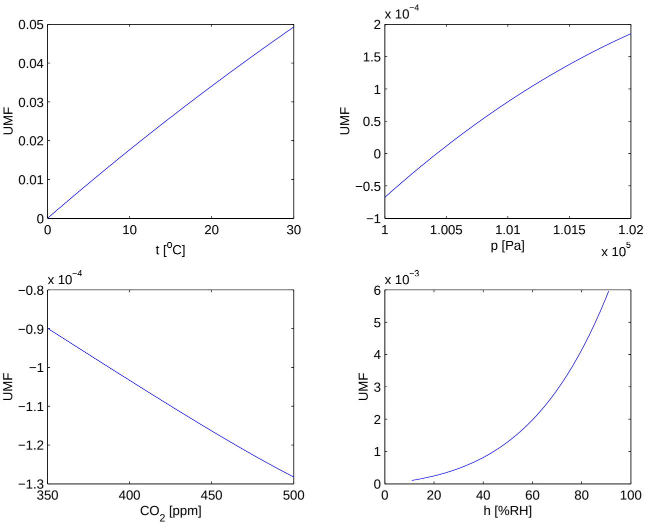

Fig. 2. Uncertainty magnification factors (UMFs) for each of the variables temperature, pressure, carbon dioxide concentration and humidity, all appearing in (2).

From figure 2 it can be seen that temperature has the largest uncertainty magnification factor, within the parameter range in this study. It can also be seen that the uncertainty magnification factors from humidity is about one order of magnitude greater than the uncertainty magnification factors for both carbon dioxide mole fraction and humidity. Note that the uncertainty magnification factor for humidity becomes of the same order as the uncertainty magnification factor for carbon dioxide mole fraction for small values of %RH. Thus, it is concluded that the dominating source of uncertainty for the uncertainty limit δ_{c0} is uncertainty in temperature. Furthermore, the uncertainty limit is also influenced by humidity, carbon dioxide mole fraction and pressure, in order of significance.

By using the same analysis method it is also found that the dominating source of uncertainty for the uncertainty limit δ_c is the uncertainty for the distance, L, between the transducers. In order to decrease the uncertainty limit δ_c, the precision in the calibration procedure of the gas flow meter has to increase. The uncertainties from the transit time measurements, t_1 and t_2, are found not to contribute to any significant degree to the uncertainty limit δ_c.

4. RESULTS AND DISCUSSION

Measurements were performed for two different temperature ranges, 20 °C - 21 °C and 26 °C - 46 °C. The results from the measurements as well as theoretically calculated speed of sound are presented graphically in figure 3 and 4. The figures show that theoretical and experimental values on c_0 are in a close agreement. It is also noted in figure 3 that the difference between measured and calculated speed of sound increases with increasing temperature. This deviation could be attributed to the temperature limit in the application range of (2), which is only valid up to 30°C, Cramer [3]. In figure 3 it is seen that there is a small systematical difference in speed of sound of about ~0,05 m/s. This error is about ten times smaller than the uncertainty limit for the meter, δ_c, and therefore considered as insignificant.

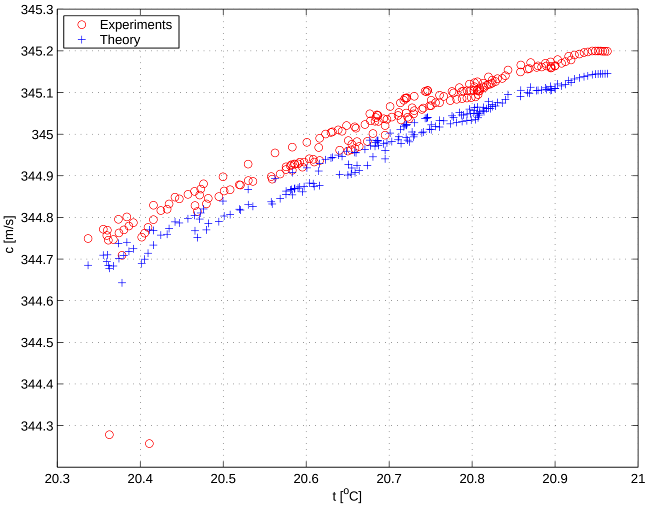

Fig. 3. Speed of sound as a function of temperature. The temperature is varied between 20 °C to 21 °C.

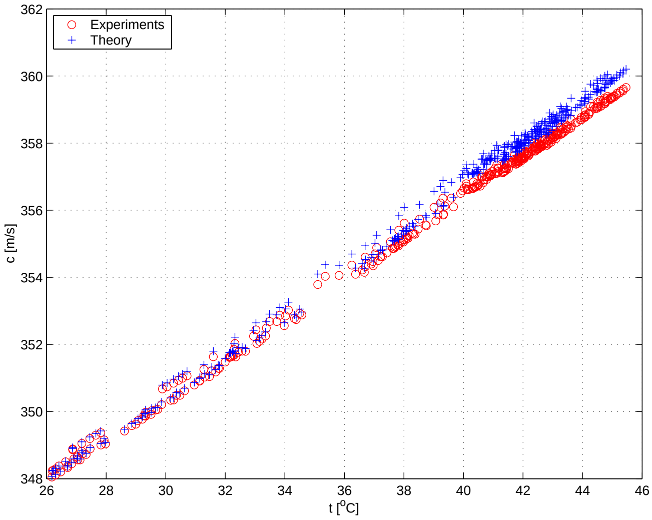

Fig. 4. Speed of sound as a function of temperature. The temperature is varied between 26 °C to 46 °C.

In the uncertainty analysis above, it was found that the speed of sound was influenced, in order of significance, by variations in temperature, water vapor mole fraction and carbon dioxide mole fraction. From figure 2 it is seen that pressure variations, within the limits of validity of (2), does not affect the speed of sound in any significant degree, which is also shown by Cramer [3].

