

*XVII IMEKO World Congress  
Metrology in the 3rd Millennium  
June 22–27, 2003, Dubrovnik, Croatia*

## **ACHIEVING 0.25 mK UNCERTAINTY WITH AN INTEGRATED-CIRCUIT RESISTANCE THERMOMETER READOUT**

*Rick Walker<sup>1)</sup>, Norman Willgress<sup>2)</sup>*

<sup>1)</sup> Hart Scientific, American Fork, Utah, USA

<sup>2)</sup> Hart Scientific, Norwich, UK

**Abstract** - Resistance bridges, when used for temperature measurements with SPRTs, are able to achieve uncertainties better than 1 ppm. However, they have several shortcomings that prohibit their use in many applications. Among these are cost, size, slow speed, and limited range. An endeavor was made by the author to design a readout for resistance thermometers that achieves less than 1 ppm uncertainty in resistance ratio while overcoming some of the problems of resistance bridges. A new approach was taken with a design that uses the latest integrated-circuit analog-to-digital converters. This allows the instrument to have lower cost, smaller size, the capability of increased speed, and additional features. Special effort was made to reduce errors caused by component drift, thermoelectric EMF, component offset, electrical noise, and nonlinearity. The new resistance thermometer readout was tested to identify and evaluate sources of measurement uncertainty. The combined uncertainty was calculated for resistance ratio and  $W(T_{90})$  measurements of an SPRT with self-heating corrections. Measurements made with the resistance thermometer readout were compared with measurements made with a resistance bridge. The results show that the standard uncertainty of the new resistance thermometer readout is about  $0.34 \cdot 10^{-6}$  in measuring resistance ratio at  $25\Omega/100\Omega$  and about  $0.68 \cdot 10^{-6}$  in measuring  $W(T_{90})$  near the triple point of water.

### 1. INTRODUCTION

In the past, resistance bridges have been used with standard platinum resistance thermometers (SPRTs) whenever temperature measurements with uncertainties below 5 mK were required. High-quality resistance bridges can often achieve temperature uncertainties less than 0.1 mK. However, their high cost, large size, slow speed, and limited flexibility discourage their use in many applications. Other types of resistance measurement devices, such as digital multimeters, are available for much lower cost and have other advantages over resistance bridges but are unable to achieve uncertainties less than about 5 mK. There was a need for a resistance readout with uncertainties approaching those of bridges but available for much lower cost. In an attempt to meet this challenge, the author set out in 1992 to develop a new resistance readout that overcame the accuracy limitations of digital multimeters but avoided the use of

precision ratio transformers that made bridges so large and expensive. By taking advantage of recent advances in analog-to-digital converters (ADCs) and carefully considering sources of error in typical digital multimeters, a readout was produced that achieved temperature uncertainty as low as 1 mK and incorporated integrated circuits rather than ratio transformers. Besides lower cost, additional benefits of this design were smaller size, lighter weight, the capability of faster measurements, and wider measurement range. Continued research led to refinements in the design that made possible even better accuracy. This latest version of the resistance thermometer readout is marketed by Hart Scientific as Model 1590.

### 2. DESIGN OF THE RESISTANCE THERMOMETER READOUT

The author's design for the resistance readout combines elements of bridge designs with elements of digital multimeter designs. Fundamentally, the resistance thermometer readout measures the ratio of resistance between a resistor (an SPRT) and a reference resistor. The primary components of the measurement system are the current source, switch, amplifier, analog-to-digital converter (ADC), and controller (see Figure 1). As with AC resistance bridges, the current source drives current through both the SPRT and the reference resistor, which are connected in series. As with many digital multimeters, the readout uses an analog-to-digital converter to measure the electromotive force (EMF) across the SPRT. The amplifier increases and conditions the signal passed to the ADC to improve the signal-to-noise ratio. The switch allows the EMF across the reference resistor to also be measured. Dividing the voltage measured from the SPRT by the voltage from the reference resistor gives the ratio of the two resistances. Assuming the current remains constant and is the same through both the SPRT and the reference resistor and the amplifier and ADC gains remain constant, any uncertainty in the value of the current or gains will not affect the resistance ratio measurement.

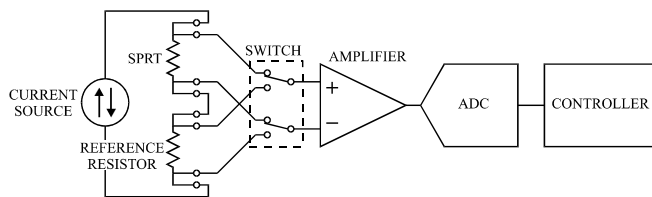


FIGURE 1:  
Simplified schematic diagram of the resistance thermometer readout

To cancel errors from thermoelectric EMF and offsets in the amplifier and ADC, the resistance ratio measurement is repeated with the current in the opposite direction and the two measurements are, in essence, averaged to make a complete resistance ratio measurement. As long as thermoelectric EMF and offsets are constant, they also will not affect the resistance ratio measurement.

In summary, it takes four EMF samples from the ADC to make a complete resistance ratio measurement: (1) a sample of the SPRT EMF with forward current, (2) a sample of the SPRT EMF with reverse current, (3) a sample of the reference resistor EMF with forward current, and (4) a sample of the reference resistor EMF with reverse current. With an ADC that can sample in less than 0.5 seconds, a complete resistance ratio measurement can be obtained in about two seconds.

Variations in the current, gains, thermoelectric EMF, and offsets will produce errors in measurements that appear as random variations between readings. Careful design of the electrical circuits can reduce noise from these sources to a negligible amount. Error in the measurements is primarily due to electrical noise and imperfect linearity of the ADC and other components. Careful design and selection of components reduces these errors to a small amount. Further improvement in linearity is gained by having the controller apply a slight cubic correction function to the ADC readings.

### 3. PERFORMANCE OF THE RESISTANCE THERMOMETER READOUT

The resistance thermometer readout using this design offers several advantages. First, the components required are fairly inexpensive (compared to the precise ratio transformers required in resistance bridges). Second, the components are smaller and lighter allowing the readout to be more portable. Third, this resistance thermometer readout can make measurements in much shorter time than is possible with resistance bridges. No nulling is required. An independent measurement can be obtained in about two seconds. This makes the device well suited for applications where different temperatures, using different sensors, must be measured in a short period of time. Fourth, the resistance thermometer readout is not limited to a small range of resistance ratios as are bridges. It is capable of measuring resistance ratios from 0 to 50, or higher, with resistances of 0 to 500 kΩ or higher.

The primary goal of the project, however, was to develop a resistance thermometer readout using integrated circuits that can achieve uncertainties approaching those of resistance bridges. The readout's measurement uncertainty must be evaluated. Uncertainty can be considered to arise from two types of error: systematic error—error that remains constant while conditions remain constant—and random error—error that varies unpredictably with each measurement. In this resistance thermometer readout, systematic error is primarily caused by linearity errors. Random error is largely due to electrical noise in the components. These two types of error and their contributions to the uncertainty of an average of measurements will be examined.

#### 3.1. LINEARITY

Of the sources of systematic error, linearity error of the analog-to-digital converter is expected to be most significant. This error is dependent on the EMF at the input of the ADC and thus is a function of the resistances being measured, excitation current, and amplifier gain.

The resistance thermometer readout (after careful calibration to adjust the coefficient of the ADC's linearity correction) was tested for linearity using the Aeonz Model RBC400 Resistance Bridge Calibrator [1]. The calibrator incorporates a set of four resistors that are switched in various series or parallel combinations to produce resistances ranging from approximately 30Ω to 400Ω. The uncertainty of the calibrator for testing linearity is less than 0.1 ppm. The ratio of resistance between the calibrator and a 100Ω reference resistor was measured with the resistance thermometer readout for each of 16 resistor combinations. For each resistance, four minutes of measurements were recorded and averaged to reduce the uncertainty due to measurement noise. The current used was 1 mA. A least-squares method was used to solve for the four basic resistances and the measurement error at each ratio. The absolute errors were divided by the measured ratios to give relative errors. The test was repeated four times and the average of the linearity errors at each resistance ratio computed. The average relative errors of the resistance thermometer readout at various measured ratios are shown in Figure 2. The root-mean-square of the linearity errors is 0.24 ppm. The standard uncertainty of the test (due to calibrator uncertainty and measurement noise) is estimated to be about 0.12 ppm.

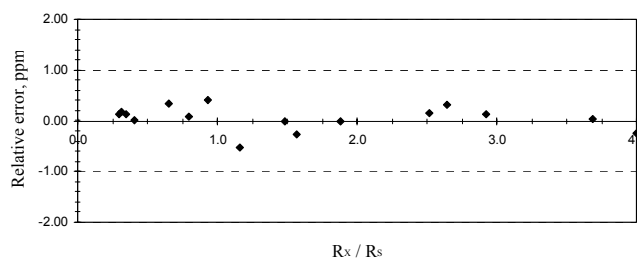


FIGURE 2:  
Linearity error of the resistance thermometer readout

### 3.2. NOISE

Electrical noise in the resistance thermometer readout causes random error in the readings. The most significant source of noise is expected to be the ADC. If so, the standard deviation of the noise error will depend on the resistances being measured, current, and gain of the amplifier. Assuming the noise is independent between measurements and is constant (referred to the input of the ADC) the standard deviation of the average of  $n$  measurements of the ratio of SPRT resistance  $R_X$  to the reference resistance  $R_S$  can be approximated by the following equation:

$$\sigma(\text{noise}) \approx q \sqrt{\frac{1}{2n} \left[ \left( \frac{R_X}{R_S} \right)^2 + \left( \frac{R_S}{R_X} \right)^2 \right]} \quad (1)$$

where  $q$  is a noise factor (that depends on the current) and is estimated by measurement to be 0.95 ppm with 1 mA current. Predicted standard deviation of noise from this model is compared with the actual standard deviation (calculated from at least 50 samples) of the average of  $n$  measurements in Table 1 for several resistance ratios and values of  $n$ . (The values are in ppm of the ratio. The current is 1 mA and the reference resistance is approximately 100Ω.)

$R_X / R_S$	$n$	$\sigma$ , predicted	$\sigma$ , measured
0.25	1	2.7	2.7
0.25	5	1.2	1.2
1.00	1	0.95	0.93
1.00	90	0.10	0.10
4.00	1	2.7	2.6
4.00	5	1.2	1.1

TABLE1:

Comparison of predicted and measured noise of the resistance thermometer readout

### 3.3. UNCERTAINTY ANALYSIS

The uncertainty of the average of a quantity of resistance ratio measurements is a combination of linearity uncertainty and noise uncertainty:

$$u\left(\frac{R_X}{R_S}\right) = \frac{R_X}{R_S} \left[ u^2(\text{linearity}) + u^2(\text{noise}) \right]^{1/2} \quad (2)$$

As shown previously, the noise uncertainty depends on the resistance ratio and  $n$ , the number of readings averaged. Table 2 shows the estimated standard uncertainties of resistance ratios that would be measured at various ITS-90 fixed-point temperatures for two values of  $n$ . It is assumed the SPRT has a resistance of 25.5Ω at 273.16 K, the excitation current is 1 mA, and the reference resistor is approximately 100Ω.

Self-heating effects in the SPRT are often corrected by measuring the resistance at two levels of excitation current. Specifically, for a 25.5Ω SPRT, the resistance is measured with 1 mA ( $r_1$ ), then measured with 1.414 mA ( $r_2$ ), and then measured again with 1 mA ( $r_3$ ). The resistance at zero power is calculated from these measurements:

$$r(0 \text{ power}) = r_1(1 \text{ mA}) - r_2(1.414 \text{ mA}) + r_3(1 \text{ mA}) \quad (3)$$

$T_{90}$	$R_X / R_S$	$u$ , $n=1$	$u$ , $n=120$
Ar	0.055	$2.6 \cdot 10^{-6}$	$0.25 \cdot 10^{-6}$
Hg	0.215	$2.6 \cdot 10^{-6}$	$0.31 \cdot 10^{-6}$
TPW	0.255	$2.7 \cdot 10^{-6}$	$0.34 \cdot 10^{-6}$
Ga	0.285	$2.7 \cdot 10^{-6}$	$0.36 \cdot 10^{-6}$
In	0.41	$2.7 \cdot 10^{-6}$	$0.46 \cdot 10^{-6}$
Sn	0.48	$2.7 \cdot 10^{-6}$	$0.52 \cdot 10^{-6}$
Zn	0.66	$2.9 \cdot 10^{-6}$	$0.67 \cdot 10^{-6}$
Al	0.86	$3.4 \cdot 10^{-6}$	$0.86 \cdot 10^{-6}$

TABLE 2:

Estimated standard uncertainty of  $R_X / R_S$

Each of the resistance measurements has uncertainty but some correlation exists between the uncertainties. The linearity error, with variance  $u^2(\text{linearity})$ , is considered the same for the three measurements since the ratios do not differ appreciably. The noise uncertainty, with variance  $u^2(\text{noise})$ , is independent in each measurement. Thus, the combined uncertainty for the zero-power resistance can be evaluated using the expression

$$u^2(0 \text{ power}) = u^2(\text{linearity}) + 3u^2(\text{noise}) \quad (4)$$

The most accurate temperature measurements are made in comparison with the temperature of a triple point of water (TPW) cell (273.16 K). The ITS-90  $W(T_{90})$  ratio for an SPRT is calculated as the ratio of its resistance at the temperature of interest to the resistance at 273.16 K. The combined uncertainty of  $W(T_{90})$ , using self-heating correction, can be evaluated by the expression

$$u(W(T_{90})) = W(T_{90}) \left[ 2u^2(\text{linearity}) + 3u^2(\text{noise}, T_{90}) + 3u^2(\text{noise}, 273.16 \text{ K}) \right]^{1/2} \quad (5)$$

The calculated standard uncertainties of  $W(T_{90})$  at several fixed-point temperatures are shown in Table 3. The noise uncertainties are based on averages of 120 readings (taken over four minutes). The values of standard uncertainty in temperature, based on  $dW/dT$  for an SPRT, are also shown in the table. Again, a 25.5Ω SPRT, a 100Ω reference resistor, and 1 mA current are assumed.