In order to determine the humidity of the flowing air we have to consider the uncertainties of the ultrasonic flow meter, the temperature sensor and the zero frequency speed of sound, (2). Based on the findings from the uncertainty analysis, it is reasonable to assume that the carbon dioxide mole fraction as well as the pressure could be regarded as constants, which leaves the zero frequency speed of sound, c_0, as a function of temperature and water vapor mole fraction. Thus, by measuring the speed of sound in the flowing air as well as its temperature it is theoretically possible to determine the water vapor mole fraction. But, in order to achieve this, the ultrasonic flow meter and the temperature sensor has to be precise enough.

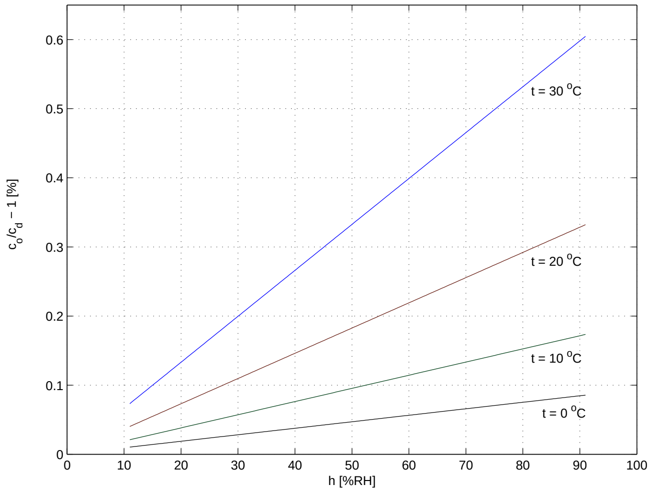

Fig. 5. The zero frequency speed of sound in air, from (2). c_0 , is normalized with c_d , the zero frequency speed of sound in air at 0 %RH. The pressure is set to 101 325 Pa.

By comparing the uncertainty limit for the ultrasonic flow meter, $\delta_c \sim 0{,}16$ %, with the data presented in figure 5, it is concluded that it is possible, within a limited range of %RH and temperature, to use the ultrasonic flow meter in its present configuration in determining the humidity in air. In order to have the flow meter resolving larger variations in humidity level in the range used in this study, the flow meter's uncertainty limit δ_c has to be decreased.

From the uncertainty analysis, we obtained an uncertainty limit for the zero frequency speed of sound for the present experimental set-up. It is found that its uncertainty limit is $\delta_{c0} \sim 0{,}23$ %. From figure 5 it is then seen that it is, in a limited range of temperature and %RH, possible to determine the humidity level using the zero speed of sound, (2), as described above. However, by using a temperature sensor with smaller uncertainty, it is possible to resolve larger variations in humidity. It is also noted, that for low temperatures and low %RH, the uncertainty magnification factor for humidity will be of the same order as for the one for carbon dioxide, which will result in a violation of the assumption above.

Thus, under the assumptions that the carbon dioxide mole fraction and pressure can be regarded as constants and in the parameter range in this study, it could be possible to a reasonable degree of accuracy to use an ordinary, properly calibrated, transit-time, ultrasonic flow meter and a high precision temperature sensor in determining the humidity level, or the water vapor mole fraction, in flowing air. The experimental verification of the proposed method has not yet been performed.

5. CONCLUSION

This study show that the experimentally measured speed of sound corresponded well with the speed of sound obtained from theory. Hence, an ultrasonic gas flow meter operating in sing-around mode is well suited to objectively measuring the speed of sound in flowing, humid air at varying temperature, pressure and humidity. It is also proposed that, at atmospheric pressure and room temperature conditions, it is possible use concurrent measurements of speed of sound and temperature in estimating the water vapour mole fraction in flowing air. This could be of importance in, for example, health-care and anaesthetic applications.

REFERENCES

[1] C. M. Harris, "Effects of Humidity on the Velocity of Sound in Air", *J. Acoust. Soc. Am.*, vol. 49, pp. 890-893, 1969.

[2] C. L. Morfey and G. P. Howell, "Speed of Sound in Air as a Function of Frequency and Humidity", *J. Acoust. Soc. Am.*, vol. 68, pp. 1525-1527, 1980.

[3] O. Cramer, "The Variation of the Specific Heat Ratio and the Speed of Sound in Air with Temperature, Pressure, Humidity and CO_2", *J. Acoust. Soc. Am.*, vol. 93, pp. 2510-2516, 1993.

[4] L. C. Lynnworth, "Ultrasonic measurements for process control – theory, techniques, applications", Academic Press, 1989.

[5] C. Carlander, "Installation effects and self diagnostics for ultrasonic flow measurement", PhD Thesis, 2001:11, Luleå University of Technology, Luleå, Sweden, 2001.

[6] H.W. Coleman and W.G. Steele, "Experimentation and uncertainty analysis for engineers." 2^{nd} ed., John Wiley & Sons, 1999.

Authors: Torbjörn Löfqvist, Department of Computer Science and Electrical Engineering, EISLAB, Luleå University of Technology, 971 87 Luleå, Sweden. Tel.: +46 920 491777, fax: +46 920 492082, E-mail: tlt@sm.luth.se

Kęstutis Sokas, Department of Computer Science and Electrical Engineering, EISLAB, Luleå University of Technology, 971 87 Luleå, Sweden. SokasK@one.lt

Jerker Delsing, Department of Computer Science and Electrical Engineering, EISLAB, Luleå University of Technology, 971 87 Luleå, Sweden. Tel.: +46 920 491896, fax: +46 920 492082, E-mail: jerker.delsing@sm.luth.se