$T_{90}$	$R_X / R_S$	$u(\text{noise})$	$u(\text{lin.-ity})$	$u(W(T_{90}))$	$u(T_{90})$
Ar	0.055	1.11	0.24	$0.43 \cdot 10^{-6}$	0.10 mK
Hg	0.215	0.29	0.24	$0.62 \cdot 10^{-6}$	0.15 mK
TPW	0.255	0.24	0.24	$0.68 \cdot 10^{-6}$	0.17 mK
Ga	0.285	0.22	0.24	$0.73 \cdot 10^{-6}$	0.19 mK
In	0.41	0.15	0.24	$0.96 \cdot 10^{-6}$	0.25 mK
Sn	0.48	0.13	0.24	$1.11 \cdot 10^{-6}$	0.30 mK
Zn	0.66	0.10	0.24	$1.45 \cdot 10^{-6}$	0.42 mK
Al	0.86	0.09	0.24	$1.89 \cdot 10^{-6}$	0.59 mK

TABLE 3:  
Calculation of standard uncertainty of  $W(T_{90})$  and  $T_{90}$  for the resistance thermometer readout

### 3.4. MEASURED ERROR

Measurements of  $W(T_{90})$  were made with the resistance thermometer readout and the results compared with measurements made with a Measurements International Model 6010B direct current comparator resistance bridge. The differences between  $W(T_{90})$  (with self-heating corrections) from the resistance thermometer readout measurements and  $W(T_{90})$  from the bridge at several fixed-point temperatures are shown in Table 4. The zero-power  $W(T_{90})$  were calculated from the averages of 120 measurements taken over four minutes at 1 mA, 1.414 mA, and again at 1 mA as explained previously. The differences between the measurements in terms of temperature based on  $dW/dT$  for an SPRT are also shown in the table. The resistance bridge  $W(T_{90})$  has an estimated combined standard uncertainty, in temperature, of 0.06 mK or less.

$T_{90}$	$W(T_{90})$ , readout	$W(T_{90})$ , bridge	$\Delta W(T_{90})$	$\Delta T_{90}$
Hg	0.84415741	0.84415646	$0.95 \cdot 10^{-6}$	0.22 mK
Sn	1.89260364	1.89260384	$-0.20 \cdot 10^{-6}$	-0.05 mK
Zn	2.56881289	2.56881190	$0.99 \cdot 10^{-6}$	0.28 mK

TABLE4:  
Comparison of  $W(T_{90})$  measurements of the resistance thermometer readout and bridge

### 4. CONCLUSIONS

The new resistance thermometer readout exhibits unprecedented accuracy for a readout designed with integrated circuits rather than ratio transformers. The standard uncertainty in measuring resistance ratio is about  $0.34 \cdot 10^{-6}$  at  $25\Omega/100\Omega$ . The standard uncertainty in measuring  $W(T_{90})$  is about  $0.68 \cdot 10^{-6}$  near the triple point of water and  $1.5 \cdot 10^{-6}$  at the zinc point. The resistance thermometer readout also has advantages of lower cost, smaller size, the capability of increased speed, and wider measurement range.

### ACKNOWLEDGEMENTS

Tom Wiandt at Hart Scientific was most helpful in providing equipment and assisting in measurements. Much appreciation goes to Xumo Li at Hart Scientific for his inspiration and expertise. Many others at Hart Scientific took part in the development of the resistance thermometer readout and the preparation of this paper.

### REFERENCES

- White D. R., *A method for calibrating resistance thermometry bridges*, Proceedings of TEMPMEKO '96, 1997, 129-134

---

**Author:** Rick Walker, Engineering Manager/Senior Design Engineer, Hart Scientific, 799 E. Utah Valley Dr., American Fork, UT 84003, USA, Tel: 801-763-1600, E-mail: rick\_walker@hartscientific.com, Web: www.hartscientific.com

**Presenter:** Norman Wilgress, Temperature Calibration Specialist, Hart Scientific, 52 Hurricane Way, Norwich, Norfolk, NR6 6JB, United Kingdom. E-mail: [norman.wilgress@hartscientific.com](mailto:norman.wilgress@hartscientific.com). Web: [www.hartscientific.com](http://www.hartscientific.com)